The Astrophysical Journal

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS

Agents
THE CAMBRIDGE UNIVERSITY PRESS
LONDON AND EDINBURGH

THE MARUZEN-KABUSHIKI-KAISHA
TOKYO, OSAKA, KYOTO

KARL W. HIERSEMANN
LEIPZIG

THE BAKER & TAYLOR COMPANY
NEW YORK

THE

ASTROPHYSICAL JOURNAL

An International Review of Spectroscopy and Astronomical Physics

VOLUME XXXIX

JANUARY–JUNE, 1914

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS

Published
January, March, April, May, June, 1914

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VOLUME XXXIX | JANUARY 1914 | NUMBER 1

VICTOR SCHUMANN

By THEODORE LYMAN

Victor Schumann was born near Leipzig in the year 1841. He received his early education at Leipzig and later, from 1860 to 1864, he was at the Gewerbeschule at Chemnitz. It must have been during this period that he acquired that extraordinary mechanical technique which characterized all his scientific work. For a time he was employed as a designer by Hartmann & Zimmermann; later he was engaged in the manufacture of machinery for the book industry; finally, he became a partner with Mr. A. Hogenforst in the machine business in which he remained actively engaged until 1892 and by means of which he was able to accumulate the funds which he spent in scientific work.

He was more than forty years of age before he was able to turn from his business to scientific pursuits. Even then, his investigations were conducted in the evening or at such odd times as he could spare from his regular profession. Photography first attracted him; one of his earliest papers, published in 1885, is on the sensitization of photographic plates. Almost immediately after this, however, he took up the pursuit of spectrum analysis, to which he devoted himself for the remainder of his active life. His first paper on the spectrum of hydrogen and upon the effect of the presence of impurities on the spectrum of mercury appeared in

1886. He must have been inspired by the idea of penetrating into the region of the extreme ultra-violet very early in his scientific studies, for it was but four years later that the article which marks the beginning of his attack on the region of the most refrangible rays appeared. Guided by the work of Stokes, Soret, and Miller, he began by instituting a very careful comparison of the relative advantages of fluorite and quartz, and, becoming convinced of the superiority of fluorite as a transparent medium for rays of the shortest wave-length, he employed this substance for his prisms and lenses. Thus equipped, he followed the spark spectra of more than twenty metals to the region λ 1820. He next set himself the problem of determining the factors which caused the common limit in the spectrum of so many substances. His knowledge of photographic phenomena led him to recognize the part played by the absorption of gelatine, while his familiarity with the work of Cornu drew his attention to the absorption of the air. He put these ideas to the test by the construction of a vacuum spectroscope and by the use of special photographic plates whose emulsion was nearly free from gelatine. His efforts were almost immediately crowned with success, for a very considerable extension of the spectrum followed the use of the new apparatus. Brief accounts of this work appeared between 1890 and 1893, while a more detailed description of these researches was published in the *Proceedings* of the Vienna Academy in the latter year. It was during this period that Schumann gave up his business interests to devote himself entirely to his spectroscope. During the next ten years he went steadily forward, accurately and surely adding to his methods one improvement after another as the results of experiments showed the way, until he finally arrived at the limit of the spectrum set by the absorption of fluorite near λ 1200.

But as early as 1897 his health began to give way. Never of a robust constitution, he had submitted to considerable privations in early life in order to obtain the funds for the purchase of books. He undoubtedly still further undermined his constitution by the arduous labors entailed in the construction of his apparatus. In 1905 he was forced to give up nearly all experimental work. He died on September 1, 1913.

Many of Schumann's results are to be found summed up in the *Smithsonian Contributions to Knowledge*, No. 1413. His first considerable contribution to science was the investigation of the absorption of the air. The existence of the region which bears his name having been once established, he set himself to study the absorption of a number of gases and demonstrated that it was to oxygen that the high absorbing power of the air was due. He next proved that hydrogen possessed great transparency in the most refrangible region and made use of this fact to improve the action of his spectroscope by flushing the interior with this gas. On turning his attention to emission spectra, he obtained valuable information on the radiations from oxygen, carbon monoxide, carbon dioxide, and nitrogen; but it was in the study of the spectrum of hydrogen that his finest results were obtained. He showed that this gas possessed a multitude of lines extending from near λ 1650 to the limit set by fluorite, and by great technical skill and keen experimental insight he succeeded in producing spectrograms of hydrogen which probably will never be surpassed for definition and finish.

Labor spent in the extension of human knowledge is never wasted, no matter how remote from active human interest the field of such labor may appear. The work of Schumann is a brilliant example of the truth of this statement. For the region into which he penetrated reveals day by day to those who explore it greater and greater possibilities for results of fundamental scientific importance. The biologist may watch wonderful changes in living organisms if he will illuminate the field of his microscope with the extreme ultra-violet rays from a hydrogen tube; the student of spectral series may find the key to his fascinating problem on the more refrangible side of λ 1500, and the mathematical physicist who seeks to verify the quantum theory by photo-electric action will find an important test for his hypothesis in the Schumann region.

It has been said that genius consists in an illimitable power of taking pains. Schumann's genius belonged to this character, but the observer who, having marveled at the intricate construction of his spectroscope, obtains the impression that its mechanical perfection represents the highest mental attainment of its maker

entirely misses the truth. Schumann took up the pursuit of science at a time of life when initiative and perseverance in most men are no longer active qualities. The extent of his studies was cut short by the failure of his health. It was never given to him to explore thoroughly the promised land which he discovered. But Schumann possessed the mind of the true investigator; his inductive reasoning was without a flaw. What he did, he did excellently. The final results of his labors are established so firmly that they will never be shaken.

Jefferson Laboratory
Harvard University
December 4, 1913

TERTIARY STANDARDS WITH THE PLANE GRATING
THE TESTING AND SELECTION OF STANDARDS[1]

SECOND PAPER

By CHARLES E. ST. JOHN and L. W. WARE

APPARATUS AND METHOD

In this second paper we have examined the international secondary standards from λ 4282 to λ 5506 as to their consistency among themselves, and have determined the wave-lengths in international units of a series of 198 lines in the arc spectrum of iron from λ 4118 to λ 5506. The region from λ 5371 to λ 5506 is common to the 1912 and 1913 investigations, but an entirely new series of plates was made for the common region. The Pasadena plates were taken by Mr. Babcock and one of the writers. The spectrograph used was the 30-foot instrument in the Pasadena laboratory, which had been completely readjusted by Mr. Babcock after an examination and testing of the conditions requisite to the best performance. The grating was ruled by Anderson and is of the highest quality—a plane grating having a ruled surface 64×73 mm with 590 lines to the millimeter. The performance of the instrument is most satisfactory. The diffraction pattern in the case of the sharp lines is perfectly symmetrical on the two sides of the line with the first minimum absolutely black, by both visual and photographic tests. Each portion of the ruled surface is consistent with any other portion, so that errors arising from varying illumination are reduced to a minimum. The series of Mount Wilson plates was made with the 75-foot Littrow spectrograph used in conjunction with the 150-foot tower telescope. The grating (by Michelson) has an available ruled surface 57×124 mm and gives excellent definition for both bright and dark lines. In both series the Pfund arc[2] was used upon a 110-volt direct-current circuit. The length of the arc was 5–6 mm. An enlarged image was thrown upon the slit which was placed transverse to the axis

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 75.

[2] *Astrophysical Journal*, **27**, 296, 1908.

of the arc near its middle point. The focal lengths of the projecting lenses were such that the gratings were always filled. The grating in the 75-foot spectrograph was in the axis of a light cone 1.5 m in diameter at the grating level. The current was maintained at approximately 6 amperes. The scale of the Pasadena plates, second order, is 1 mm=0.88 A; of the Mount Wilson plates, first order, 1 mm=0.72 A. They were measured either upon a Toepfer comparator with a screw 300 mm long or upon a comparator constructed in the Observatory instrument shop with which a plate 500 mm long can be measured without readjustment. The periodic errors of the screws are negligible. The errors of run have been determined and the necessary corrections have been applied to the scale readings. The plates have been measured red right and red left. At least four settings have been made on every line in the two positions, and in the case of the international secondaries, the settings have frequently been doubled, particularly when these standards have offered more than usual difficulties in the determination of their position. The comparator microscopes are equipped with single and double cross-wires. That used depends upon the character of the line under consideration; frequently settings have been made by both methods and the means taken of the closely agreeing readings. A gain in convenience of measurement and in accuracy was made by shading the strong lines on the laboratory plates during a part of the exposure so that all the lines were of more nearly uniform intensity. This proved particularly helpful with lines that have a tendency to widen unsymmetrically, and in the case of very strong lines the edges of which become more or less fringed when the exposure time is such as to bring out the weaker lines.

The method of reduction was fully described in our first paper.[1] In brief, the reduction factors obtained by dividing the intervals in angstroms between each two successive international standards by the measured distance between the lines were plotted as ordinates with the mean wave-lengths of the corresponding intervals as abscissae, and as smooth a curve as possible was passed through

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 61; *Astrophysical Journal*, 36, 19, 1912.

the points. The smoothness of the curves has been used as a test of the consistency of the international secondaries. In the case of standards 50 angstroms apart a change of 0.002 A in the wave-length of one of them produces a marked irregularity in the curve. The accuracy can be pushed to a high degree by repeated measurements of the plates. We have combined the results from different plates to separate what was accidental from what was constant in the course of the curves. In the series of overlapping plates, a line has in general occurred at three different positions so that the standards have been combined in various ways and are interlocked throughout the region investigated. The plates are 43 and 60 cm long, so that with the scales employed a number of standards occur on each plate, amply sufficient to determine the course of the curve, which in each case is nearly linear. Each line has been referred to both the standards between which it stands, but the course of the curve at the points from which the factors are taken for the reduction depends upon four standards so that the wave-length is intimately interlocked with the standards.

THE SELECTION OF STANDARD LINES

In the discussion of our results it will appear that they are closely related to the classification of the arc lines of iron proposed by Gale and Adams[1] and based upon the behavior of the lines under pressure.

To the five classes determined by the characteristics of the lines we have added a sixth:

1. Lines which are symmetrically reversed.

2. Lines which are unsymmetrically reversed.

3. Lines which remain bright and fairly narrow under pressure.

4. Lines which remain bright and symmetrical but become wide and diffuse under pressure..

5. Lines which remain bright and are widened very unsymmetrically toward the red.

6. Lines which remain bright and are widened very unsymmetrically toward the violet.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 58; *Astrophysical Journal*, **35**, 10, 1912.

Gale and Adams suggested a grouping of these lines according to pressure-shift, as follows:

Group a.—This includes the flame lines; these show small pressure displacements.

Group b.—This is a large group including all lines of moderate displacement and is probably complex.

Group c.—This consists of lines showing much larger displacements than those of group *b* and includes two fairly distinct classes.

Group d.—This is made up of lines showing immense displacements and unsymmetrical widening toward the red under pressure. We were led to divide group *d*, calling the separated portion *sub-d*. These lines are similar in behavior but show much smaller displacements and much less dissymmetry than those assigned to *d*.

Group e.—To the above groups we have added a group consisting of lines that are greatly displaced and unsymmetrically widened toward the violet under pressure.

The pressure-shift of the iron lines is still under examination by Gale and Adams. We have had access to the preliminary results of this investigation and so have been able to assign more lines to their probable groups.

The lines that remain symmetrical and reverse symmetrically under pressure are best fitted for standards; such are the lines of groups *a* and *b* which belong to classes 1, 3, and 4. The international standards of the second order from λ 4282 to λ 4531, with the exception of λ 4494, belong to these two groups and are lines that can be measured with high precision. λ 4494 and probably the secondaries λ 4547 to λ 4789 belong to group *c*, class 4, but from λ 4850 to λ 5001 they belong to group *c*, class 5. The lines of group *c*, class 4, are next in quality to those of groups *a* and *b*. Those of group *c*, class 5, are troublesome as standards. It is difficult to obtain consistent results with them, as they have a tendency to asymmetry toward the red. From λ 5012 to λ 5167 the lines belong to group *a* and are of excellent quality but from λ 5192 to λ 5324 they are of class 5 and belong to group *d* or *sub-d*, probably the latter. These are lines that are still more troublesome than those of group *c*, class 5. The selection of these lines for secondary standards was particularly unfortunate, as lines of group *a* of

excellent quality that could replace them occur in this region, as follows:

Present secondaries: group *sub-d*, λ 5192, λ 5232, λ 5266, λ 5302, λ 5324
Suggested secondaries: group *a*, λ 5194, λ 5216, λ 5227, λ 5270 λ 5328, λ 5341

The only space that cannot be filled by *a* lines is that occupied by λ 5302, but this line is so weak that it has been very difficult to obtain it strong enough for measurement without overexposing the other lines. In fact, we have not used it as a standard but have determined its wave-length as for an unknown line. The gain in having dependable standards of the second order would far outweigh any lack in desirable spacing. From λ 5371 to λ 5506 the international standards belong to group *a* and are again of excellent quality. From λ 5569 to λ 5763 they are of class 5 and belong to group *sub-d*. These lines cannot be depended upon when the highest precision is required and should always be employed with caution. From λ 6027 to λ 6494 the secondaries are all of group *b*, and with these as with those of group *a*, a precision of 0.001 A can be obtained. Fortunately no lines of groups *d* or *e* occur among the secondaries. These, in our judgment, based upon considerable experience, are entirely unsuitable for standards of any order, and happily they can be omitted without serious inconvenience.

It is of vital importance for standard wave-length determinations to fix the cause of large discrepancies.[1] From his remarks at the meeting of the British Association at Dundee,[2] one must judge that Professor Kayser is somewhat discouraged with the outcome of the work of wave-length determinations. He had hoped a few years ago, he said, to establish the third decimal place. He now finds uncertainties in the second decimal, depending upon the part of the arc used and other obscure causes. Goos[3] raises the question of producing a light-source suitable for obtaining consistent results. He used the interferometer method for checking

[1] The effects of varying conditions on the arc and of using its different parts are now being investigated in the observatory laboratory.

[2] *The Observatory*, **35**, 383, 1912.

[3] *Astrophysical Journal*, **37**, 48, 1913.

some results obtained with the plane grating. He found that with a short arc, 3 to 4 mm long, certain lines gave very poor or no interference phenomena, but with the middle of a 10 mm arc, using the same current it was possible to obtain fair interference rings for these lines. He went to Kayser's laboratory in Bonn, where he again tried the short and long arc with the concave grating and found for certain lines a marked difference between the two light-sources. Now it is interesting to notice that all these lines for which he found differences in the two sources were shorter in the denser arc, and, so far as they are common to our lists, belong to group *e*. The characteristics of the lines of this group are displacements to the violet under pressure and great unsymmetrical widening on the violet edge when the pressure or density of the emitting vapor is increased. In the short arc, carrying the same current as the long arc, the density and possibly the pressure also in the central portion would be the greater and the wave-length would be shortened from one or both causes.

In writing the first paper based upon the Mount Wilson investigations, in a paragraph omitted because it was in a portion reserved for a later contribution but given in substance in a paper read at the Cleveland meeting of the American Physical Society,[1] it was said:

> An increase of intensity and consequent unsymmetrical widening may be an insidious cause of error in measuring such lines. This was clearly shown when the results of our measurements were finally assembled. Upon Plate 530 taken on the mountain there are five exposures of equal length. This plate was measured once by each of us and again unintentionally by one of us. It was noticed when the measurements were finally assembled that the wave-lengths of λ 6003 and λ 6008 were longer for exposure 3 than for any of the other exposures. Upon the plate, besides the lines belonging to group *d*, displaced and widened to the red under pressure, were lines of group *e*, displaced and widened to the violet under pressure, and lines belonging to the same group as the standards, namely group *b*.
>
> Table I shows the differences obtained by subtracting the mean of the values given by the four other exposures from the corresponding values obtained from the third exposure.
>
> The systematic differences are striking, and at first were very misleading.

[1] "Standards of Wave Length and Desirable Data for Them and for Other Lines," *Physical Review*, 1, 67, 1913.

At the time the plate was measured we did not know of the existence of a group with the characteristics of group *e*. It was only after a long wandering in the dark that we convinced ourselves that the differences in the measurement in lines belonging to groups *d* and *e* were inherent in the characteristics of the lines, and that the differences between their wave-lengths on Mount Wilson and in Pasadena were due to the enormous pressure-shifts that these lines undergo. There was nothing in the appearance of Plate 530 to put us on our guard, and this is particularly true when a solar comparison is on each side of the narrow arc spectrum on this plate. Exposure 3, however, is stronger than the others owing to the increased production of vapor in the arc during that exposure, and it may be a question whether the displacements were the effects of unsymmetrical broadening due to increased density or of a temporary increase in pressure due to a rapid evolution of vapor. In the case of lines so sensitive to pressure changes as those of groups *d* and *e*, it may be a question whether rapid changes in the rate of vapor production in the arc may not cause temporary changes in pressure sufficient to displace these lines appreciably. A mean change of pressure of one twenty-fifth of an atmosphere during the short exposure could produce a measurable effect.

TABLE I

EFFECT OF VARYING DENSITY IN THE ARC UPON LINES OF GROUPS *d* AND *e*

Group *b*		Group *d*		Group *e*	
λ 6020	+0.001 Å	λ 5983	+0.006 Å	λ 5984	−0.000 Å
λ 6127	− .001	λ 6003	+ .016	λ 5987	− .003
λ 6136	− .001	λ 6008	+ .017	λ 6042	− .006
λ 6151	.000	λ 6021	+0.011	λ 6055	− .011
λ 6157	−0.002			λ 6078	−0.007
Mean...	−0.0006 Å		+0.012 Å		−0.007 Å

As has been mentioned, the effect of varying arc conditions is now being investigated in the Pasadena laboratory. It can be stated from the preliminary examination that, if a difference of pressure exists in the arc, it is not of a magnitude sufficient to account for the large discrepancies in the case of lines of groups *d* and *e*. The main cause of these differences is found, we believe, in the unsymmetrical widening of these lines as the negative pole is approached. Pressure-shift *per se* is not troublesome when once it has been determined, but the great sensitiveness of these lines to density changes is, in our opinion, the seat of the difficulty. The integrating action of the concave grating due to its astigmatism

in the usual mounting spreads the polar dissymmetry over the whole length of the line and would affect the measurement of such lines to a high degree. We would recommend an arc 5–6 mm long, carrying a current of 5–7 amperes, and preferably the Pfund form because of its great convenience and steadiness of action; that an image enlarged at least two to three times be used, and that the slit be placed at right angles to the axis of the arc at the middle point of the image. The lines with a short slit then have good edges and uniform widths throughout their length and good lines are capable of very exact determination. Under these conditions by keeping the lines of moderate intensity, quite consistent results can be obtained even in the case of lines of group *c*, class 5, and of group *sub-d;* that is, one can expect a precision of 0.002 to 0.003 A. In the case of lines of groups *a*, *b*, and *c* 4, the ordinary conditions of the arc are in great probability without measurable influence.

RESULTS

The general results of our examination of the secondary standards and our measurements of the tertiary standards from λ 4118 to λ 5371 are shown in Table II.

The lines, as far as possible, have been grouped and classed according to the scheme proposed by Gale and Adams, with the subdivision of group *d* and the addition of group *e*. In fo ming the means for the eighth column the mountain value of lines of groups *d*, *sub-d*, and *c* 5, when referred to the standards of groups *a* and *b*, have been given a positive correction of from 0.001 to 0.005 A, and the mountain values for the lines of group *a*, referred to standards of groups *c* and *sub-d*, have been decreased by 0.003 A in the formation of the means. To allow for the pressure difference of one-fifth of an atmosphere between Pasadena and Mount Wilson the value of the correction is taken from the published and unpublished determinations of Gale and Adams and from the correction which we were obliged to apply in order to make the mountain standards consistent with the Pasadena values when standards of different groups were used together. In the case of lines of group *e*, only the Pasadena values are used in comparison with Kayser and Goos as indicated by "P" in the last column. In this column

we have also given some indications of the degree of dependence that can be placed upon the results obtained from the use of the lines as standards. In regions where secondary standards of groups *a*, *b*, and *c* 4 are available, we would place little reliance upon the lines indicated as unreliable and would not employ them at all if a high accuracy were desired. If only a fair precision is required they will be useful, if care is taken to have these lines of moderate intensity. This is best done by using only the middle of the arc. The difficulties inherent in the lines of groups *d* and *sub-d* increase with the wave-length; those of group *d* are practically unusable below λ 5000. For the region of shorter wave-length, no clear distinction can, as yet, be made between groups *d* and *sub-d*. All lines of group *e* are, according to our experience, unusable as standards. When a line is so weak upon our plates that the determination is not of high weight, we have indicated the fact by the word "faint." The figures in parentheses give the number of times the lines have been measured.

DISCUSSION

In Table III are assembled the wave-lengths of all the standards of the second order for which a deviation from the international value exceeding 0.002 A has been found by any observer. The second column contains the maximum deviations from the accepted value in the case of the th ee independent determinations of the standards by the interferometer method. In the three succeeding columns are the deviations found by the three observers using the spectrographic method. Our results seem to indicate a remarkable degree of consistency among the international standards, and we are confident that the method we have employed when pushed to its limiting accuracy is capable of detecting deviations of 0.002 A in the relative wave-lengths of a series of standards. The average deviation for the interferometer method is 0.0015 A, and that shown by Kayser's results is 0.0045 A. It seems to us so improbable that errors of this latter magnitude can exist in the secondary standards and escape detection by the method we have employed that we are forced to the conclusion that some obscure source of error affects the determinations published by Professor Kayser. The deviations from the accepted international values exceeding

TABLE II

Group	Class	International Wave-Length: Pasadena	International Wave-Length: Mt Wilson	Mean Deviation: P	Mean Deviation: Mt	P − Mt	Mean Wave-Length	M − L	M − K	M − G	Remarks
		(4118.553)							(000)		
		4121.809 (11)		2			4121.809 (11)		+0.002		
		4127.615 (11)		1			4127.615 (11)		000		
b	1	4132.062 (11)		2			4132.062 (11)		− .002		
b	4	4134.685							+ 001		From λ4134 to λ4531, except λ4404, the lines used as secondary standards belong to groups *a* and *b* and are first-class in quality
		4137.002 (11)		2			4137.002 (11)		− 001		
		4143.422 (11)		1			4143.422 (11)		+ .001		
b	1	4143.872 (11)		2			4143.872 (11)		− .001		
		4147.676							+ .002		
		4153.914 (4)	.917 (7)	1	3	−0.003	4153.916 (11)				
		4154.505 (11)	504 (9)	1	1	+ .001	4154.505 (20)		− .001		
		4154.815 (4)	816 (8)	1	2	− .001	4154.816 (12)				
		4156.805 (11)	.805 (10)	1	1	.000	4156.805 (21)		− 001		
		4157.804 (7)	.806 (9)	5	4	− .002	4157.805 (16)				
		4170.906 (17)	907 (7)	3	1	− .001	4170.906 (24)		.000		
		4172.126 (4)	.126 (5)	1	2	.000	4172.126 (9)				
		4174.918 (1)	.917 (5)	0	2	+ .001	4174.917 (6)				
		4175.639 (17)	.640 (10)	3	2	− .001	4175.639 (27)		− 003		
		4176.563 (5)	.566 (7)	4	4	− .003	4176.565 (12)				
		4177.595 (5)	598 (6)	2	2	− 003	4177.597 (11)				
		4181.758 (17)	.757 (9)	3	2	+ .001	4181.758 (26)				
		4184.895 (17)	895 (10)	4	1	000	4184.895 (27)		+ 001		
		4187.051 (11)	.049 (9)	2	1	+ .002	4187.050 (20)		.000		
		4187.806 (8)	805 (9)	2	2	+ 001	4187.805 (17)		− 002		
		4191.441							000		
e		4191.671 (7)	.679 (6)	3	1	− .008			− .006		P. Unusable. Faint
		4195.342 (4)	.344 (7)	0	1	− .002	4195.343 (11)				
		4198.306 (11)	.307 (11)	1	2	− .001	4198.307 (22)		.000		
		4199.099 (14)	.098 (12)	3	1	+ .001	4199.099 (26)		.000		
b	1	4202.031 (14)	.032 (12)	3	2	−0.001	4202.031 (26)		+0.001		

b	3	4203.988 (4)	.986 (8)	0	3	+0.002	4203.987 (12)				
c	5	4210.358 (11)	.360 (12)	1	2	− .002	4210.359 (23)		+0.002		
b	3	4216.187 (14)	.188 (11)	3	3	− .001	4216.187 (25)		+ .002		
		4217.558 (2)	.561 (8)	1	1	− .003	4217.560 (10)				
		4219.365 (14)	.366 (12)	2	3	− .001	4219.365 (26)		.000		
		4222.223 (11)	.225 (12)	2	3	− .002	4222.224 (23)		+ .004		
		4224.177 (4)	.177 (8)	1	2	.000	4224.177 (12)				
		4225.466 (4)	.469 (8)	2	2	− .003	4225.468 (12)				
d	5	4227.446 (11)	.445 (12)	7	8	+ .001	4227.447 (23)		+ .007		Unreliable. Edges hazy. Mt. +0.002
d	5	4233.613 (11)	.615 (13)	2	6	− .002	4233.615 (24)		+ .005		Unreliable. Mt. +0.002
d	5	4235.948 (11)	.950 (12)	3	7	− .002	4235.950 (23)				Unreliable. Mt. +0.002
		4247.439 (15)	.444 (14)	2	6	− .005	4247.441 (29)		+ .004		Unreliable. Probably group *e*
c	5	4250.132 (17)	.133 (12)	3	7	− .001	4250.133 (29)		+ .004		Mt. +0.001
b	2	4250.791 (17)	.793 (12)	2	7	− .002	4250.792 (29)		+ .003		
c	2	4260.486 (17)	.484 (12)	4	8	+ .002	4260.485 (29)		+ .001		Measurements difficult
		4267.833 (8)	.833 (6)	3	1	.000	4267.833 (14)		+ .005		Difficult of exact determination
		4271.168 (17)	.166 (12)	6	5	+ .002	4271.167 (29)		+ .004		
b	1	4271.765 (11)	.764 (12)	1	2	+ .001	4271.764 (23)		+ .010		Reversed .765. Unreversed .764. Always good
b	1	4282.408						0.000	+ .004	0.000	
b	2	4294.131 (11)	.131 (26)	1	1	.000	4294.131 (37)		+ .007	+ .001	
d	5	4299.250 (11)	.248 (27)	5	4	+ .002	4299.250 (38)		+ .007	+ .003	Unreliable. Mt. +0.002
b	1	4307.906 (14)	.909 (23)	3	3	−0.003	4307.908 (37)		− .037	− .004	Reversed .909. Unreversed .907. Difficult when unreversed
b	3	4315.090						+0.001	+0.001	+0.001	

TABLE II—*Continued*

Group	Class	International Wave Length: Pasadena	International Wave Length: Mt. Wilson	Mean Deviation: P	Mean Deviation: Mt	P − Mt	Mean Wave-Length	M − L	M − K.	M. − G.	Remarks
b	1	4325 766 (14)	766 (27)	4	4	0 000	4325 766 (41)		+0 003	+0 002	Reversed .768. Unreversed .765. Difficult when unreversed
b	3	4337 051 (11)	052 (26)	1	1	− 001	4337 052 (37)		− .001	.000	Excellent line
b	3	4352 730						−0.002	− .001	− .002	
		4367 581 (2)	.583 (7)	4	1	− .002	4367.582 (9)			− .002	
b	3	4369 777 (11)	775 (19)	1	1	+ .002	4369.776 (30)		+ .009	− .002	
a	3	4375 934						.000	+ .004	.000	
b	1	4383 548 (14)	549 (27)	3	4	− 001	4383 549 (41)		− .003	− .002	Reversed .551. Unreversed .547. Difficult when un reversed
		4388 414 (1)	412 (1)	0	0	+ .002	4388 413 (2)			− .007	
b	1	4404 754 (14)	754 (26)	2	3	.000	4404 754 (40)		+ .001	− .001	Reversed .756. Unreversed .753. Difficult when unreversed
c	4	4407.714 (15)	717 (7)	3	2	− .003	4407 715 (22)			− .001	Faint
c	4	4408.420 (10)	420 (12)	1	2	.000	4408.420 (22)			.000	
b	1	4415 128 (11)	.127 (26)	2	2	+ .001	4415.127 (37)		+ .002	− .002	Unreversed. Good line
b	3	4422 572 (11)	572 (16)	2	2	.000	4422.572 (27)		+ .010	− .002	
a	3	4427.314						0 000	.000	.000	
c	4	4430.622 (5)	.620 (13)	1	2	+ .002	4430.621 (18)		+ .003	− .002	
c	4	4442.345 (5)	344 (13)	0	1	+ 001	4442.344 (18)		+ .001	− .005	
b	3	4443 198 (5)	.196 (13)	1	2	+ .002	4443.197 (18)			.000	
c	4	4447.724 (5)	.723 (14)	1	1	+ .001	4447.723 (19)		.000	− .002	
b	3	4454 387 (5)	.384 (8)	0	2	+ .003	4454.385 (13)		+ .003	+ .001	
c	4	4459 124 (5)	.123 (12)	1	2	+0.001	4459.123 (17)		+0.003	−0.004	

a	3	4461.657 (5)	655 (14)	1	2	+0.002	4461.656 (19)		+0.004	−0.004	
b	4	4466.556						0.000	+ .003	+ .001	
		4469.386 (7)	.382 (4)	1	2	+ .004	4469.385 (11)		+ .003	− .011	
b	4	4476.025 (7)	.027 (9)	6	1	− .002	4476.026 (16)			+ .002	Unreliable. Hazy on red edge
		4482.164 (6)	173 (5)	2	3	− .009	4482.168 (11)		+ .005	− .005	Measures influenced by closeness
		4482.270 (6)	.260 (6)	3	2	+ .010	4482.265 (12)		− .001	− .005	
		4484.236 (7)	.235 (1)	1	0	+ .001	4484.236 (8)			− .005	
a	3	4489.740 (7)	.744 (1)	1	0	+ .002	4489.746 (8)		+ .002	− .001	
		4490.090 (7)		1			4490.090 (7)		+ .002	+ .005	
c	4	4494.572						.000	+ .001	− .001	
		4514.192 (2)	.192 (2)	2	0	.000	4514.192 (4)		.000	− .004	
		4517.532 (2)	.535 (4)	0	1	− .003	4517.534 (6)		+ .004	− .009	
		4525.150 (7)	.151 (8)	2	2	− .001	4525.151 (15)		− .001	− .004	
c	4	4528.620 (8)	.623 (10)	1	2	− .003	4528.622 (24)		.000	.000	
b	3	4531.155						.000	− .004	+ .001	
		4547.853						.000	− .006	− .001	
		4552.551 (1)	.548 (1)	0	0	+ .003	4552.550 (2)			− .002	Faint
e		4556.128 (7)	131 (12)	3	3	− .003			+ .001	+ .003	P. Unusable. Hazy
		4574.726 (1)	.727 (3)	0	0	− .001	4574.727 (4)		− .002	− .005	Faint
		4581.522 (2)	522 (4)	4	1	.000	4581.522 (6)		− .005	− .019	
		4592.658						.000	+ .002	− .002	
		4598.132 (4)	.130 (5)	2	1	+ .002	4598.131 (9)		− .005	− .013	Faint
		4602.008 (5)	.009 (4)	1	1	− .001	4602.008 (9)		+ .004	− .009	Faint
		4602.947						0.000	+ .002	+ .002	From λ4547 to λ4789 except λ4707 and λ4736 secondaries belong probably to group c 4; second-class in quality
		4607.663 (7)		2			4607.663 (7)				
		4611.292 (7)	.292 (6)	1	2	.000	4611.292 (13)		− .003	− .006	
		4613.219 (3)	.217 (5)	4	2	+ .002	4613.218 (8)		− .007	− .018	Faint
		4619.207 (7)	.208 (5)	2	3	− .001	4619.207 (12)		+ .002	− .003	
d?		4625.062 (7)	056 (6)	3	4	+ .006	4625.059 (13)		−0.005	−0.014	Unreliable. Hazy and faint
		4630.130 (7)	128 (5)	2	2	+0.002	4630.129 (12)				

TABLE II—*Continued*

Group	Class	International Wave-Length		Mean Deviation		P.—Mt.	Mean Wave-Length	M.—I	M.—K.	M.—G	Remarks
		Pasadena	Mt. Wilson	P.	Mt.						
d?		4637.522 (7)	515 (6)	3	2	+0.007	4637.519 (13)		+0.002	−0.009	Unreliable
		4638.018 (7)	017 (6)	2	4	+ .001	4638.018 (13)		− .001	− .008	
		4647.430						0.000	+ .003	.000	
		4654.503 (7)	503 (4)	1	1	.000	4654.503 (11)		+ .008	− .011	Measures influenced by closeness
		4654.638 (7)	630 (4)	2	1	+ .008	4654.635 (11)		− .004	− .010	
		4667.459 (7)	462 (6)	2	3	− .003	4667.460 (13)		+ .003	+ .003	
d?		4668.151 (7)	147 (6)	2	2	+ .004	4668.149 (13)		+ .001	− .008	Unreliable
		4673.169 (6)	171 (4)	3	2	− .002	4673.170 (10)		+ .003	− .008	Faint
c	4	4678.857 (7)	857 (6)	1	3	.000	4678.857 (13)		+ .003	+ .001	
c	4	4691.417						.000	+ .002	+ .003	
c	5	4707.288						.000	− .001	− .001	
		4710.287 (8)	288 (9)	2	4	− .001	4710.288 (17)		+ .008	+ .007	
d?		4727.438 (8)	432 (7)	4	4	+ .006	4727.435 (15)		− .003	+ .010	Unreliable
		4733.594 (13)	598 (8)	3	2	− .004	4733.596 (21)		+ .004	+ .009	
c	5	4736.786						.000	− .001	− .003	
		4741.532 (13)	534 (6)	1	3	− .002	4741.533 (19)		+ .003	+ .004	
		4745.806 (11)	.807 (5)	3	4	− .001	4745.806 (16)		+ .004	+ .014	Faint
		4754.048 (14)	045 (9)	2	2	+ .003	4754.047 (23)		− .002	+ .001	
		4757.582 (2)	582 (1)	4	0	.000	4757.582 (3)		+ .006	+ .010	Faint
		4762.375 (14)	.374 (8)	1	3	+ .001	4762.375 (22)		+ .004	+ .003	
		4765.865 (13)	864 (6)	2	2	+ .001	4765.865 (19)			+ .004	
		4766.425 (13)	426 (7)	2	1	− .001	4766.425 (20)		+ .003	+ .005	
		4772.818 (12)	.818 (5)	2	2	.000	4772.818 (17)		+ .002	+ .006	
c	5	4783.435 (14)	431 (9)	3	2	+ .004	4783.433 (23)		− .003	− .004	
c	4	4786.813 (13)	812 (7)	1	2	+ .001	4786.813 (20)		+ .002	+ .003	
c	4	4788.764 (4)	.764 (2)	1	2	.000	4788.764 (6)				
c	4	4789.657						.000	+ .006	+ .003	
c	5	4823.527 (7)	.522 (9)	2	2	+ .005	4823.524 (16)		+ .001	− .002	
		4839.552 (2)	553 (4)	2	1	−0.001	4839.553 (6)		+ .007	+ .007	
c	5	4859.758						0.000	+0.004	+0.002	

c	5	4871.331 (7)	.333 (11)	3	2	−0.002	4871.332 (18)		−0.001	+0.003	
c	5	4872.152 (6)	.152 (11)	4	2	.000	4872.152 (17)		− .003	− .003	From λ4859 to λ5001 the secondaries belong to group c, class 5, and are third-class in quality
c	5	4878.225						0.000	− .004	− .001	
		4881.725 (1)	.724 (2)	0	2	+ .001	4881.724 (3)		.000	+ .006	
		4885.440 (1)	.443 (2)	0	1	− .003	4885.442 (3)		− .003	+ .001	
c	5	4890.767 (7)	.769 (11)	3	1	− .002	4890.768 (18)		− .003	+ .008	
c	5	4891.504 (7)	.506 (11)	4	2	− .002	4891.505 (18)		− .005	− .005	
c	5	4903.325						.000	+ .002	.000	
		4910.036 (4)	.036 (3)	3	2	.000	4910.036 (7)		+ .005	+ .001	
c	5	4919.007						.000	− .007	+ .001	
c	5	4920.517 (15)	.514 (11)	3	2	+ .003	4920.516 (26)		− .007	− .002	
a		4924.775 (6)	.779 (6)	3	3	− .004	4924.776 (12)		− .001	+ .008	Mt. −0.003
c		4938.827 (15)	.827 (11)	3	1	.000	4938.827 (26)		− .003	.000	
a		4939.688 (19)	.691 (21)	3	2	− .003	4939.688 (40)			+ .009	Mt. −0.003
		4946.404 (10)	.403 (5)	2	1	+ .001	4946.404 (15)		− .004	− .002	
c	5	4957.310 (14)	.309 (11)	3	3	+ .001	4957.310 (25)		+ .007	+ .004	
c	5	4957.609 (15)	.608 (11)	3	3	+ .001	4957.609 (26)		.000	− .004	
c	5	4966.104						.000	− .001	− .001	
		4973.115 (3)	.114 (4)	2	3	+ .001	4973.114 (7)			.000	
c		4978.614 (1)	.616 (2)	0	2	− .002	4978.615 (3)		− .005	− .004	
d		4982.532 (12)	.528 (8)	5	4	+ .004	4982.532 (20)		+ .003	− .009	Mt. +0.005
c		4985.268 (14)	.269 (7)	1	2	− .001	4985.268 (21)		− .006	− .001	Apparently uninfluenced by closeness
c		4985.560 (14)	.564 (7)	1	2	− .004	4985.561 (21)		− .008	− .003	
a		4994.133 (19)	.138 (22)	3	2	− .005	4994.134 (41)		− .005	+ .003	Mt. −0.004
c		5001.881						.000	− .003	− .001	
c		5005.730 (15)	.726 (16)	2	3	+ .004	5005.730 (31)		− .002	− .002	Mt. +0.004
c		5006.135 (15)	.131 (17)	3	3	+ .004	5006.135 (32)		− .001	− .002	Mt. +0.004
a		5012.074						+ .001	+ .004	+ .004	
c		5014.960 (11)	.958 (12)	3	2	+ .002	5014.961 (23)		+ .003	.000	Mt. +0.004
c		5022.253 (9)	.249 (9)	3	3	+ .004	5022.253 (18)		− .005	+ .004	Mt. +0.004
a		5028.131 (1)	.129 (2)	0	0	+ .002	5028.130 (3)		− .006	+ .004	Faint
a		5041.077 (4)	.077 (9)	2	3	.000	5041.077 (13)		− .002	+ .005	
a		5041.761 (8)	.760 (12)	1	3	+ .001	5041.760 (20)		+ .002	+ .002	
a		5049.827						0.000	+ .002	− .001	
a		5051.639 (8)	.640 (6)	2	2	−0.001	5051.639 (14)		−0.001	−0.001	

TABLE II—*Continued*

Group Class	International Wave-Length: Pasadena	International Wave-Length: Mt. Wilson	Mean Deviation: P	Mean Deviation: Mt	P − Mt	Mean Wave-Length	M − I	M − K	M − G	Remarks
c	5068.783 (4)	778 (6)	3	4	+0.005	5068.782 (10)		−0.004	−0.004	Mt. +0.004
c	5074.730 (6)	741 (9)	4	5	−.002			−.001	+.017	P. Unusable. Hazy
a	5079.229 (8)	226 (6)	2	5	+.003	5079.228 (14)		−.003	+.002	From λ5012 to λ5167 the secondaries belong to group *a* and are first-class in quality
a	5079.745 (8)	741 (5)	1	2	+.004	5079.743 (13)		−.008	+.002	
a	5083.344						0.000	−.002	−.002	
a	5098.708 (13)	704 (22)	2	2	+.004	5098.705 (35)			+.009	
	5105.546 (2)	544 (3)	2	3	+.002	5105.545 (5)			−.002	
a	5107.454 (8)	455 (6)	2	3	−.001	5107.454 (14)		−.008	−.014	Measures influenced by closeness
a	5107.647 (8)	646 (6)	3	4	+.001	5107.647 (14)		−.003	+.004	
a	5110.415						.000	+.003	+.004	
a	5123.727 (8)	729 (6)	1	2	−.002	5123.728 (14)		−.001	−.004	
a	5127.366 (8)	370 (6)	2	2	−.004	5127.368 (14)		+.002	+.001	
c	5133.656 (9)	674 (10)	7	5	−.018			−.010	+.010	P. Unusable. Very hazy
c	5139.269 (8)	270 (6)	3	3	−.001	5139.271 (14)		+.003	−.007	Mt. +0.003. Apparently uninfluenced by closeness. Mt. +0.003
c	5139.484 (8)	481 (6)	4	6	+.003	5139.484 (14)		+.002	−.009	
	5142.933 (8)	937 (6)	1	2	−.004	5142.935 (14)		+.001	+.001	
a	5150.845 (14)	849 (8)	2	3	−.004	5150.846 (22)		+.005	−.001	
a	5151.916 (8)	920 (5)	2	3	−.004	5151.918 (13)		+.003	+.002	
d	5162.336 (4)	319 (9)	3	7	+.017			+.020	−.011	P. Unusable. Hazy
a	5167.492						.000	+.004	+.002	
	5168.904 (4)	905 (3)	0	1	−.001	5168.904 (7)				
	5171.600 (8)	603 (11)	2	3	−.003	5171.602 (19)			.000	
c	5191.474 (10)	470 (10)	2	1	+.004	5191.474 (20)		−.003	+.005	Mt. +0.004
sub-d	5192.363						0.000	+.001	−.001	
a	5194.949 (18)	951 (9)	4	2	−.002	5194.949 (27)		+.003	−.002	Mt. −0.003
	5198.719 (14)	.721 (4)	4	3	−.002	5198.719 (18)		+.002	−.001	
a	5202.341 (11)	.343 (17)	4	4	−0.002	5202.340 (28)		−0.001	−0.001	Mt. −0.003

sub-d		5208 612 (15)	.612 (5)	2	2	0 000	5208.612 (20)		−0 007	−0.002	
sub-d		5215 199 (17)	.201 (4)	3	1	− .002	5215.199 (21)		+ .006	+ 002	
a		5216.282 (10)	.283 (9)	5	2	− 001	5216 281 (28)		+ .006	+ .002	Mt. −0 003
c		5217.407 (15)	.410 (4)	2	3	− 003	5217.408 (19)		+ .005	− .001	
c		5226 881 (17)	.870 (11)	1	2	+ .002	5226 880 (28)		+ .006	+ 007	
a	4	5227.193 (19)	.193 (11)	4	4	000	5227.192 (30)		+ .007	− .002	Mt. −0 003
c		5229.863 (3)	865 (4)	2	2	− .002	5229 864 (7)		+ .021	+ .002	
sub-d	5	5232.957						0 000	− .003	+ .002	
a		5242.497 (5)	.499 (6)	2	2	− .002	5242 496 (11)		+ 001	+ .001	Mt. −0 003. Faint
		5250 652 (7)	652 (3)	2	2	000	5250 652 (10)			+ .008	
sub-d		5263 320 (5)	318 (4)	2	2	+ 002	5263.319 (9)		.000	+ 001	
sub-d	5	5266 569						.000	− .005	− 001	
a	1	5269 541 (21)	.542 (33)	3	4	− 001	5269 540 (54)		+ 005	+ .008	Mt. −0 003. Measures difficult
a	4	5270 360 (11)	359 (13)	3	3	+ 001	5270 358 (24)		+ 004	+ 005	Mt. −0 003
sub-d		5273 179 (6)		2			5273 179 (6)		+ 003	+ 001	
sub-d		5273 382 (6)		2			5273 382 (6)		+ 004	+ 011	
sub-d		5281 805 (11)	803 (11)	2	2	+ .002	5281.804 (22)		− 002	− 004	
sub-d		5283 636 (11)	633 (13)	1	2	+ 003	5283 634 (24)		− 001	− 001	
sub-d		5302 318 (11)	313 (22)	2	2	+ 005	5302.315 (33)	000	+ 001	− 001	Faint. Too weak for good secondary
		5307 366 (4)		2			5307.366 (4)		+ 002	+ 002	
sub-d	5	5324 194						− .002	− 003	− 001	
a	1	5328 046 (11)	043 (11)	3	3	+ 003	5328 044 (22)		+ 004	+ 004	
a	4	5328 538 (11)	536 (11)	2	2	+ 002	5328 537 (22)		+ 005	+ 002	
a	4	5332 908 (6)		2			5332 908 (6)		+ 004	− 004	
sub-d	5	5339 945 (10)	942 (9)	2	2	+ 003	5339 945 (19)		− 002	− 004	Mt. +0 003
a	4	5341 030 (11)	028 (11)	2	4	+ 002	5341 029 (22)		+ .002	− 006	
c		5364 860 (7)	862 (16)	7	4	− 002			− 002	+ .009	P. Unusable. Hazy
a		5365 407 (6)	404 (6)	3	4	+0 003	5365 405 (12)		+ 001	+ 001	
a	1	5371 495						0 000	+0 005	0 000	

NOTE.—From λ 5192 to λ 5324 the secondaries belong to group *sub-d* and are fourth-class in quality. For this region lines belonging to group *a* and first-class in quality are available for secondaries

0.002 A shown by Goos's results are nearly all associated with lines that are inherently difficult of measurement, while in the Kayser series, the secondary standards of excellent quality, groups *a* and *b* show the same order of deviations as the poorer lines of groups *c* and *d*.

TABLE III

DEVIATIONS FROM THE INTERNATIONAL VALUES GREATER THAN 0.002 A

International Wave-Lengths	Maximum Deviation	I − P.	I. − G.	I − K.
4282.408	0.001	0.000	0.000	+0.004
4375.934	.001	.000	.000	+ .004
4466.556	.002	.000	+ .001	+ .003
4531.155	.000	.000	+ .001	− .004
4547.853	.001	.000	− .001	− .006
4647.439	.002	.000	.000	+ .003
4691.417	.002	.000	+ .003	+ .002
4736.786	.001	.000	− .003	− .001
4789.657	.001	.000	+ .003	+ .006
4859.758	.002	.000	+ .002	+ .004
4878.225	.001	.000	− .001	− .004
4919.007	.001	.000	+ .001	− .007
5001.881	.004	.000	− .001	− .003
5012.073	.001	− .001	+ .003	+ .003
5110.415	.001	.000	+ .004	+ .003
5167.492	.001	.000	+ .002	+ .004
5192.363	.001	.000	− .001	+ .001
5232.957	.001	.000	+ .002	− .003
5266.569	.001	.000	− .001	− .005
5371.495	.003	.000	.000	+ .005
5569.633	.003	− .001	.000	+ .003
5615.661	.003	.000	− .002	− .006
5658.836	.002	.000	+ .001	− .010
6027.059	.000	.000	+ .002	− .003
6065.492	.001	.000	.000	+ .003
6430.859	0.004	0.000	−0.001	+0.011
Sum of deviations	0.041	0.002	0.036	0.111
Mean deviation	0.0016	0.0001	0.0014	0.0043

PAIRS OF LINES

In fixing the standards, it becomes important to determine the exactness with which certain types of lines can be measured. They are of little use as standards if different observers set upon them differently, or if there is something in the character of the lines tending to produce a bias in the observer's mind or a conscious effort to avoid a possible error. In the lists of tertiary standards published there are nine pairs of lines with intervals from

TABLE V

MEASUREMENT OF CLOSE PAIRS OF LINES

Pasadena λ	Pasadena λr−λv	Mt. Wilson λ	Mt. Wilson λr−λv	Kayser λ	Kayser λr−λv	Goos λ	Goos λr−λv	Max Range λ	Max Range λr−λv	Group
4482 164	0.106	.173	0.087	163	0.103	.173	0.097	0 010	0 019	
4482 270		.260		.266		.270		.010		
4654 503........	.135	503	.127	.495	.144	.514	131	019	.017	
4654 638.		630		.639		.645		.015		
4057 310	290	.309	.299	.303	.306	.306	.307	.007	.008	*c*
4957 609		608		.609		613		.005		*c*
4985 268	292	.269	295	.274	.295	.269	.295	006	.003	*c*
4985 560		.564		.569		.564		.009		*c*
5107 454	193	.455	191	.462	188	.468	.175	.014	018	*a*
5107 647		.646		.650		.643		007		*a*
5130 269	.215	.270	0 211	.268	214	.278	215	010	004	*c*
5139 484		.481		482		.493		.012		*c*
5273 170	203			176	.202	.178	103	003	010	*sub-d*
5273 382				.378		371		011		*sub-d*
5462 957	313			.964	308	052	301	012	012	
5463 270				272		253		019		
5476 297	0 286			294	0 287	300	0 286	006	0 001	*a*
5476 583				.581		.586		0 005		*d*

0.1 to 0.3 A. In determining the separation of such pairs, that is, in the measurement of such short intervals, neither the absence of complete normality in the spectra, nor the method of reduction, nor small errors in the reduction factor can have an appreciable effect. The agreement or the lack of agreement between the values obtained will indicate the degree of exactness with which settings can be made upon such lines by different observers, and will give a criterion for judging the fitness of such lines for standards. The data relative to these nine pairs are shown in Table IV. In the third column from the end is given the range in the separate determinations of the wave-lengths. This reaches very high values such as 0.012, 0.014, 0.015, and 0.019 A, much higher than for 18 lines taken at random. The range in the intervals between the pairs is perhaps the most striking, attaining as it does a maximum of 10 per cent of the quantity measured, and a mean value of 0.010 A. It seems quite evident that the disturbing effect due to the presence of the near-by line is sufficient to render the settings upon both uncertain to a high degree. This makes the exact determination of the wave-lengths extremely difficult, and seems to preclude their use in a series of standards aiming at an accuracy of 0.001 A. Even if their wave-lengths should be well determined from the means of a great number of measurements, their employment as standards would not be advisable because of the probability of a great error even in the means of a large number of settings by a single observer, dependent upon the dispersion, the cleanness of the lines, and the personal equation.

RELATIVE WAVE-LENGTHS PASADENA—MOUNT WILSON

Gale and Adams have shown that the pressure displacements of iron lines increase as the cube of the wave-length. Therefore the changes in relative wave-length between Pasadena and Mount Wilson would be less in the case of the present series, λ 4118–λ 5371, than for the first, λ 5371 λ 6404, in a 1:2 ratio, and the effect due to a decrease of one-fifth of an atmosphere would be less in evidence. A few lines of groups *c* and *sub-d* have been referred to standards of group *a* both in Pasadena and on Mount Wilson, and some lines of group *a* have been referred to standards of

groups *c* and *sub-d* in the two series. In the first instance the wave-lengths of lines of groups *c* and *sub-d* are relatively shortened at the higher elevation, and in the second instance the lines of group *a* are apparently lengthened on the mountain due to the lessened wave-lengths of the standards. The measurements are difficult in both cases, as in the first instance the lines to be measured are of poor quality, and in the second the reference lines are poor, though the lines to be measured are of high quality. The lines of group *e* will be displaced to the red under the lessened pressure at the higher elevation and hence the wave-lengths will be actually longer whatever secondary standards are used. The results in the three cases are shown in Table V.

TABLE V

CHANGE IN WAVE-LENGTH BETWEEN PASADENA (244 M) AND MOUNT WILSON (1704 M)

a LINES AGAINST *c* AND *sub-d* STANDARDS		*c* AND *sub-d* LINES AGAINST *a* STANDARDS		*e* LINES AGAINST *a*, *b*, AND *d* STANDARDS	
λ	P.−Mt.	λ	P.−Mt.	λ	P−Mt
4924	−0.004	5005	+0.004	4191	−0.008
4939	− .003	5006	+ .004	4556	− .003
4994	− .005	5014	+ .002	5074	− .002
5194	− .002	5022	+ .004	5133	− .018
5202	− .002	5068	+ .005	5364	−0.002
5216	− .001	5139	− .001		
5227	.000	5139	+ .003		
5242	− .002	5191	+ .004		
5269	− .001	5339	+0.003		
5270	+0.001				
Mean..	−0.0019		+0.0031		−0.007

REVERSED LINES

We have given in the last column of Table II the measurements made upon the reversals and upon the lines when unreversed. In the case of the strong lines λλ 4307, 4325, 4383, and 4404, the measurements upon the reversals are the more reliable, as these lines when not reversed are broad with edges often much fringed. The line λ 4271 is a good line under all conditions, and λ 4415 is an excellent line with sharp edges. It was not reversed upon our plates. These lines are referred to standards of good quality and if measured

as reversals any large variations could hardly be due to errors in the settings. The conspicuous differences between our results and Kayser's for λλ 4271 and 4307 we are unable to explain. An appeal to the Rowland wave-lengths gives no new light as the differences, Rowland−Kayser, for these two lines are still inconsistent with the other Rowland−Kayser differences. He gives a strong *Ca* line near λ 4307 which has never appeared upon our plates, and which if present at a distance of 0.073 A from the strong *Fe* line would make an exact determination impossible.

TABLE VI

COMPARISON OF THE WAVE-LENGTHS OF LINES WHEN REVERSED AND UNREVERSED

λ	Reversed		Unreversed		Wt Mean	Mean−Kayser	Mean−Goos	Rowland −Mean	Rowland −Kayser
	P.	Mt	P	Mt.					
4271	.765	.766	.765	.763	.764	+0.010		0.170	0.180
4307	.910	909	.904	909	.908	− 037	−0.004	.173	.136
4325	.769	767	.764	.766	.766	+ .003	+ .002	.173	.170
4383	551	.550	546	547	.549	− 003	− 002	.171	.168
4404	.757	.755	752	754	.754	+ .001	− 001	.173	174
4415	....		128	127	.127	+0.002	−0.002	0.166	0.168

THE PRECISION OF THE MEASUREMENTS

The internal agreement of the series of 1912 was indicated by a mean probable error of 0.0007 A and 0.0006 A for the Pasadena and mountain plates, respectively, in the case of good lines. The two series of 1913 contain fewer plates but both are of better quality, particularly the Pasadena series. In the series of 1913 all lines (many of poor quality) have been included. The lines have been measured on the average 9 times; the mean probable errors for a single line are 0.0006 A and 0.0007 A respectively, for the Pasadena and mountain series. If the lines of groups *d*, *sub-d*, and *e* were omitted, as in the case of the 1912 series, the probable errors would be less than for that series, even with the smaller number of measurements per line, as would be anticipated from the better quality of the plates. Other lines grouped about λ 4000 and belonging to group *e*, class 5, are accountable in part for the large residuals. In the case of these lines, as for those belonging to group *sub-d*, it is difficult to reach a precision greater than 0.002 A.

A short region, λ 5371–λ 5506, is common to the 1912 and 1913 series of determinations. It is of interest to see what degree of agreement, in the case of lines belonging to the better group *a*, may be obtained by the same observers working upon two different lots of plates. The second series of Pasadena plates was made with a new plane grating of the highest quality. The mountain plates were obtained with the 75-foot Littrow spectrograph of the 150-foot tower telescope. The comparison in shown in Table VII. The practically complete agreement between the two series in regard to both secondary and tertiary standards shows that the plane grating spectrograph of long focus can be considered an instrument of high precision and seems to justify its employment when the purpose is the interpolation between standards distributed through the spectrum, as are the international standards of the second order.

TABLE VII

STANDARDS COMMON TO THE TWO SERIES

1912	1913	1912 − 1913
5371.495	.495	0.000
5420.702	.702	.000
5434.529	.529	.000
5446.919	.921	−.002
5455.614	.614	.000
5462.957	.957	.000
5463.268	.270	−.002
5473.917	.915	+.002
5476.296	.297	−.001
5497.522	.522	.000
5501.470	.471	−.001
5506.784	.784	0.000

SUMMARY

1. The results of our investigation of the secondary standards from λ 4282 to λ 5371 are as favorable as for the yellow-red region. Of the 32 standards of the second order in the blue-green region we find adjustments indicated in the case of four lines only, amounting to 0.001 A for two lines and to 0.002 A in the case of two. Summing up the two investigations, we conclude that there are no errors n relative wave-length of the 53 secondary standards exceeding 0.002 A.

2. The secondary standards belong to groups *a*, *b*, *c* 4, *c* 5, and *sub-d*, and the quality of the lines of the several groups is in the order given for the groups.

3. It would be a gain if all the lines of group *sub-d* could be replaced by lines of better quality. This is easily feasible for the region λ 5192–λ 5324, where excellent lines of group *a* are well distributed.

4. From a comparison of the measured intervals between close pairs of lines it appears that the wave-lengths of the components cannot be determined with sufficient accuracy to fit them for standards.

5. In the case of reversed lines, measurements made upon the sharp reversals are the more reliable.

6. The change in relative wave-length between Pasadena and Mount Wilson is less easy to substantiate in the blue-green region, but it appears distinctly in the means.

7. The large discrepancies between observers in the case of lines of groups *d* and *e* appear to us attr butable to the marked unsymmetrical broadening of these lines near the negative pole of the arc.

8. To eliminate the effect of this polar dissymmetry we recommend that the slit of the spectrograph be placed at right angles to the axis of the arc at the middle point of the enlarged image.

9. The consistency of the various series of our investigation we attribute to the two constant factors, the complete analyzing action of the plane grating and the uniform arc conditions.

Mount Wilson Solar Observatory
June 20, 1913

WAVE-LENGTHS IN THE SPECTRUM OF THE CALCIUM ARC *IN VACUO*

By HENRY CREW AND GEORGE V. McCAULEY

The establishment of the angstrom in terms of the meter by Michelson and Benoit, followed by the determination of secondary interferometer values by Fabry and Buisson, Eversheim, Pfund, and Burns, marks a new era in quantitative spectroscopy. Kayser's interpolation of tertiary iron standards between these interferometer values completes, in a certain sense, a third step and leads to what is apparently the next problem in spectroscopy; namely, the measurement of all the lines in each of the elements in terms of the international unit of wave-length. A considerable number of such measurements has already been made. A list of these, brought up to date, is given by Kayser;[1] besides these there are certain elements for which the work has been done by interferometer methods, with greater accuracy, but with less completeness. As new spectroscopic sources come into use and new series formulas are suggested, the importance of these determinations increases. It was with this idea in mind that the following measures upon the calcium arc spectrum were undertaken. About the time of their completion appeared Holtz's[2] measures upon the same element. But since his arc was used at atmospheric pressure, and since this pressure is sufficient to widen asymetrically many calcium lines and completely to fuse together certain doublets in one of the important series—as illustrated by Plate I, it has seemed advisable to publish our values which were obtained from an arc working at the lowest practical pressure—about one centimeter of mercury—and giving sharp lines, as already demonstrated by Barnes[3] and Saunders.[4]

[1] *Zeitschrift fur wissenschaftliche Photographie*, **12**, 300, 1913.

[2] *Ibid.*, **12**, 101–122, 1913.

[3] *Astrophysical Journal*, **30**, 14, 1909.

[4] *Ibid.*, **32**, 154, 1910.

SOURCE

The electrodes were of metallic calcium, prepared by electrolysis; their cross-section was about 0.35 cm^2; the average separation between electrodes—length of arc—was about 0.5 cm.

The vacuum chamber containing the arc, and shown in Fig. 1, is similar to those used by Barnes[1] and Saunders.[2] The two electrodes pass through asbestos packing at opposite ends of a cylindrical brass chamber, thus enabling the operator to start the arc readily and to adjust for collimation. This chamber consists of two parts which unite, by means of flanges and clamps, at

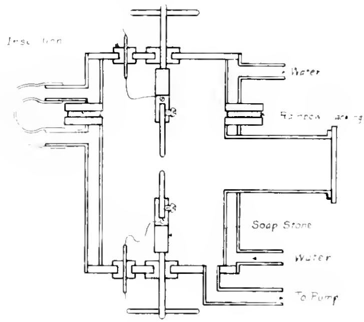

FIG. 1.—Vacuum Arc Chamber

a horizontal plane passing just above the center. In order to secure an air-tight joint a gasket of "rainbow packing" is clamped between these flanges. A side tube permits the light of the arc to pass to the image-lens. A second side tube, at right angles to the first, enables the operator to examine the arc conveniently while exposures are in progress. The chamber is cooled by a small stream of tap-water flowing, in series, through two cylindrical jackets, one about each half of the chamber.

The pressure in the chamber (approximately one centimeter of mercury) was maintained by a Geryk pump and measured

[1] Astrophysical Journal, 27, 153, 1908. [2] Loc. cit.

with a closed-end mercury manometer. As a spectroscopic source, free from noise, annoying vapors, and unsteadiness, the calcium arc under the above conditions leaves little to be desired.

For a comparison spectrum, the middle of a Pfund[1] iron arc was employed.

SPECTROGRAPH

Our photographs were obtained from a ten-foot Rowland concave grating, 9 cm wide, ruled with a total of 50,000 lines, and mounted on a steel triangle in the Rowland method. This triangle was carried upon three brick piers, completely detached from the building and resting directly upon the ground; these three piers were further isolated by being placed in a room inclosed only by interior walls; the steel triangle was isolated from the piers by blocks of soft rubber. From λ 6200 to λ 2800 the second-order spectrum was used; outside this region the first-order was employed.

The photographic plates employed were Cramer "Crown" and Cramer "Spectrum." They were uniformly exposed in the following manner. Immediately in front of the sensitive plate was placed a brass screen with a single horizontal slot 3 mm wide. Through this slot the same part of the plate was exposed in succession to the iron arc during *half* of its proper exposure, then to the calcium arc during the *whole* of its proper exposure, then to the iron arc during the remaining *half* of its proper exposure. Throughout these three exposures nothing about the spectrograph was touched or changed except the source of light. The spectrum strip thus obtained was the one used for measurement. On the upper side of it and in immediate juxtaposition with it was photographed the spectrum of iron alone; on the lower side, in like manner, was photographed the spectrum of calcium alone. These two outside spectra were exposed a little longer than the corresponding spectra on the middle strip and were used merely for the visual identification of *Ca* and *Fe* lines.

During all the five exposures above described the images of the arc covered the same portion of the slit. It is to be noted

[1] *Astrophysical Journal*, **27**, 296, 1908.

also that the passage from the top to the bottom spectrum on each plate was made by moving the plate and not the metal screen; therefore in all three strips one used the same central portion of the spectral band produced by the grating. In any given region of the spectrum three times of exposure were used for the calcium, viz., one just long enough to yield the strong lines as sharp as possible, one long enough to make lines of medium intensity as sharp as possible, and one to give the weak lines as great density as possible.

MEASUREMENT OF PLATES

All the second-order and some of the first-order plates were measured on a Société Genévoise dividing engine. The screw of this engine is 35 cm long and has a pitch of one millimeter. By means of an extra gear wheel, a Veeder cyclometer, and a correcting bar—a combination due to our colleague, Professor Tatnall—this engine enables one to read directly to hundredths of an angstrom; and since a division of the head, indicating one-hundredth of an angstrom, is approximately 2 mm long, one can easily estimate to thousandths of an angstrom.

The wave-lengths from the first-order plates were also obtained by means of a very superior measuring engine made by Mr. F. Küng, the university mechanician. The screw of this engine was cut with a pitch which differed by only one part in ten thousand from a length equal to 10 angstroms on the first-order plates. On the head of this screw one-hundredth of an angstrom corresponds to 0.35 mm in length.

The wave-lengths were determined as follows: The reading microscope and cyclometer were so adjusted that the reading of the engine for a standard iron line near one end of the plate was identical with its wave-length. The successive positions of all the lines appearing in the calcium spectrum and of a sufficient number of standard iron lines were read. This process was repeated twice. The mean of these three sets, considered as a single determination, was then corrected as follows: A curve was plotted in which wave-lengths were used as abscissae, while the differences between the wave-lengths of the iron standards and our engine readings for these same lines were employed as ordinates. A

TABLE OF WAVE-LENGTHS

(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
2103 239[2])	2	1	+0 14	P_1	3078 240.	2			t
12 763[2])	2	.7	+0 06	P_1	3706 022.	9	028	−0 006	P_2
13 10[3])	1*	.	.	P_1	36 903.	9	905	−0 002	P_2
50 78[3])	1		.	SL_1	48 374[1])	1			t
97 791[2])	3	.8	−0 01	P_2	50 340.	1			t
2200 76[3])	1	5	+0 26	SL_1	53 367[1])	1			t
08 606[2])	3	7	−0.09	P_2	3870 506.	2			t
75 493[3])	1	5	−0 01	SL_1	72 552.	3			t
2398 582.	2	587	−0 005	SL_1	75 807.	4			t
2994 953.	4	961	−0 008	T	89 141.	1			SL_3
97 309.	4	.311	−0 002	p	3933 664.	10R	674	−0 010	PH
99 651[1])	4	653	−0 002	T	46 05.	0			SL_2
3000 865[1])	4	878	−0 013		48 899.	3	914	−0 015	T_2
06 864.	5	859	+0 005	p	52 84.	000			
09 212.	4	.210	+0.002	T	57 054.	5	063	−0 009	T_2
3102.36.	0	.		T_2	68 465.	10R	479	−0 014	PH
07.388.	1	. . .	.	T_2	72 578.	1*			SL_3
17 656.	1			T_2	73 716.	4	718	−0 002	T_2
36.003.	2	5.868	+0 135	T_1	4058 912.	0			SL_2
40.782.	2	.720	+0 062	T_1	92 649.	2	690	−0 041	t
41 164.	0*	.	. . . .	T_1	94 044.	3	083	−0 039	t
50 747.	4	.714	+0 030	T_1	98 552.	4	575	−0 023	t
51 280.	1*		.	T_1	4108 554.	1		. .	SL_3
58 877.	10	866	+0.011	P_1	4226 731.	10R	728	+0 003	
64 618.	1	.	.	T_2	40 455.	2	437	+0 018	SL_2
69 854.	1	. .	.	T_2	83 008.	9	006	+0 002	p
79.340.	10	332	+0 008	P_1	89 363.	9	362	+0 001	T
80 521.	2*	. .	.	T_2	98 089.	8	088	+0 001	T
81.283.	6	274	+0 009	P_1	4302 525.	9	528	+0 001	p
3209 930.	3	892	+0 038	T_1	07 738[1])	7	744	−0 006	
15 145.	3	126	+0 019	T_1	18 648.	9	645	+0 003	T
15 334.	1*			T_1	55 099.	5	14	−0 041	SL_3
25 883[1])	5	862	+0 021	T_1	4425 428.	9	449	−0 021	T_1
26 129[1])	1*			T_1	34 948.	9	963	−0 015	T_1
69 090.	1	139	−0 049	T_2	35 673.	8	682	−0 009	T_1
74 661.	2	703	−0 034	T_2	54 765.	9	781	−0 016	T_1
86 060.	3	100	−0 040	T_2	55 875.	5	893	−0 018	T_1
3344 508.	5	491	+0 017	T_1	56 612.	3	623	−0 011	T_1
50.198.	5	188	+0 010	T_1	4507 854.	*owh*			
50 361.	2*		.	T_1	09 446.	0		.	
61.918.	6	904	+0 014	T_1	12 281.	1		.	
62 131.	2*	. .	. . .	T_1	26 044.	4	952	−0 008	SL_2
62 28.	0*	.	. . .	T_1	78 570.	4	588	−0 018	t
3468 484.	2	.489	−0 005	T_2	81 414.	5	445	−0 031	t
74 774.	3	.779	−0 005	T_2	85 868.	6	908		t
87 611.	5	613	−0 002	T_2	85 923.	2			t
3624 107.	6	106	+0 001	T_1	4685 264.	2	273	−0 009	
30 749.	6	739	+0 010	T_1	4807 54.	*oowh*		.	
30 973.	2	958	+0 015	T_1	23 08.	*oowh*			
44 400.	7	403	−0 003	T_1	33 94.	*oowh*			
44 760.	3	757	+0 003	T_1	47 202.	2			
44.990.	0*	. .	. . . .	T_1	78 132[1])	5	168	−0 036	SL_3
73 448.	1	. .	.	t	5001 489.	1			vp
75 307.	2		.	t	19 981.	2			vp

TABLE OF WAVE-LENGTHS—*Continued*

(1)	(2)	(3)	(4)	(5)
5021 141.	0	.	.	vp
41 613[1]	3	634	−0 021	SL_2
5188 840.	3	854	−0 008	
5200 375.	1	387	−0 012	
61 701. .	3	698	+0 003	
62 238. .	3	239	−0 001	
64 237. .	3	235	+0 002	
65 559. .	5	551	+0 008	.
70 272[1]	5	265	+0 007	
5349 470...	5	467	+0 003	
5512 978.	4	935	+0 043	
81 973.	6	956	+0 017	
88 746. .	9	743	+0 003	
90 109.	5	099	+0 010	
94 464.	8	447	+0 017	
98 484.	8	467	+0 017	.
5601 283. .	5	264	+0 019	
5602 829[1] .	5	844	−0 015	
5857 476. .	8	494	−0 018	
67 578. .	1	61	−0 032	
6102 716. .	8	736	−0 020	T_2
22.216. .	8	232	−0 016	T_2
6161 309...	2	.321	−0 012	..
62 177...	9	.196	−0.019	T_2
63 749...	2	.797	−0 048	.
66 443...	2	486	−0 043	..
69 034...	2	.081	−0.047	..
69 576...	3	.605	−0 029	..
6417 71....	00	.	.....	..
39 086...	9	.061	+0.025	..
49 811...	7	.794	+0 017	p
55 606...	3	560	+0 046	p
62 576. .	9	.550	+0 026	..
71 659...	5	.644	+0 015	..
93 789...	8	.762	+0 027	p
99 648...	4	.624	+0 024	p
6508 742...	1	. .	... ...	..
72 783. .	3	71	+0 073	..
6707.866...	1	.81	+0 056	..
17 688. .	5	.70	−0.012	..
7148 123...	3	.	......	p
7202 161...	2	.	..... .	p
7326 099...	1	. .	..... .	..

[1] Too near iron line for accurate measurement; measured with reference to other calcium lines.

[2] Photographed in the third-order and measured with reference to second-order iron lines.

[3] Measured by interpolation between those calcium lines designated by "2)."

smooth curve drawn through the points thus located gave us graphically the correction needed to reduce the readings of each line to the scale of the iron standards; i.e., to terms of the international unit.

RESULTS

In this manner were obtained the wave-lengths of the calcium arc *in vacuo* given in the first column of the above table; in column 2 follow the intensities on the customary scale running from 1 to 10; in column 3 are the values obtained by Holtz for the arc at atmospheric pressure, using $CaCO_3$ in cored carbons; in column 4 are placed the divergences between our values and those of Holtz; in column 5 is indicated in Saunders'[1] notation the series to which the line belongs.

Abbreviations used in the table are as follows: *R* denotes a reversed line; *wh.* one which is wide and hazy; an asterisk indicates a line hitherto unobserved.

[1] *Astrophysical Journal*, **32**, 178, 1910.

DISCUSSION

The lines measured include all those found by Kayser and Runge,[1] with the exception of λλ 4624.71, 3166.95, and 3101.87. These last two are to be identified probably with our lines at 3164.62 and 3102.38, which satisfy more nearly the series formula proposed by Kayser and Runge. Of the new lines discovered by Saunders in the region studied, the following eighteen were not observed by us:

1837.1	2083.2	2216.7	2335.0	5342.4	6784.35
1839.8	2097.8	2249.8	2354.7	6261.7	6789.6
2072.8	2118.99	2287.9	2373.3	6395.4	6798.9

For the sake of completeness, reference is also made to the lines of calcium measured by Paschen[2] and Hermann[3] in the infra-red, and to those measured by Lyman[4] in the ultra-violet.

Of the eleven new lines observed by us, all belong to the first subordinate series of main triplets, except λλ 3180.52, 3972.58, and 2113.19. The line at 3180.52 probably belongs to the second subordinate series of main triplets, replacing λ 3181.28 employed by Kayser and Runge,[5] and fills the place left vacant in Saunders' classification,[6] which makes λ 3181.28, from its close relationship with the pair λλ 3158.88 and 3179.34, a satellite of λ 3179.34. The line at 3972.58 appears to be the missing member of Saunders'[7] series SL_3, while λ 2113.91 appears to be a satellite of the longer line in the pair λλ 2103.24 and 2112.76 of the first subordinate series of pairs. It is therefore the line analogous to λ 3181.28. The new lines in the structure of the first subordinate series of main triplets are analogous to λλ 4435.67, 4455.88, and 4456.62, as may be readily seen from the table. λ 3362.28 may be one of the spurious lines accompanying λ 3361.92 and due to this grating, which has the peculiarity of giving several sharp companions to each strong line, all within the first ghost distance. However, on the two plates from which λ 3362.28 was measured, these

[1] *Annalen der Physik*, **43**, 391, 1891.

[2] *Ibid.*, **20**, 651, 1909.

[3] *Ibid.*, **16**, 684, 1905.

[4] *Astrophysical Journal*, **35**, 341, 1912.

[5] *Loc. cit.*

[6] *Astrophysical Journal*, **32**, 169, 1910.

[7] *Ibid.*, **32**, 157, 1910.

spurious lines were not prominent and seemed to be clearly separated from the line in question. We therefore give this wavelength as a "candidate" for the position in the calcium triplet at this point analogous to each of the positions occupied by λ 4456.61 and λ 3644.99 in the longer triplets of this same series.

The divergences between our measures and those of Holtz are seldom greater than one-hundredth of an angstrom; the principal exceptions are those lines characterized by Holtz as being hazy or widened either to the red or to the violet. A few noticeable cases in which the foregoing statement does not apply are worthy of mention. The divergence in the measures of λ 3286.06 is explained by ascribing to it the same physical character as to the other two lines, λλ 3274.66 and 3269.09, of this triplet. A similar statement holds in the case of the "narrow triplet," λ 4578.57, λ 4581.41, and λ 4585.87, which should have the same appearance—a widening to the red—as the next triplet, λ 4092.65, λ 4094.94, and λ 4098.55, in the same series. The large discrepancy in λ 5512.98 and the systematic differences in the measures of all lines longer than λ 5800, we are unable to explain.

The average divergence between our own measures, made from different plates, and by different observers, was only six- or seven-thousandths of an angstrom unit. This seemed to warrant the use of the third decimal in the final values except for a few very weak and difficult lines which have been entered in the table with two decimal figures only.

The identity of λ 6707.87 has not been satisfactorily established. Adams[1] thinks that it may be an impurity, possibly lithium; while Holtz[2] concludes positively that it is due to calcium, since other lithium lines of equal intensity do not appear on any of his photographs. We have included the line in our list without undertaking at this time to ascertain its origin.

PRINCIPAL IMPURITIES

In the calcium were found as impurities: magnesium, strontium, manganese, silicon, sodium, iron, and aluminium.

[1] *Astrophysical Journal*, **30**, 92, 1909.

[2] *Loc. cit.*

Reliable measures were made on the following lines:

Mg		*Sr*	*Mn*	*Na*
2705.540	3838.296	4077.723	4030.754	5889.963
2852.132	4481.104	4215.525	4033.064	5895.920
3829.361	5172.642	4607.336	4034.484	
3832.308	5183.598			

UNIDENTIFIED LINES

The following twenty-six lines which appeared repeatedly on the photographs are unidentified: it is, of course, not impossible that some of them belong to calcium.

2721.646	3045.751	3099.341	3705.618	4506.624
3018.551	3055.321	3461.896	4100.825	4507.417
3024.027	3071.575	3108.577	4110.330	4547.878
3034.524	3076.988	3594.083	4137.922	6250.753
3041.046	3080.819	3683.714	4496.158	6456.007
		3694.108		

COMPARISON OF ARC AND SPARK SPECTRA

A comparative study of the spark at reduced pressure was made from a series of photographs of the arc and spark taken side by side. The same electrodes and same vacuum chamber were used for the spark as for the arc. Step-up transformers were employed to furnish the potential needed for the spark. One of these has a closed magnetic circuit and transforms from 100 to 5000 volts; the other has an open magnetic circuit and transforms from 100 to 2500 volts; but the latter furnishes much greater current in the spark circuit than can be obtained with the former. The spectrum of the spark under a pressure of one centimeter of mercury was found to be relatively weak in calcium and strong in cyanogen and nitrogen bands. The heads of these bands appear stronger in the spark, while the "structure" lines are stronger in the arc. At a pressure of 48 cm the photographs are entirely free from bands of cyanogen and nitrogen, and more calcium lines make their appearance.

The only lines appearing in the spark and not in the arc were identified either as air lines or "spark" lines of impurities. Even with exposures that bring out the faint lines λλ 3644.76, 3644.99,

3350.36, and 3344.51 of the first subordinate series of main triplets, no trace is found of the lines listed by Eder and Valenta[1] as characteristic of the spark. Nor did λ 2373.27 and λ 4132.7 of Exner and Haschek[2] appear.

BAND SPECTRA

In addition to the bands already studied by Olmstead[3] and Eagle,[4] the arc spectrum of calcium, at reduced pressure, shows several bands which make their appearance only during the first half-hour of exposure with fresh calcium electrodes. These lie in the region λ 6700–λ 7000 and at λ 3500, and present a structure of fairly sharp lines with no continuous background. The origin of these bands we have left undetermined.

PHYSICAL LABORATORY
NORTHWESTERN UNIVERSITY
September 27, 1913

[1] *Denkschriften der Wiener Akademie*, **67**, 1898.

[2] *Die Spectren der Elemente*, Wien, 1911.

[3] *Astrophysical Journal*, **30**, 231, 1909.

[4] *Ibid.*, **27**, 66, 1908.

MEASURES OF VARIABLE RADIAL VELOCITIES OF STARS

By OLIVER J. LEE

In the course of measuring Bruce spectrograms the ten stars given in the first list below were found to be binaries. In the fourth column, "Observer," A=Adams; Ar=Arbogast; B=Barrett; F=Frost; L=Lee; M=S. A. Mitchell. F. R. Sullivan has assisted in observing, as usual.

89 f Piscium ($\alpha = 1^h 13^m$; $\delta = +3°5'$; Mag. = 5 3)

Plate	Date	G M T.	Taken by	No. of Lines	Velocity	Quality
					km	
IB 2108	1909 Aug. 20	$21^h 37^m$	L	6	+27	g.
2120....	Aug. 27	20 50	L	9	+16	g.
2494 ...	1910 Aug. 25	20 6	L	6	+18	v.w.
2569.........	Nov. 28	14 40	F	5	−12	w.

g = good; v. = very; w = weak.

The spectrum is classified as of type A2. It has numerous metallic lines which are diffuse and on all the plates give the effect of unresolved components of unequal intensity. On plate No. 2120 the edges of the lines were sharp and supposed components were measured, giving −30 km and +73 km from 8 and 6 lines respectively. *Mg* λ 4481 alone shows a range of velocities of 45 km when referred to the sun.

73 ξ^2 Ceti ($\alpha = 2^h 23^m$; $\delta = +8°1'$; Mag. = 4 3)

Plate	Date	G M T.	Taken by	No of Lines	Velocity	Velocity λ 4481	Quality
					km	km	
IB 2123.....	1909 Aug. 30	$19^h 43^m$	B	9	+ 4	+ 4	v.g.
2132.....	Sept. 10	19 59	L	7	+ 6	+ 9	g.
2547.....	1910 Oct. 28	17 31	B, Ar	4	+20	+24	v.g.
2559....	Nov. 7	15 10	F, Ar	5	+13	+13	v.g.

The spectrum is of class A. The *Mg* line λ 4481 is by far the best on our plates and velocities derived from it were reduced to the

sun and tabulated as above after duplicate measures on it had been made.

125 Tauri ($\alpha = 5^h 34^m$; $\delta = +25°50'$; Mag. = 5.0)

Plate	Date	G M T	Taken by	No of Lines	Velocity	Quality
					km	
1B 2887	1911 Dec. 3	$17^h 57^m$	L	7	+13	v.g.
3104	1912 Dec. 10	17 36	M	6	+63	g.
3223	Dec. 30	15 45	B	3	+ 1	v.g.

This star has a spectrum of type B3. The lines are simple.

40 Aurigae ($\alpha = 6^h 0^m$; $\delta = +38°30'$; Mag. = 5.3)

Plate	Date	G M T	Taken by	Stronger Comp		Weaker Comp		Quality
				Velocity	No of Lines	Velocity	No of Lines	
				km		km		
1B 3512	1913 Oct. 2	$22^h 11^m$	L	−58.5	12	+130.8	9	g.
3517	3	21 24	L	−35.6	16	+ 92.9	10	g.
3519	6	21 31	B	+22.8	7			w.

This star, of spectral type A, has numerous good metallic lines. No. 3519 shows the two components superposed. The orbits will be discussed when sufficient data have been obtained.

24 Canum Venaticorum ($\alpha = 13^h 30^m$; $\delta = +40°32'$; Mag. = 4.6)

Plate	Date	G M T	Taken by	Center		Violet Comp		Red Comp		Measured by	Quality
				No of Lines	Velocity	No of Lines	Velocity	No of Lines	Velocity		
					km		km		km		
1B 1983	1909 Feb. 5	$21^h 24^m$	L	6	−32	2	+ 8	5	+34	Ar	g.
				4	− 4	5	−38	4	+58	L	
2337	1910 May 20	16 56	B	7	−26	4	−51	4	+40	Ar	g.
				10	−22	3	−60	3	+26	L	
2662	1911 Jan. 23	20 57	B	6	+ 1	1	−45	1	+72	Ar	w.
				7	+14	4	−41	4	+65	L	
2668	Jan. 27	22 4	Ar	5	+18	11	−33	6	+64	L	g.
2694	Mar. 3	19 30	Ar	14	− 8	1	−58	1	+20	Ar	g.

This spectrum, of type A3, has numerous lines which are diffuse and hard to analyze when complicated by doubling. The com-

ponents are of nearly equal brightness. On No. 2668 Arbogast measured the middle points of 4 lines, the mean of which reduced to the sun is −23 km. On No. 2694 my velocity for the center is −16 km from 8 lines. Our measures were made independently and the binary nature of the star was first noted by Arbogast.

33 Bootis ($\alpha = 14^h 35^m$; $\delta = +44^\circ 50'$; Mag. = 5.4)

Plate	Date	G.M.T.	Taken by	No. of Lines	Velocity	Quality
					km	
IB 3282	1913 Feb. 5	$22^h 31^m$	L	5	+15	g.
3331	Mar. 31	18 42	L, M	4	− 6	v.g.
3348	Apr. 16	17 26	B	4	−21	v.g.

This is an A-type spectrum. The metallic lines are faint but measurable.

27 β Librae ($\alpha = 15^h 12^m$; $\delta = -9^\circ 1'$; Mag. = 2.7)

Plate	Date	G.M.T.	Taken by	No. of Lines	Velocity	Quality
					km	
IB 337	1904 May 10	$14^h 58^m$	F, B	3	−18	v.g.
2308	1910 Mar. 14	21 2	B	4	−29	v.g.
2334	May 13	19 16	L	4	−53	g.
2340	May 20	19 51	L	3	−65	v.g.
2698	1911 Mar. 3	23 23	Ar	4	−12	v.g.

This spectrum is of type B8 and the hydrogen lines alone are measurable. Low dispersion and fine-grained plates should be used in further investigation of it.

B.D. 25° 4165 ($\alpha = 20^h 11^m$; $\delta = +25^\circ 17'$; Mag. = 4.8)

Plate	Date	G.M.T.	Taken by	Center		Violet Comp.		Red Comp.		Quality
				No. of Lines	Velocity	No. of Lines	Velocity	No. of Lines	Velocity	
					km		km		km	
IB 47	1903 June 13	$10^h 22^m$	A	6	−14					v. strong
59	July 24	20 14	F	3	−23	3	−85	3	+30	v.g.
69	Sept. 5	15 22	A	3	−15	1	−80	1	+40	v.g.
78	Sept. 18	13 31	A	2	+20	2	−40	2	+88	v.g.
372	1904 June 18	20 10	B	5	−11	6	−58	5	+55	g.

This star has a spectrum of type B3. The lines are diffuse and hard to analyze. The line *Mg* λ 4481 is widely double on No. 372 and is certainly single on No. 59. Without doubt this star is a binary showing two spectra. The spectrum shows striking changes, possibly due to causes other than those of velocity.

33 ι *Aquarii* ($\alpha = 22^h 1^m$; $\delta = -14°21'$; Mag. = 4.4)

Plate		Date		G M T	Taken by	Stronger Component	Weaker Component	Quality
						km	km	
IB	1755	1908	Sept. 25	14^h36^m	B	−68	+ 46	g.
	1762		Sept. 29	14 47	L	+ 9	−130	g.
	1766		Oct. 2	16 35	B	−87	+ 48	w.
	1780		Oct. 9	13 10	L	−97	+ 97	g.
	1789		Oct. 12	12 27	L	−66	+103	g.

The measures given above are on the *Mg* line λ 4481 alone, this being the only interpretable line in the spectrum. The character of the line changes greatly. The period may be short. The spectrum is of type B8.

18 λ *Piscium* ($\alpha = 23^h 37^m$; $\delta = +1°14'$; Mag. = 4.6)

Plate		Date		G M T.	Taken by	No. of Lines	Velocity	Measured by	Quality
							km		
IIB	3	1903	Sept. 19	16^h45^m	F	5	+11	F	p.
						11	+ 5	L	
IB	91		Sept. 25	16 36	F	9	+19	F	g.
						10	+23	L	
	199		Dec. 1	12 29	F, A	7	+ 6	F	v.g.
						10	+ 8	L	
	217		Dec. 25	12 19	A, F	7	+10	F	g.
						13	+ 9	L	
	585	1905	Sept. 18	17 11	F	6	+36	L	w.
	1238	1907	Nov. 22	12 51	B, L	10	+12	L	v.g.
	2764	1911	July 14	20 55	B	11	+20	L	v.g.

This spectrum is of type A5. It has numerous good lines which seem to be overlapping components. An attempt was made to measure them, but similar lines did not show the proper consistency and the results are therefore not given. The dispersion of two and three prisms can be used to advantage on this star. Duplicate measures on No. 585 gave +34 and +38 km from 4 and 6 lines respectively.

The following eighteen stars have been previously announced here or elsewhere as spectroscopic binaries. The plates here measured were obtained for the most part in the course of the regular program with the Bruce spectrograph, generally prior to the announcement of the variable velocity of the stars by other observers. The director has removed all of these stars from the observing program for the present.

15 κ Cassiopeiae ($\alpha = 0^h 27^m$; $\delta = +62°23'$; Mag. = 4.2)

Plate	Date		G M T	Taken by	No. of Lines	Velocity	Quality
						km	
IB 1718	1908	Sept. 7	$16^h 25^m$	B	9	− 3	g.
1727		Sept. 8	15 10	L	10	− 8	g.
1772		Oct. 5	15 7	F, L	8	+ 2	v.g.
2475	1910	Aug. 8	18 38	L, B	5	−16	w.
2482		Aug. 12	19 40	L	7	−10	g.

The variable velocity of this star was announced in the *Lick Observatory Bulletin*, **6**, 141, 1911.

29 π Andromedae ($\alpha = 0^h 32^m$; $\delta = +33°10'$; Mag. = 4.4)

Plate	Date		G M T	Taken by	No. of Lines	Velocity	Quality
						km	
IB 1177	1907	Sept. 23	$19^h 6^m$	L	7	+67	v.g.
1185		Sept. 24	18 29	F, L	6	+68	v.g.
1197		Oct. 11	17 28	B	5	+26	g.
1230		Oct. 22	15 41	F	7	+14	v.g.

This binary was announced by Frost and Adams in this *Journal*, **18**, 384, 1903, from their measurement of three earlier spectrograms.

46 ω Cassiopeiae ($\alpha = 1^h 48^m$; $\delta = +68°12'$; Mag. = 5.0)

Plate	Date		G M T	Taken by	No. of Lines	Velocity	Quality
						km	
IB 2142	1909	Sept. 20	$18^h 45^m$	L, B	3	−35	w.
2799	1911	Aug. 18	20 12	B	4	−28	g.
2862		Nov. 20	12 9	L	6	− 7	w.
2863		Nov. 25	13 49	B	5	− 1	g.

This spectrum is of type B8 and the hydrogen and helium lines are strong and fairly sharp. The variable velocity was announced by Adams in *Publications of the Astronomical Society of the Pacific*, **24**, 129, 1912.

4 g Persei ($\alpha = 1^h 56^m$; $\delta = +54°0'$; Mag. = 5.0)

Plate	Date	G.M.T.	Taken by	No. of Lines	Velocity	Quality
					km	
IB 1171	1907 Sept. 21	$20^h 44^m$	B	4	− 6	v.g.
1187	Sept. 24	20 47	L	3	−17	g.
1283	Dec. 11	11 47	L, B	3	0	g.
1288	Dec. 16	12 54	B	5	+ 1	g.
1328	1908 Jan. 14	16 50	B	4	+ 2	g.

The variable radial velocity of this star was announced by Frost and Adams in this *Journal*, **19**, 152, 1904, where will be found their measures of four earlier plates.

33 τ^5 Eridani ($\alpha = 3^h 49^m$; $\delta = -24°54'$; Mag. = 4.8)

Plate	Date	G.M.T.	Taken by	No. of Lines	Velocity	Quality
					km	
IB 639	1905 Dec. 15	$15^h 37^m$	F	7	+27	g.
649	Dec. 25	15 34	F	5	+22	g.
657	1906 Jan. 26	14 0	F	6	+20	v.g.
885	Oct. 19	19 20	B	7	+ 8	v.g.
903	Nov. 1	18 57	F	3	+ 8	g.
1320	1908 Jan. 13	15 42	L	4	+ 3	g.

The binary character of this star was announced by Frost in this *Journal*, **25**, 64, 1907, from preliminary measures on the first five plates given above. My measures of them and of No. 1320 are to be considered as definitive. Mr. Ichinohe assisted in securing four of these plates.

45 ϵ Persei ($\alpha = 3^h 51^m$; $\delta = +39°43'$; Mag. = 3.0)

Plate	Date	G.M.T.	Taken by	No. of Lines	Velocity	Quality
					km	
IB 1176	1907 Sept. 23	$18^h 25^m$	L	3	−22	v.g.
1188	Sept. 24	21 31	L	6	+13	v.g.
1193	Oct. 7	22 33	L	5	Max. −71	v.g.
				4	Min. +67	

Measures of five earlier plates of ϵ *Persei* were published in this *Journal*, **19**, 152, 1904, by Frost and Adams. These observers commented upon the complexity and changes in the lines but did not measure components.

94 τ Tauri ($\alpha = 4^h 36^m$; $\delta = 22°46'$; Mag. = 4.3)

Plate	Date	G M T	Taken by	No. of Lines	Velocity	Quality
					km	
IB 443	1904 Nov. 8	$17^h 42^m$	F	6	+ 6	v.g.
466	Dec. 30	14 35	F	8	+30	v.g.
588.........	1905 Sept. 18	19 58	B	2	−17	w.
1286	1907 Dec. 11	14 50	L	6	+51	g.

This helium star was announced as a binary by Frost and Adams in this *Journal*, **17**, 246, 1903, from the measures of three early high-dispersion plates taken with a short camera. The orbit has been published by Parker in *Journal of the Royal Astronomical Society of Canada*.

20 τ Orionis ($\alpha = 5^h 13^m$; $\delta = -6°57'$; Mag. = 3.7)

Plate	Date	G M T	Taken by	No. of Lines	Velocity	Quality
					km	
IB 931	1906 Dec. 17	$17^h\ 3^m$	F	4	+29	g.
968	1907 Feb. 2	15 18	Fox	8	+26	v.g.
1399	1908 Feb. 2	16 29	Jordan	3	+25	w.

The binary nature of this helium star was announced in this *Journal*, **25**, 64, 1907, by Frost.

30 τ Canis majoris ($\alpha = 7^h 15^m$; $\delta = -24°47'$; Mag. = 4.4)

Plate	Date	G M T	Taken by	No. of Lines	Velocity	Quality
					km	
IB 712.........	1906 Mar. 23	$14^h 19^m$	F	2	+78	v.w.
714	Mar. 30	14 4	F	5	+48	g.
722	Apr. 2	13 30	F	7	+38	g.
974 .	1907 Feb. 7	15 48	F	2	+ 0	v.w.
1991.........	1909 Mar. 1	15 52	F	8	+92	g.

The variable velocity of this star was announced by Frost in a footnote on p. 265 of the *Astrophysical Journal*, **23**, 1906. Measures of four Lick plates are given in *Lick Observatory Bulletin*, **6**, 145, 1911.

8 π Virginis ($\alpha = 11^h 56^m$; $\delta = 7°10'$; Mag. = 4.6)

Plate	Date	G.M.T.	Taken by	No. of Lines	Velocity	Quality
					km	
IB 733	1906 Apr. 20	$15^h\ 4^m$	B	9	−33	g.
755	May 11	14 24	B	9	−20	g.
1040	1907 Apr. 26	16 14	F	10	− 8	g.
1979	1909 Feb. 1	21 43	B	7	+ 6	g.

This star was announced as a binary by Albrecht in *Lick Observatory Bulletin*, **5**, 175, 1910.

21 ι Boötis ($\alpha = 14^h 13^m$; $\delta = +51°50'$; Mag. = 5.1)

Plate	Date	G.M.T.	Taken by	No. of Lines	Velocity	Quality
					km	
IB 2704	1911 May 1	$17^h 20^m$	F, Ar	2	+ 3	Poor
3009	1912 Mar. 6	19 49	B	7	− 3	v.g.
3015	Mar. 8	19 30	B	6	−16	v.g.

The spectrum of this star contains numerous rather poor lines. Measures by Jordan are recorded in the *Publications of the Allegheny Observatory*, **2**, 123, 1911. There can be no doubt of the actual variation in radial velocity.

9 α² Librae ($\alpha = 14^h 45^m$; $\delta = -15°38'$; Mag. = 2.9)

Plate	Date	G.M.T.	Taken by	No. of Lines	Velocity	Quality
					km	
IB 3010	1912 Mar. 6	$21^h 33^m$	B	9	− 4	v.g.
3016	Mar. 8	20 52	B	13	−10	v.g.
3021	Mar. 5	19 16	B	7	+26	v.g.

The binary character of this star was reported by Slipher in the *Lowell Observatory Bulletin*, **1**, 57, 1904. The hydrogen lines on our Bruce plates are apparently complex, but there is no evidence of the second component in the numerous metallic lines.

14 ι Coronae Borealis ($\alpha = 15^h57^m$; $\delta = +30°8'$; Mag. = 4.9)

Plate	Date	G.M.T.	Taken by	No. of Lines	Velocity	Quality
					km	
IB 2705	1911 May 1	18^h35^m	F, Ar	4	− 3	w.
2723	June 9	15 1	B	8	−13	g.
3035	1912 Mar. 29	20 59	B	8	−20	v.g.

The variable velocity was announced by Baker in *Publications of the Allegheny Observatory*, **1**, 121, 1909.

55 α Ophiuchi ($\alpha = 17^h30^m$; $\delta = +12°38'$; Mag. = 2.1)

Plate	Date	G.M.T.	Taken by	Stronger Comp.		Weaker Comp.		Quality
				Velocity	Quality	Velocity	Quality	
				km		km		
IB 1165	1907 Sept. 16	15^h54^m	L	− 16	g.	+120	w.g.	v.g.
1168	21	14 7	F	+ 4	g.	− 98	g.	v.g.
1174	23	15 5	B	+ 69	...	.	....	v.g.
1182	24	13 23	F	− 30	g.	+125	g.	v.g.
1204	Oct. 18	13 10	B	− 79	g.	+ 38	g.	v.g.
1213	20	14 13	F	− 54	g.	+ 86	w.	v.g.
1219	21	13 0	F, L	+127	g. wide	− 77	g. wide	v.g.
1228	22	13 56	F	− 18	Distorted	−140	w. doubtful	v.g.
1546	1908 Mar. 20	23 38	B	+105	Fair	−113	w. wide	v.g.

The spectrum of this star is given as A5. The *Mg* line λ 4481 shows the components best on our plates and the measures given above were made on this line alone. On No. 1168 the components are equally strong and on No. 1174 the line could not be resolved. These observations seem to be roughly satisfied with a period of about 2 days. The binary nature of the star was discovered by Frost and announced in *Astronomische Nachrichten*, **177**, 174, 1908.

102 Herculis ($\alpha = 18^h 4^m$; $\delta = +20°48'$; Mag. = 4.3)

Plate	Date	G.M.T.	Taken by	No. of Lines	Velocity	Quality
					km	
IB 123	1903 Oct. 17	$13^h 42^m$	A	12	−16.5	v.g.
388	1904 July 26	17 12	F	10	−16.3	g.
1636	1908 July 20	16 15	B	11	−15.6	v.g.
1645	July 24	18 25	B	11	−16.1	v.g.

This star, No. 8382 of Burnham's *General Catalogue*, with a 12th magnitude companion, was one of the twenty helium stars observed by Frost and Adams (*Publication of the Yerkes Observatory*, **2**, 1903). From four plates they obtained a mean velocity of −10.8 km, with a range of 3.9 km. This value is given with revised wave-lengths of the silicon lines as −8.8 km in this *Journal*, **32**, 85, 1910. The binary character of the star was detected by Albrecht (*Lick Observatory Bulletin*, **6**, 147, 1911) with value ranging from −11 to −18 km. The spectrum is of type B2, and numerous lines are well measurable with a dispersion of one prism.

111 Herculis ($\alpha = 18^h 43^m$; $\delta = +18°4'$; Mag. = 4.4)

Plate	Date	G.M.T.	Taken by	No. of Lines	Velocity	Quality
					km	
IB 2340	1910 May 23	$19^h 50^m$	B	11	−36.2	v.g.
2405	June 27	17 35	L	12	−52.1	v.g.
2443	July 25	15 38	L	12	−42.2	v.g.
2463	Aug. 5	15 24	B	11	−43.1	v.g.

The variable radial velocity of this star was discovered by Wright and it was announced in *Lick Observatory Bulletin*, **5**, 63, 1908.

14 Pegasi ($\alpha = 21^h 45^m$; $\delta = +29°42'$; Mag. = 5.0)

Plate	Date	G.M.T.	Taken by	No. of Lines	Velocity	Quality
					km	
IB 2423	1910 July 8	$19^h 15^m$	L	7	−30	v.g.
2432	July 18	20 3	B	8	−20	v.g.
2467	Aug. 5	18 0	B, L	5	−10	g.

The variable velocity of this star was announced in the *Lick Observatory Bulletin*, **6**, 149, 1911.

48 γ Aquarii ($\alpha = 22^h 16^m$; $\delta = -1°53'$; Mag. = 4.0)

Plate	Date	G.M.T.	Taken by	No. of Lines	Velocity	Quality
					km	
IB 804	1906 July 13	$20^h 55^m$	F	4	−26	v.g.
1139	1907 Aug. 12	19 15	B	6	− 6	v.g.
2453	1910 July 29	18 27	B, L	8	−10	v.g.

The variable radial velocity of this star was announced by Plaskett in the *Journal of the Royal Astronomical Society of Canada*, **2**, 272, 1908.

YERKES OBSERVATORY
October 1913

THE FIRST DESLANDRES' GROUP OF THE POSITIVE BAND SPECTRUM OF NITROGEN, UNDER HIGH DISPERSION

By RAYMOND T. BIRGE

This paper is a preliminary discussion of the First Deslandres' Group of the band spectrum of nitrogen, based mainly upon photographs taken by the author in the second order of a 21-foot concave grating, from λ 5000 to λ 6800. The bands are almost completely resolved into lines, and the discussion in this paper is concerned with the relations between the lines forming the three principal heads of the bands.

INTRODUCTION

The positive band spectrum of nitrogen has been the subject of a large number of investigations. Under low dispersion the apparent regularity of the bands, both in position and in appearance, is very striking. The violet end of this spectrum can easily be photographed under high dispersion, but because of the relatively low intensity in the longer wave-lengths, this portion had not previously been resolved into its component lines in a satisfactory manner. Von der Helm[1] made the latest attempt. The two objects of his investigation, with his success in accomplishing them, are stated as follows:

> 1. Übersicht über den gesamten Teil des in Frage kommenden Spectrums, inbesondere über die Lage der Bandenköpfe.
>
> 2. Genaueres Studium einzelner Banden.
>
> Den ersten Teil der Aufgabe darf ich als gelöst betrachten; am zweiten bin ich leider fast völlig gescheitert.

Von der Helm gives a complete discussion of relevant previous work on nitrogen, together with the possible results of such an investigation, and the first eight pages of his article could well form the introduction to this paper.

The First Deslandres' Group of bands extends from λ 5100 out into the infra-red. Measurements on the heads of individual

[1] *Zeitschr. f. wiss. Phot.*, 8, 405, 1910.

bands have previously been made to λ 9100. The results seem to show that the spectrum consists of a series of band groups, each of which is most intense at the center, and diminishes in intensity toward either side. Kayser's *Handbuch*[1] gives only three groups, which he calls *a*, *b*, and *c*. Other groups of longer wave-length have since been found, and it appears now that there are six groups in all, which will be designated *a* to *f* respectively.

Group *a* 1.06 μ (?)
Group *b* 9101 (?) to 7887
Group *c* 7887 to 7059
Group *d* 7059 to 6185 (Kayser's *a* group)
Group *e* 6186 to 5485 (Kayser's *b* group)
Group *f* 5632 to 5126 (Kayser's *c* group)

Group *f* is quite different from the others. It has two intensity maxima, one at λ 5200 and the other at λ 5475. This would indicate two groups, but as the spacing is the same in both, it has been customary to classify them together. This group also overlaps considerably on group *e*.

The author obtained, besides the exposures on the large grating, one on a Hilger constant deviation spectroscope, extending to λ 7650. From this point to λ 9100 we have only the measurements of Croze.[2] Coblentz,[3] in connection with other infra-red work, has recorded positions of maximum intensity at 0.546, 0.667, 0.75, 0.90, and 1.06 μ. These are very evidently the approximate positions of maximum intensity in the several band groups. The reading at 1.06 μ points to the existence at this point of another group, which we have called group *a*.

In making this investigation the author had two objects in view: (1) to determine whether or not the bands in any one group were identical; (2) to determine, in case there were any similarities, whether corresponding lines in successive bands would fit into a Deslandres' series or other arithmetical relation.

The results of the study made thus far indicate that out of the 250 or more lines composing each band, at least 50 of the strongest are related to corresponding lines in other bands, and that the relationship is approximately that expressed by Deslandres' Law:

$$v=a+b(m+c)^2$$

[1] *Handbuch der Spectroscopie*, **5**, 828.

[2] *Comptes rendus*, **150**, 860, 1910.

[3] *Physical Review*, **22**, 1, 1906.

where a, b, and c are constants, and m takes successive integral values.

EXPERIMENTAL ARRANGEMENTS

Atmospheric nitrogen, free from oxygen, carbon dioxide, and water-vapor, was used as a source. Hence the inert gases of the atmosphere were present, but the only lines due to them which have thus far been noted are a few of the stronger argon lines of the red spectrum. There is no trace of helium λ 5876. Traces of mercury diffused into the spectrum tube from the pressure gauge, but only the three strong lines at λ 5790, λ 5769, and λ 5461 appear, the last enormously overexposed.

The nitrogen was electrically excited in a Goetze "Type C" spectrum tube. The emission from the capillary of such a tube, in a "head-on" direction, appears to be the most intense, per unit cross-section, now obtainable. The electrical excitation was furnished by the secondary of a large induction coil, the primary being run on 110 volts A.C., 1.5 amperes. The nitrogen was introduced at about 5 mm pressure and used until the pressure fell to about 1 mm, low enough to cause a slight diminution of the radiation. Refilling of the tube was necessary only once in 24 to 36 hours.

The tube was placed accurately "head-on" to the slit of the grating, 60 cm away. A double convex lens of 15 cm focus produced on the slit a sharp image of the end of the capillary, somewhat more than 1 mm in diameter. This usual arrangement was now varied by introducing, at a distance of 12 cm from the slit, a double concave cylindric lens of 12 cm focus, placed with its axis horizontal. This caused the circular image on the slit to be drawn out into a vertical line some 2 cm in length. The use of such a cylindric lens in spectrum work has been advocated by Humphreys,[1] but I know of no definite statement of the advantages and disadvantages incident to its use.

The action of the cylindric lens is greatly to reduce the vertical aperture of the cone of rays proceeding from the slit. With the particular lenses used, it is possible, with a source of light less than approximately 2 mm in diameter, to reduce the vertical aperture, at the grating, to less than the length of the grating rulings. Thus

[1] *Astrophysical Journal*, **18**, 324, 1903.

the cross-section of the cone of light at the grating, instead of being a 75-cm circle, is reduced (roughly) to an ellipse of 75 cm horizontal diameter, but with a vertical diameter of 5 cm or less. The gain in intensity of the middle point of the astigmatic image at the camera is theoretically $\frac{75 \text{ cm}}{5 \text{ cm}} = 15$. The actual increase, determined experimentally, was thirteen fold.

If now the source is made 4 mm in diameter, instead of 2, the amount of light actually striking the grating, using the cylindric lens, is scarcely increased at all. But with the ordinary arrangement, the amount would practically be doubled. Hence the advantage of the cylindric lens is proportionally decreased. For sources more than 2 cm in diameter, there is no appreciable advantage in using a cylindric lens.

The chief disadvantage attendant upon its use is the necessity of accurate adjustment. The centers of the tube, convex lens, concave lens, and slit should all lie accurately in the horizontal plane formed by the center of the grating and of the camera. With this condition fulfilled, and the cone of light falling symmetrically upon the grating, a raising or lowering of the cylindric lens of even one-tenth of a millimeter is sufficient to throw an appreciable portion of the light entirely below or above the rulings of the grating.

Because of the excess of radiation in a "head-on" direction, the illumination of the grating is far from uniform; but this is true even when the cylindric lens is not used. Such a non-uniformity is liable, however, to cause a shift of the lines of the comparison spectrum relative to those under investigation. The actual shifts found in many cases, between the iron and nitrogen lines, are believed to be due primarily to this cause.

As a comparison source I used an iron arc of the Pfund[1] type, run on 200 volts, 5 amperes, with iron and carbon electrodes. It worked in a very satisfactory manner. The exposures were made in the second order, and both the second-order and coincident third-order international iron normals were used, the measurements in the ultra-violet being those of Buisson and Fabry,[2] not yet officially adopted as standards.

No relative shift of orders could be detected on those plates where both the second- and third-order normals were present.

[1] *Astrophysical Journal*, **27**, 296, 1908. [2] *Ibid.*, **28**, 169, 1908.

Whenever two normals fell near together and were both of suitable intensity for an accurate setting, the agreement was perfect. When one or both lines were overexposed the disagreement might be anything from 0.007 A down. This was taken to indicate that the secondary international normals, when overexposed, do not necessarily broaden symmetrically. The much greater uniformity in intensity of the normals between λ 3500 and λ 4500 thus makes them preferable for use, and this fact, coupled with the great faintness of the normals from λ 5900 into the red, caused the author to use only the coincident third-order normals in the region λ 5900 to λ 6800.

In order to eliminate the exceedingly strong violet bands of nitrogen, an 8 per cent solution of potassium chromate 5 mm thick was employed. The absorption of this solution sets in at about λ 5200 and this accounts for the rapid decrease in intensity below this point. (See Plate III.) Although the head of the λ 3576 band is a thousand times as intense, photographically, as that of any band under investigation, no trace of it appears on the exposures. Fluorescein was tried as an absorbent and found quite ineffective.

For the exposures from λ 5000 to λ 5900 the Cramer Instantaneous Isochromatic plates were employed, while from λ 5800 to λ 6900 both Cramer "Spectrum" and Wratten & Wainwright "A" Panchromatic were used. For the one exposure on the Hilger spectroscope, from λ 6800 to λ 7700, I used a Wratten & Wainwright "B" Panchromatic plate.

The strongest portion of the spectrum, from the photographic standpoint, is that from λ 5700 to λ 5800. The λ 5804 band is fully three times as intense as that at λ 6023, the only one which von der Helm appears to have obtained sufficiently intense for measurement. The region from λ 5500 to λ 5900 was accordingly photographed first, using $12 \times 1\frac{1}{4}$ inch plates, and the usual Rowland type of comparison shutter. All other exposures were made with $18 \times 2\frac{1}{2}$ inch plates, using a comparison shutter, mounted independent of the camera.

In making exposures several days in length, the greatest problem is a proper control of temperature. Fortunately for the author, the large grating of the University of Wisconsin is mounted inside a double-walled room, built in turn entirely inside an ordinary room.

The temperature in this outer room was kept constant within a few tenths of a degree by suitable electrical heating. This enabled the temperature of the grating to be kept constant within a few hundredths of a degree. The grating temperature was read on an accurate mercury thermometer, mounted in metallic contact with the side of the grating. Other thermometers were laid in a slot in the iron beams forming the slit-grating-camera triangle. A small change of temperature in this triangle is immaterial, so long as all parts remain at an equal temperature.

For the grating, however, a constant temperature is indispensable, the change of wave-length at a given point on the camera plate being proportional, to first-order effects, to the change in the width of the grating space.[1] Holtz[2] seems to question this, and spends some time searching for other causes for the observed shift of lines with temperature. The mounting of the grating at the University of Wisconsin is such as to exclude the chief sources of error which he mentions, and it was found experimentally that the shift was exactly that computed from the change of temperature and the coefficient of expansion of the grating.

A change of 0°01 C. in the grating temperature will shift a line (at λ 5000) about 0.001 A. During the exposures the temperature was never allowed to leave a 0°1 C. range, and during any one exposure the average variation from the mean temperature varied, in different exposures, from 0°015 to 0°035 C. The broadening of the lines was thus always less than 0.01 A.

Not only the temperature, but the barometric pressure as well, causes a shift of the spectrum. A change of 1 mm in pressure will shift the lines 0.002 A. With frequent total pressure variations of 2 cm, sufficient to cause a 0.04 A broadening of the lines, it becomes necessary to eliminate this change also. This was done by arbitrarily changing the temperature. A 1 cm rise of pressure is compensated by a 0°15 lowering of temperature. The mean temperature mentioned above, which I endeavored to hold constant, refers to the initial temperature, properly corrected for subsequent change in barometric pressure.

The time of exposure varied from 66 to 120 hours. The slit-

[1] See Baly, *Spectroscopy*, p. 241, 1912 edition.

[2] *Zeitschr. f. Wiss. Phot.*, **12**, 101, 1913.

width varied from 0.01 to 0.04 mm, being usually 0.02 mm. The theoretical resolving power of the grating (a 6-inch, 14,438-line grating), for the slit-width used, was actually obtained on all exposures except those in the red where, in the second order, the grating has a somewhat poorer definition.

The spectrum was photographed on eight different plates, two for each region. These regions were (1) λ 6900–λ 6300, (2) λ 6400–λ 5800, (3) λ 5900–λ 5500, (4) λ 5600–λ 5000. For regions (1) and (2), one was a Cramer plate, the duplicate a Wratten & Wainwright plate. No plates were exact duplicates, as the slit-width and time of exposure were varied. One 85-minute exposure was made on a Hilger spectroscope, for the region λ 6800–λ 7700. A one-minute exposure is sufficient, on this instrument, for the shorter wave-lengths. The spectroscope was calibrated with the argon spectrum, and the readings obtained for nitrogen are probably correct to 1 A. All of the plates obtained with the large grating are usable save one in the λ 6300–λ 6900 region which dried very unevenly. The duplicate plate, however, is the best that I have, and the readings obtained from it are believed to be as trustworthy as those in any portion of the spectrum.

The work that has thus far been completed is as follows:

1. The lines in the immediate vicinity of the three conspicuous "heads" of each band have been measured, and their wave-lengths computed, on all plates.

2. The regions λ 5500–λ 5900 and λ 6300–λ 6900 have been completely measured and computed.

There are about 6400 lines between λ 5000 and λ 6800, and 274 in the λ 6623 band, in which von der Helm measured 119. There appear to be fully as many in all the other bands, although in most cases the number actually measured is much less, owing to the smaller intensity and shorter length of the bands.

The measurements were made on a 55-cm Geneva dividing engine. The screw was carefully calibrated by the author and is believed to have no unknown errors greater than 0.002 mm. In order to test the evenness of drying of the plates, the international secondary standards were first corrected for non-normality of the dispersion and errors of the screw, and were then fitted as nearly as possible to a linear scale. Only standards of suitable intensity

were used, those overexposed being evidently untrustworthy. In the case of one plate in the λ 6300–λ 6900 region, the average deviation of all the normals from a linear scale was less than 0.002 A. This was taken to indicate that the screw had been correctly calibrated. On other plates there was a general drift from such a linear scale, very evidently due to uneven drying. It seldom exceeded 0.015 A and by drawing a smooth curve through the plotted readings of the normals, the correction for this was easily made.

When the wave-length determinations of one plate were compared with those of a duplicate plate, there generally appeared a constant difference between them. This difference varied from 0.01 A to 0.04 A on different sets of plates. It was considered due to the uneven illumination of the grating, as already explained. Fortunately, however, we have interferometer measurements of the three mercury lines present on my plates. By means of the ghosts and satellites of these lines, it was possible to determine their position with great accuracy, in spite of their overexposure. This settled the absolute wave-lengths from λ 5100 to λ 5900. One plate in each of the other two regions was then found to agree perfectly in the overlapping portions. I thus had a full set of plates in complete agreement, and the duplicate plates were then given the proper constant correction to make them also agree.

The values of the wave-length of any one line, as determined on different plates, then seldom differed by more than 0.01 A. Several settings were made on each line, and as the nitrogen lines are fairly sharp, the average experimental error of setting scarcely exceeds 0.003 A. It is hoped, therefore, that the relative error of all save very faint or hazy lines is less than 0.005 A, and that the absolute wave-lengths are in general correct to 0.01 A.

Table I gives the wave-lengths of 872 lines forming the three principal heads of the bands. The lines in the vicinity of all the heads given by von der Helm were measured, although in several cases there is no real head present. Several other heads not given by von der Helm were noted and measured. These so-called "heads" are caused by the proximity of several heavy lines, accompanied by more or less continuous radiation. The measurements, in all cases, cover this region of continuous radiation, which is indicated in the table by braces. Frequently the haze is due

merely to the scattering of light in the photographic film, but in most cases it is apparently a true radiation.

The three main heads of a band, out of the five that appear with low dispersion, are designated I, II, and IV. The bands themselves are designated in two ways: first, by the division into groups (*a* to *f*), the individual bands of each group, from red to violet, being designated by Arabic numerals; the second method of designation is that proposed by Cuthbertson[1] and formulated mathematically by Deslandres.[2] In this arrangement the position of the first head of each of the entire set of 57 bands is given as a function of two independent parameters, p and n. The value of these parameters, for each band, is given immediately below the designation of the band according to the first arrangement. The first integer refers to the value of p, the second to n—the values being those of Deslandres.[3]

The three columns in the table are:

(1) Intensity; lines marked "?" are so faint as to preclude an accurate determination of wave-length; (2) wave-length—on the International System (I.A.), at 15° C., 760 mm; (3) character of the line. In this regard the following abbreviations are used:

- s., especially sharp.
- b., broad.
- b.d., broad, probably double.
- d., certainly double.
- h., hazy.
- h.r., haze on the red side (due to one or more fainter components on that side).
- h.v., haze on violet side.
- n.s., a non-symmetric line due to two or more components of unequal intensity. The setting was made on the center of gravity of the system.
- k., the line at which a "head" apparently starts.
- a., argon.

Von der Helm's value for the wave-length *in air* for the general position of the head, together with the frequency *in vacuo*, is given to the right of the designation of the head.

[1] *Phil. Mag.* (6), 3, 348, 1902. [2] *Comptes rendus*, 134, 747, 1902.

[3] See Baly, *Spectroscopy*, p. 620, 1912 edition.

TABLE I

1	2	3
I *d* 4 48–53	6787 91 17,728 1	
1	6788 614	k.
1	243	
1	101	
1	87 970	
2	834	
2	712	
1	515	
2	270	
II *d* 4 48–53	6778 35 14,748 9	
1	6779 972	
1	78.821	
1	623	k.
2	448	
2	77.949	
1	538	
1	.288	
2	76 874	
2	.661	
IV *d* 4 48–53	6758 98 14,791 2	
2	6759 243	
2	58 054	k.
1	57 807	
1	665	
2	.355	
1	.067	
1	56 721	
1	611	
1	.325	
4	55 948	
I *d* 5 47–52	6704 45 14,911 4	
3	6704 755	k.
2	.634	
1	514	
3	.363	
3	.132	
3	03 879	h.d.
3	.630	
3	376	
4	227	
II *d* 5 47–52	6694 05 14,932 6	
4	6694 011	k.
2	775	
1	553	
2	301	
3	220	n.s. h.v.
2	93 774	
1	610	b.
1	474	
3	367	
3	242	
3	92 840	
IV *d* 5 47–52	6675 01 14,977 2	
3	6674 968	k.b.
2	634	b.
4	236	
2	074	
2	73 817	
3	615	
1	446	
4	72 954	
4	852	
I *d* 6 46–51	6623 534 15,093 6	
4	6623 574	k.b.
2	417	
2	281	
4	120	b.h.
4	22 915	
2	705	
5	658	
3	305	b.h.
4	130	n.s.
4	21 971	s.
3	838	
II *d* 6 46–51	6614 023 15,115 0	
6	6614 031	k.
2	13 789	h.
4	678	
3	514	
3	6613 188	h.r.
4	061	
2	12 878	h.
2	759	h.
4	524	
4	241	s.
1	001	
3	11 722	
2	603	
IV *d* 6 46–51	6594 425 15,160 1	
5	6594 418	k.b.
4	175	
6	93 739	
2	598	h.v.
5	155	
4	92 568	
6	423	
3	91 036	
2	781	
I *d* 7 45–50	6544 81 15,275 2	
3	6544 881	k.
3	716	
1	598	
5	432	h.
4	237	
2	.095	
6	43 942	
2	714	
2	616	
4	466	
4	251	
II *d* 7 45–50	6535 50 15,296 8	
5	6535 655	s.
6	110	d.
3	34 924	
4	627	
3	482	
4	188	h.r.
3	.028	
4	33 754	h.r. h.v.
1	305	
4	127	b.

TABLE I—*Continued*

1	2	3
IV *d* 7	6516 44	
45–50	15,341 6	
2	6516 010	
4	403	h.r.k.
2	256	h.
2	166	h.
5	15 807	
6	759	
1	586	
2	407	
4	181	
6	14 687	
5	459	
I *d* 8	6468 53	
44–49	15,455 3	
4	6468 597	k.b.
4	.438	h.r.
5	144	h.v.
4	67 951	
2	.802	
6	634	
2	416	
3	280	
3	142	
4	66 913	
2	809	
2	588	
5	442	b.
II *d* 8	6459 04	
44–49	15,478 1	
6	6459 073	
2	181	
5	58 884	k.
2	726	h.
3	503	
2	385	
4	261	
2	128	
2	57 810	
4	669	
2	460	h.d.
3	086	
IV *d* 8	6440 80	
44–49	15,521 8	

1	2	3
2	6441 134	
4	40 768	k.h.d.
3	563	
2	302	
4	150	
2	39 992	
3	599	
4	270	
4	168	
5	38 887	
I *d* 9	6394 45	
43–48	15,634 2	
2	6394 628	k.
3	442	
1	284	
2	122	
3	93 991	h.v.
1	854	
5	636	
II *d* 9	6384 93	
43–48	15,657 6	
2	6386 096	
4	85 978	
2	503	
3	026	k.
3	84 900	
4	627	d.
3	451	b.
2	322	
2	081	
4	83 887	b.
IV *d* 9	6367 55	
43–48	15,700 3	
4	6367 416	h.r.
2	165	
4	66 808	
2	252	
3	107	
3	65 913	
3	564	
I *d* 10	6322 73	
42–47	15,811 6	

1	2	3
2	6322.816	k.h.
?	708	
2	594	
1	462	
1	386	
2	280	
1	162	
2	003	n.s.
4	21 797	n.s.
II *d* 10	6313 20	
42–47	15,835 5	
4	6314 420	
2	13 287	k.
2	185	h.
2	069	
1	12 957	
2	.720	h.
2	626	h.
1	.214	h.
4	11 659	
IV *d* 10	6296 03	
42–47	15,878 8	
3	6296 212	b.h.
1	026	n.s.
?	95 805	
3	606	
3	.077	
I *d* 11	6252 81	
41–46	15,988 4	
1	6253 001	b.k.
2	52 806	
?	670	h.
?	587	h.
1	494	
1	377	
1	226	b.
II *d* 11	6243 51	
41–46	16,012 2	
?	6243 688	
2	581	s.h.v.
2	207	
1	42 944	h.r.
1	.744	

TABLE I—*Continued*

1	2	3
1	6242.601	
?	.402	h.
3	.199	
IV *d* 11 41–46	6227.00 16,054.7	
2	6227.096	h.k.
3	26.078	b.h.
1	.790	h.
2	.416	
1	.079	h.
I *e* 1 49–53	6185.44 16,162.6	
?	6187.022	h.r.
1	86.733	k.
2	.297	b.h.
1	85.874	
1	.570	b.
I *d* 12 40–45		
1	6185.224	
1	.127	
?	84.937	d.
1	.750	b.
1	.516	b.
1	.313	b.
1	.120	s.
1	.012	s.
?	83.861	
II *e* 1 ? 49–53		
2	6178.549	
2	.170	
?	77.574	
?	.385	
2	.259	
?	76.800	
II *d* 12 40–45	6175.32 16,189.1	

1	2	3
2	6176.126	k.
1	75.967	
?	.701	
?	.654	
1	.523	
1	.392	
1	.251	
1	.112	
2	74.874	
3	.612	
IV *e* 1 ? 49–53		
2	6161.648	
1	.276	
1	.131	
?	60.707	
?	.580	
1	.395	b.
IV *d* 12 40–45	6160.43 16,228.2	
1	6159.844	h.k.
2	.692	h.
?	.443	
2	.114	
1	58.952	h.
1	.581	h.
2	.433	
2	57.960	d.
I *e* 2 48–52	6127.23 16,316.1	
2	6127.374	h.v.k.
1	.208	
1	.087	
4	26.044	
1	.752	
2	.471	
II *e* 2 48–52	6119.79 16,336.0	
2	6120.602	
2	.273	h.v.
1	19.840	k.
2	.720	h.v.

1	2	3
3	6119.454	
1	.312	b.
2	.129	b.
1	18.093	
IV *e* 2 48–52	6102.00 16,382.0	
2	6102.736	k.
1	.514	
3	.381	
1	.259	
2	01.537	
I *e* 3 47–51	6069.00 16,471.0	
5	6069.063	b.k.
2	.463	
5	.280	h.v.
3	.174	
2	.034	h.v.
5	68.704	
1	.506	
3	.382	
2	.250	
II *e* 3 47–51	6062.44 16,490.5	
3	6062.563	
2	.414	
2	.100	h.r.
6	61.984	b.
3	.500	h.r.
IV *e* 3 47–51	6045.55 16,536.6	
3	6045.484	k.
3	.407	
6	.173	h.r.
1	44.970	d.
2	.808	
4	.310	h.r. h.v.
I *e* 4 46–50	6013.60 16,624.4	

TABLE I—*Continued*

1	2	3
5	6013.575	k.h.r.
3	.335	
6	195	s.
4	030	
2	12.907	
3	.772	b.
4	567	
II *e* 4 46–50	6006.34 16,644.5	
4	6006.477	
2	341	
5	118	
4	05.967	
3	.834	
4	645	b.
IV *e* 4 46–50	5990.01 16,689.0	
4	5989.812	n.s.k.
5	.636	s.
3	.519	
1	.324	
2	179	
4	88.702	
I *e* 5 45–49	5959.25 16,776.0	
2	5959.220	
5	053	h.v.k.
3	58.707	
5	656	b.
2	548	
3	447	
2	342	
3	206	
II *e* 5 45–49	5952.02 16,796.4	
4	5951.935	h.k.
5	782	
4	617	
3	440	
1	332	
2	115	
2	010	

1	2	3
IV *e* 5 45–49	5935.95 16,841.9	
3	5935.920	
3	740	k.
4	660	h.r.
3	.553	
5	.426	
1	.231	
3	.092	
1	34.925	
3	.774	
5	627	
I *e* 6 44–48	5906.24 16,926.6	
5	5906.010	b.k.
1	05.900	
?	702	
6	672	h.v.b.
2	503	
3	368	
2	285	
2	126	
3	04.948	s.
3	.852	
II *e* 6 44–48	5899.19 16,946.7	
2	5899.070	
5	98.930	n.s.k.
2	631	
2	523	
3	401	b.
?	252	
1	148	
3	040	b.
3	97.874	
IV *e* 6 44–48	5883.49 16,992.0	
2	5883.517	
2	446	
5	140	k.
5	.029	
1	82.917	
3	807	

1	2	3
2	5882.615	h.
3	479	h.
3	320	
6	016	h.v. h.r.
I *e* 7 43–47	5854.69 17,075.6	
5	5854.404	b.d.k.
2	.253	
2	168	
7	.032	b.
3	53.873	
4	.666	h.r.
2	462	
4	286	
2	168	s.
II *e* 7 43–47	5847.67 17,096.1	
?	5847.740	
5	518	s.k.
1	405	
2	300	
2	243	
1	120	
2	040	
6	46.815	b.h.
1	578	h.r.
5	.477	h.r.
4	111	h.
IV *e* 7 43–47	5832.20 17,141.2	
3	5832.054	k.
6	31.881	
1	707	
3	507	
1	405	b.
3	274	
?	110	
2	30.041	
4	830	
I *e* 8 42–46	5804.28 17,224.0	

TABLE I—*Continued*

1	2	3
6	5804 135	b.k.
1	03 978	
8	776	d.
3	590	
1	482	
4	367	
2	.195	
2	.123	
3	02 980	
3	.858	
II *e* 8 42–46	5797 23 17,244 9	
5	5797 451	h.r.
?	.308	
6	.062	
1	96 969	
3	830	
?	707	
5	602	
2	.513	
2	.412	b.
2	.290	
IV *e* 8 42–46	5782 23 17,289 1	
5	5782 307	
8	059	b.k.
1	81 043	
1	849	
5	740	
3	566	
3	.426	
4	239	
6	80 984	
4	.836	
I *e* 9 41–45	5755 20 17,370 8	
2	5755 415	
4	188	h.k.
?	074	
3	54 983	
5	862	
5	746	
4	616	
1	498	
6	366	
2	195	h.v.
II *e* 9 41–45	5748 18 17,392 0	
5	5748 664	
1	289	
5	101	k.
1	.008	
3	47 883	b.
8	650	b.
6	408	s.
2	178	
2	040	
1	46 939	h.v.
8	615	
1	487	
1	250	h.
2	149	
IV *e* 9 41–45	5733 68 17,436 0	
4	5733 830	
4	555	k.
4	450	
1	369	
2	276	
4	159	s.
1	32 992	
3	830	b.
1	703	
9	462	
1	279	
2	31 921	
I *e* 10 40–44	5707 49 17,516 0	
3	5707 580	k ?
2	462	h.
1	301	
2	131	b.
?	06 944	
?	840	
3	597	h.
II *e* 10 40–44	5699 50 17,540 6	
3	5701 358	
2	00 939	
?	883	
2	5700 637	
2	512	
?	5699 830	
IV *e* 10 40–44	5685 56 17,583 7	
?	5685 490	
1	190	
1	058	b.
I *e* 11 39–43	5660 54 17,661 4	
1	5660 842	k.
?	590	
?	421	
1	331	
1	240	
1	115	
2	59 954	b.
II *e* 11 39–43	5653 31 17,683 0	
?	5653 504	s.h.r.
?	170	
?	52 676	
?	51 841	s.
IV *e* 11 39–43	5639 17 17,728 2	
?	5639 349	
?	38 868	
?	37 899	
?	775	
I *f* 1 40–52		
Haze 5632.754 to 5632 361 Very faint lines to 5627 750		
II *f* 1 40–52	5622 97 17,779 3	
?	5622 610	

TABLE I—*Continued*

1	2	3
I *e* 12 38–42	5615 00 17,804 5	
1	5615 318	k.
?	230	
2	032	b.
1	14 900	b.
1	770	
1	695	
1	573	
1	387	d.
?	230	
II *e* 12 38–42	5607.73 17,827 6	
1	5608 214	k.
?	094	
?	07.687	.
?	375	
I *f* 2 48–51	5592 57 17,876 0	
?	5593 014	
3	92 881	h.v.k.
1	657	
4	514	
2	304	h.
2	283	h.
2	130	
1	013	
?	91 888	
?	.768	
1	586	
II *f* 2 48–51		
2	5588 081	k.
?	87 989	
1	883	
2	742	b.n.s.
3	531	h.
3	440	h.
1	190	
2	064	
?	86 620	
2	400	
2	330	
IV *f* 2 48–51		
2	5573 126	s.
2	72 894	
1	.737	
2	551	a.
2	347	h.r.k.
3	.247	
2	.039	
1	71 881	
3	.780	
1	.638	
2	410	s.
2	.320	s.
I *e* 13 37–41	5570 60 17,946 5	
3	5570 777	k.h.
1	679	
4	501	b.h.
2	.354	
2	201	d.
2	013	h.v.
2	69 850	
2	786	
II *e* 13 37–41	5563 48 17,969 4	
2	5563 704	
1	571	
3	244	b.k.
?	124	
1	038	
?	62 937	
?	828	
?	696	
?	612	
1	405	b.h.
I *f* 3 47–50	5553 63 18,001 2	
2	5554 258	
2	130	
2	5553 996	
2	.924	
5	730	b.h.k.
2	600	
2	.491	
6	.362	b.h.v.
4	204	b.h.
3	52 962	
II *f* 3 47–50	5548 40 18,018 2	
2	5548 825	
3	711	
1	600	
1	515	
3	.390	
2	.242	
1	137	h.
IV *f* 3 47–50	5533 46 18,066 9	
5	5533 504	
3	.406	
1	238	
1	164	
1	041	
4	32 955	h.r.
?	824	
?	707	
4	599	h.r.
1	.473	h.
2	349	s.
2	251	s.
2	146	s.
I *e* 14 30–40	5526 84 18,088 6	
2	5527 150	k.h.
4	027	
5	26 835	s.
4	.707	
2	.610	
4	508	
4	350	
3	188	
II *e* 14 30–40	5520 11 18,110 6	

TABLE I—*Continued*

1	2	3
3	5519 682	
5	592	
?	475	
1	.398	
2	.268	
I *f* 4 46–49	5515 54 18,125 6	
1	5515 758	
8	594	k.b.
7	451	b.
5	239	b.h.
5	178	
3	081	
2	14.953	b.
3	777	h.r.
3	636	b.h.
2	496	
2	.325	b.d.
3	190	d.
II *f* 4 46–49	5510 55 18,142 3	
4	5511.096	h.r.
?	10 998	
?	929	
4	835	
4	638	d.h.v.
1	488	b.
5	.278	b.
?	177	
?	109	
?	.044	
1	09 936	
2	847	
3	736	s.
2	545	s.
4	434	
3	316	s.
IV *f* 4 46–49	5495 42 18,192 0	
1	5495 997	
4	.885	a.(?)
2	763	
3	692	s.
3	593	s.
2	430	h.
1	343	h.
4	159	h.r.
I *e* 15 35–39		
5	5484 730	
2	593	
3	488	
5	338	b.k.
3	225	
3	004	
3	018	
3	83 896	
4	686	h.
3	476	s.
I *f* 5 45–48	5478 73 18,247 4	
1	5478 575	
7	471	n.s.k.
3	266	
6	124	b.
4	007	
2	77 912	
2	804	
2	656	
2	601	
II *f* 5 45–48	5473 16 18,266 0	
2	5473 524	k.
2	468	
4	313	
3	.190	h.
4	056	h.
5	72 936	
4	590	h.v.
5	485	
2	316	
1	228	
2	131	
IV *f* 5 45–48	5458 22 18,315 9	
4	5458 870	h.k.
3	759	n.s.
2	630	h.
2	554	h.
2	451	
3	353	s.
3	273	s.
3	130	
2	008	
3	57 903	
2	797	
1	697	
4	587	
I *f* 6 44–47	5442 25 18,369 7	
2	5442 498	
2	416	
6	325	k.
4	276	
4	200	
4	104	
7	41 981	
5	932	
4	833	
3	751	
4	640	s.
II *f* 6 44–47	5437 03 18,387 3	
5	5437 328	k.d.
2	197	s.
2	027	
2	36 957	b.
2	894	b.
3	801	
3	621	
5	518	
3	300	s.
IV *f* 6 44–47	5424 17 81,430 9	
1	5423 423	
2	319	
2	209	

TABLE I—*Continued*

1	2	3
4	5423 014	
4	22 022	
3	771	
2	.088	
6	.535	b.
4	309	
6	187	b.
1	.035	
I *f* 7 43–46	5407 08 18,489.2	
2	5407 575	
3	411	b.h.
6	.129	h.r.k.
1	049	
3	06 979	
3	879	
6	785	
4	730	
3	590	h.v.
2	.528	
II *f* 7 43–46	5401 83 18,507.1	
4	5401 043	b.
3	810	k.
4	689	b.
4	.505	
5	450	
3	340	
2	189	
3	115	
1	024	
2	00 924	
6	654	
IV *f* 7 43–46	5387 82 18,555 3	
2	5388 400	
2	353	
?	230	
?	.172	
5	087	k.
5	002	
4	87 850	
4	707	
5	604	
3	518	
4	447	h.
3	385	h.
3	283	

1	2	3
1	5387 192	h.
2	050	
I *f* 8 42–45	5372 78 18,607 2	
2	5373 189	
2	72 976	
7	820	h.k.
3	.727	
7	496	b.
3	390	
3	223	b.h.
II *f* 8 42–45	5367 41 18,625 8	
4	5367 782	b.
3	654	b.
3	527	k.
3	420	
5	325	
3	236	
2	132	
3	081	
1	66 973	d.
1	853	
2	700	
IV *f* 8 42–45	5353 73 18,673 4	
3	5354 073	k.
3	53 992	
3	872	b.
1	768	
6	608	n.s.
2	493	
I *f* 9 41–44	5339 27 18,724 0	
2	5339 432	h. k.
1	303	
3	204	
1	102	
1	080	
?	38 920	d.
II *f* 9 41–44		

1	2	3
2	5334 318	d.
?	165	
?	026	b.h.
?	33.878	
3	.702	
IV *f* 9 41–44	5323 40 18,779 8	
1	5323 821	
?	523	
?	411	
?	22 983	b.h.
?	.770	b.h.
1	615	n.s.
?	483	b.h.
I *f* 10 40–43	5306.22 18,840 0	
1	5307 072	
1	004	
2	06 859	k.
2	529	
1	146	
2	05 980	
1	.690	
2	413	
II *f* 10 40–43		
2	5302 604	
1	170	
?	610	
?	524	
1	885	d.
I *f* 11 39–42	5274 35 18,953 0	
1	5275 072	k.
?	74 891	
2	777	h.h.v.
?	347	
I *f* 12 38–41	5244 05 19,063 9	
1	5244 071	k.
2	43 782	

TABLE I—*Continued*

1	2	3
I *f* 13 37–40	5213 04 19,177 4	
{2	5213.808	k.
{3	.540	h.v.
1	.021	
II *f* 13 37–40	5207 79 19,196 7	
2	5209 009	s.
2	08 669	b.h.
1	503	
3	300	h.
2	157	
1	07 885	
3	.558	h.r.
1	262	h.
2	06.894	s.
2	800	
4	.024	s.
IV *f* 13 37–40		
{3	5196 370	h.v.
{?	.196	

1	2	3
1	5196 080	
2	95 967	
1	679	
2	443	
2	346	
2	94 854	
I *f* 14 36–39	5183.84 19,285 4	
3	5184 237	b.k.
4	83 970	b.
1	843	
1	734	
1	679	
1	550	
{2	445	
{2	.394	
II *f* 14 36–39		
4	5179 876	n.s.
3	567	
1	466	
2	322	
3	.217	
2	.072	

1	2	3
1	5178 040	
IV *f* 14 36–39		
2	5167 060	k.
2	66 992	
1	872	
1	766	
2	676	s.
1	410	
1	309	
2	181	
2	058	
I *f* 15 35–38		
{2	5155 323	k.
{2	201	
1	005	
I *f* 16 34–37		
{1	5126 806	k.
{?	726	

The following table (Table II) gives the measurements of all conspicuous lines, or groups of lines, in the bands extending from λ 6800 to λ 7650, as taken on the Hilger spectroscope. The probable error is 1 A.

The three main heads are designated as before, using only the first method of grouping. The columns are: (1) wave-length *in air;* (2) frequency *in vacuo;* (3) designation of head.

Table III gives Croze's measurements of the first head of the bands from λ 7600 to λ 9100. The probable error is several angstroms.

DISCUSSION

The discussion naturally falls into three sections: (1) a brief sketch of the two methods previously proposed for grouping the heads of the nitrogen bands; (2) a quantitative test of the comparative validity of the two methods, based upon the data given

TABLE II

1	2	3	1	2	3	1	2	3
7624.8	13,111.5	I *c* 3	7261.3	13,768.1	II *c* 6	7072.8	14,134.8	
7613.5	13,131.2	II *c* 3	7254.3	13,781.3		7050.6	14,161.2	I *c* 8 I *d* 1
7589.4	13,172.9	IV *c* 3	7250.0	13,789.4				
7505.6	13,319.9	I *c* 4	7239.9	13,808.5	IV *c* 6	7048.5	14,183.7	II *c* 8
7492.2	13,343.7	II *c* 4	7233.1	13,821.6		7040.4	14,200.2	
7469.6	13,383.9	IV *c* 4	7228.5	13,830.9		7028.2	14,224.7	IV *c* 8
7445.3	13,427.6		7221.6	13,843.7		7016.5	14,248.3	
7404.8	13,501.0		7214.0	13,858.3		7001.4	14,279.1	
7386.1	13,535.3	I *c* 5	7205.2	13,875.3		6991.5	14,299.3	
7375.6	13,554.5	II *c* 5	7197.3	13,890.5		6978.6	14,325.9	
7366.8	13,570.7		7181.0	13,921.8		6968.0	14,347.6	I *d* 2
7363.0	13,577.7		7165.0	13,953.0	I *c* 7	6956.0	14,372.3	II *d* 2
7352.8	13,596.5	IV *c* 5	7153.0	13,976.2	II *c* 7	6946.0	14,392.8	
7345.6	13,609.9		7145.2	13,992.0		6938.8	14,408.1	IV *d* 2
7338.3	13,623.3		7142.2	13,997.5		6929.5	14,427.2	
7331.0	13,637.0		7132.2	14,017.2	IV *c* 7	6921.6	14,443.8	
7323.4	13,651.3		7125.0	14,031.4		6906.0	14,476.1	
7315.3	13,666.3		7120.0	14,041.2		6896.6	14,497.2	
7307.7	13,680.6		7112.3	14,056.5		6875.5	14,540.5	I *d* 3
7291.2	13,711.5		7099.3	14,082.0		6865.2	14,562.3	II *d* 3
7274.0	13,743.8	I *c* 6	7090.9	14,098.7				

TABLE III

1	2	3	1	2	3
9101	10,985	I *b* 1	8043	12,430	I *b* 7
8903	11,229	I *b* 2	7887	12,676	I *b* 8 I *c* 1
8707	11,482	I *b* 3	7742	12,913	II *c* 2 (?)
8541	11,705	I *b* 4	7628	13,106	I *c* 3
8369	11,946	I *b* 5			
8204	12,186	I *b* 6			

in the preceding tables; (3) a summary of the evidence in favor of each method, based upon (*a*) the appearance of the bands under high dispersion, and ordinary conditions of excitation (work of the author); and (*b*) the appearance of the bands under low dispersion, but unusual conditions of excitation (work of previous investigators).

SECTION I

The nitrogen lines of wave-length longer than λ 5100, comprising the First Deslandres' Group, fall into 57 similar groups of lines, called "bands." Each band contains several sets (usually five) of particularly heavy and close lines. These sets have been called the "heads" of the bands. That set in each band lying farthest to

the red usually ends abruptly on the red side, and has been called head I, the band being said to begin at this point, and to be degraded toward the violet.

In an ordinary band, such as is found in the Second Deslandres' Group of the nitrogen spectrum (λ 5060 to λ 2814) there are series of lines starting at the head, and proceeding with diminishing intensity toward the violet. Near the head, the lines of such a series are so related that successive frequency intervals form an arithmetical series. This is Deslandres' Law for band series. In the First Delandres' Group, however, there appear to be no relationships between the 250 or more lines forming each "band." This is *not*, therefore, an ordinary band spectrum.

Relationships first appear when we group together corresponding lines in successive bands, choosing one line from each band. We might take one line from the first (I) head of the λ 6623 band, another from the first head of the λ 6545 band, etc., and thus form a series. Under low dispersion the set of lines forming a head appears as a single broad line. Thus successive first heads were found to form a series satisfying Deslandres' Law—similarly successive second (II) heads, etc. Such a series extends over 10 to 15 bands, and then the interval between successive terms changes abruptly. Accordingly the ten or more bands represented in such a series have been classified as a "group of bands." The entire First Deslandres' Group is composed of five, and possibly six, such subgroups, which we have designated *a* to *f* respectively. Von der Helm decided that this was the best method for grouping the band heads, and arranged his data in this way. I shall therefore refer to it as the von der Helm arrangement, although it is not original with him.

The second arrangement of the bands was first suggested by Cuthbertson. In this the head of a band in one of the above groups is related, not to the adjacent *band*, but to a band in the adjacent band *group*. In the series thus formed we have only as many terms as we have band groups, and the spacing between terms is much greater than in the von der Helm arrangement. Since a series contains the head of only one band of a group, there are at least as many series as there are bands in a group. It is possible to form

11 such series having at least three terms each, and 6 more having only two terms each.

The reason for such a grouping is that the 17 series thus formed are identical in spacing with one another, and also with the five series into which the bands of the Second Deslandres' Group have been divided. Each series appeared to fulfil Deslandres' Law, and is known as Deslandres' First Progression. Each series, moreover, is displaced relative to the preceding one by a regularly increasing amount. Thus the corresponding terms of the several series form of themselves another set of series, also approximately obeying Deslandres' Law, and known as Deslandres' Second Progression.

Thus the entire set of the first heads of the bands in the First Deslandres' Group can be represented as a function of two parameters, p and n. The variation of n gives the First Progression, that of p the Second. Deslandres considered that both progressions obeyed his law, and wrote the complete formula

$$v = A + B(n+c_1)^2 + C(p+c_2)^2. \qquad (1)$$

In this formula we can make a linear transformation of variables

$$k = \tfrac{1}{2}(p+n) \qquad l = \tfrac{1}{2}(p-n)$$

and obtain $f(k, l)$, also of second degree in each parameter, and so giving the ordinary Deslandres' Law when one parameter alone is varied. In such a $f(k, l)$ successive integral values of k (l remaining constant) give the heads of the successive bands of one group of the von der Helm arrangement. On the other hand, l has different values for successive band groups

Fig. 1 may make this clearer. This figure gives the general position of the first head of every band, plotted with frequency as one co-ordinate and the value of p as the other. Any horizontal succession of heads, for which p=constant, gives Deslandres' First Progression. The value of n for each head is plotted beside it, and any succession of heads for which n=constant gives the Second Progression. The series l=constant indicates one of the band groups of the von der Helm arrangement.

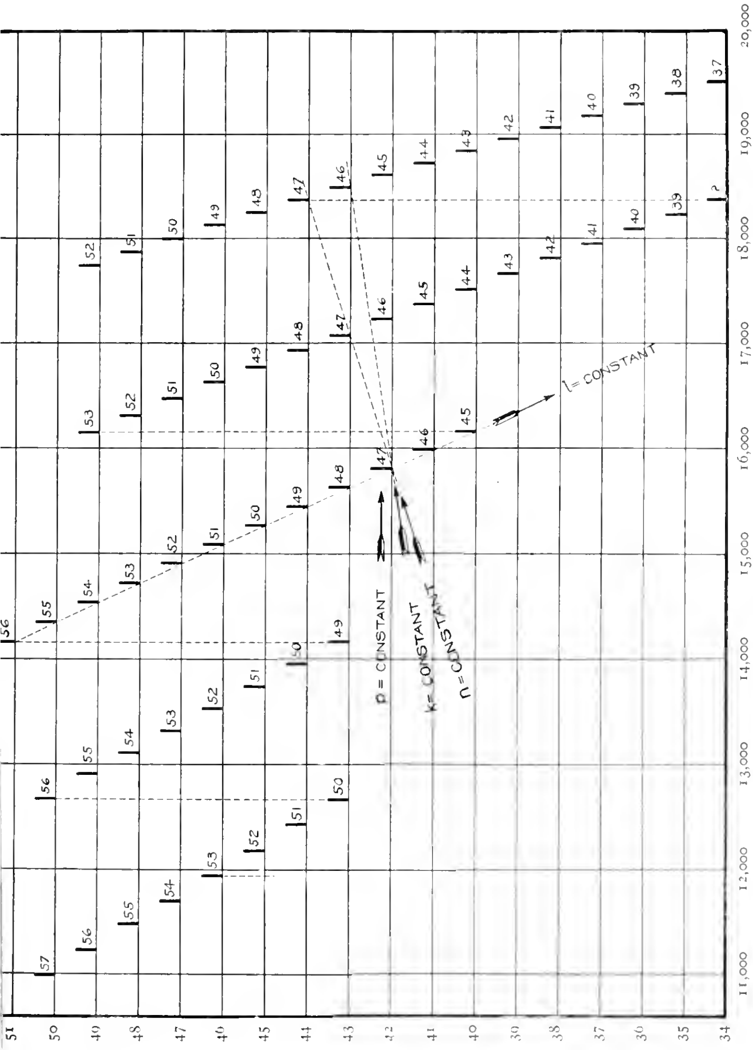

FIG. 1

SECTION II

That portion of the nitrogen spectrum under investigation appears to be formed of two superimposed spectra. One of these consists of lines of regular arrangement, the other of lines arranged irregularly. A graph of the lines of several bands of the *e* group indicates that perhaps 50 out of the 250 lines of each band belong to the regular spectrum. These sets of 50 lines have a similar appearance in each band. It is thus possible to identify corresponding lines in successive bands and to form them into series extending through one band group, and obeying Deslandres' Law as a first approximation. I shall call each of the 50 series thus formed a "simple" series.

The first heads of successive bands are composed mainly of several such series, and the general position of the first heads of successive bands, under low dispersion, forms roughly such a series. Table IV gives the simple series of longest wave-length in each band group. It is therefore composed of the "first" heavy line in each band, in the case of all the bands photographed under high dispersion. For the others the approximate position of the edge of the first head is used, as given in Tables II and III. Deslandres' Law demands that the first frequency differences, given in the fifth column, shall form an arithmetical progression, the second differences (sixth column) being a constant. The probable experimental error, in terms of frequency, varies from 0.04 at λ 5000 to 0.02 at λ 6800. Such an average error in the measurements, however, may cause an average variation four times as large in the second differences given in the last column.

Each series evidently obeys Deslandres' Law for the major portion of its extent, but deviates from this law near the violet end of the group. This is true for series in all band spectra, Deslandres' Law holding only near the head of a series. The only formula holding for an entire series is that of Thiele.[1] It contains eight undetermined coefficients and so is very difficult to work with. I have preferred to use simply Deslandres' Law, or a slight modification of it, and to note whether there was a regular deviation from this law.

[1] *Astrophysical Journal*, **6**, 65, 1897.

TABLE IV

Designation	*p n*	λ (Air)	Frequency (vacuo)	First Difference	Second Difference
f 16	34–37	5126.81	19,499.86		
				107.88	
f 15	35–38	5155.32	19,391.98		0.27
				108.15	
f 14	36–39	5184.24	19,283.83		1.22
				109.37	
f 13	37–40	5213.81	19,174.46		1.27
				110.64	
f 12	38–41	5244.07	19,063.82		1.39
				112.03	
f 11	39–42	5275.07	18,951.79		1.49
				113.52	
f 10	40–43	5306.86	18,838.27		1.32
				114.84	
f 9	41–44	5339.41	18,723.43		1.58
				116.42	
f 8	42–45	5372.82	18,607.01		1.65
				118.07	
f 7	43–46	5407.13	18,488.94		1.50
				119.57	
f 6	44–47	5442.32	18,369.37		1.62
				121.19	
f 5	45–48	5478.47	18,248.18		1.63
				122.82	
f 4	46–49	5515.59	18,125.36		1.64
				124.46	
f 3	47–50	5553.73	18,000.90		1.55
				126.01	
f 2	48–51	5592.88	17,874.89		0.52
				126.53	
f 1	49–52	5632.75	17,748.36		
e 15	35–39	5484.34	18,228.65		
				141.19	
e 14	36–40	5527.15	18,087.46		0.46
				141.65	
e 13	37–41	5570.78	17,945.81		0.69
				142.34	
e 12	38–42	5615.32	17,803.47		0.83
				143.17	
e 11	39–43	5660.84	17,660.30		1.07
				144.24	
e 10	40–44	5707.46	17,516.06		1.02
				145.26	
e 9	41–45	5755.19	17,370.80		1.23
				146.49	
e 8	42–46	5804.13	17,224.31		1.40
				147.89	
e 7	43–47	5854.40	17,076.42		1.32
				149.21	
e 6	44–48	5906.01	16,927.21		1.46
				150.67	

TABLE IV—*Continued*

Designation		p n	λ Air	Frequency (vac.)	First Difference	Second Difference
e	5	45–49	5959.05	16,776.54		1.43
					152.10	
e	4	46–50	6013.57	16,624.44		1.52
					153.62	
e	3	47–51	6069.66	16,470.82		1.51
					155.13	
e	2	48–52	6127.37	16,315.69		1.41
					156.54	
e	1	49–53	6186.73	16,159.15		
d	12	40–45	6185.22	16,163.09		
					175.18	
d	11	41–46	6253.00	15,987.91		1.35
					176.53	
d	10	42–47	6322.82	15,811.38		1.03
					177.56	
d	9	43–48	6394.63	15,633.82		1.21
					178.77	
d	8	44–49	6468.60	15,455.05		1.36
					180.13	
d	7	45–50	6544.88	15,274.92		1.34
					181.47	
d	6	46–51	6623.57	15,093.45		1.28
					182.75	
d	5	47–52	6704.75	14,910.70		1.43
					184.18	
d	4	48–53	6788.61	14,726.52		1.8
					186.0	
d	3	49–54	6875.5	14,540.5		
					192.9	
d	2	50–55	6968.0	14,347.6		
					186.4	
d	1	51–56	7059.6	14,161.2		
c	8	43–49	7059.6	14,161.2		
					208.2	
c	7	44–50	7165.0	13,953.0		
					209.2	
c	6	45–51	7274.0	13,743.8		
					208.5	
c	5	46–52	7386.1	13,535.3		
					215.4	
c	4	47–53	7505.6	13,319.9		
					208.4	
c	3	48–54	7624.8	13,111.5		
					198.5 (?)	
c	2	49–55	7742 (?)	12,913 (?)		
					237 (?)	
c	1	50–56	7887	12,676		
[illegible]	8	43–50	7887	12,676		
					246	

TABLE IV—*Continued*

Designation			λ Air	Frequency	First Difference	Second Difference
		p *n*				
b	7	44–51	8043	12.430		
					244	
b	6	45–52	8204	12.186		
					240	
b	5	46–53	8369	11.946		
					241	
b	4	47–54	8541	11.705		
					223	
b	3	48–55	8707	11.482		
					253	
b	2	49–56	8903	11.229		
					244	
b	1	50–57	9101	10.985		

In groups *d* and *e* it is occasionally doubtful what line forms the beginning of a new band. In the first heads of the *f* group, however, there appears an extremely heavy doublet, the successive pairs of lines having not only the same relative intensity, but also a constant frequency difference. The doublets thus form two simple series, of which that of longer wave-length has been used for the *f* group of Table IV. In *f* 1 only one member of the doublet is present—that of shorter wave-length. Hence it does not fit well with the other lines in Table IV. I give in Table V the simple series formed from the more refrangible member of the doublet.

The first nine terms of this series can be fitted into the ordinary Deslandres' formula

$$\nu = A + B(m + c)^2 \qquad (2)$$

with an average difference between observed and computed values of 0.005 A. For the *less* refrangible member of the doublet the corresponding average difference is 0.006 A, and the constants for this latter series are:

$$A = 22{,}000.627$$
$$B = -\quad 0.8000$$
$$c = +\quad 0.260$$
$$m = 80 \text{ to } 72$$

The beginning of the deviation from Deslandres' Law occurs, in both series, at a point of minimum intensity at *f* 10 (λ 5306).

TABLE V

Designation		λ (Air)	Frequency (vacuo)	First Difference	Second Difference
	p n				
f 1	49–52	5632 754	17,748 361		
				127 701	
f 2	48–51	5592 514	17,876 062		1 669
				126 032	
f 3	47–50	5553 362	18,002 094		1 597
				124 435	
f 4	46–49	5515 239	18,126 529		1 629
				122 806	
f 5	45–48	5478 124	18,249 335		1 607
				121 199	
f 6	44–47	5441 981	18,370 534		1 615
				119 584	
f 7	43–46	5406 785	18,490 118		1 571
				118 013	
f 8	42–45	5372 496	18,608 131		1 584
				116 429	
f 9	41–44	5339 089	18,724 560		1 544
				114 885	
f 10	40–43	5306 529	18,839 445		1 479
				113 406	
f 11	39–42	5274 777	18,952 851		1 379
				112 027	
f 12	38–41	5243 782	19,064 878		1.448
				110 579	
f 13	37–40	5213 540	19,175 457		1 197
				109 382	
f 14	36–39	5183 070	19,284 839		1 364
				108 018	
f 15	35–38	5155 095	19,392 857		

At this same point the frequency difference of the doublet also begins to diminish. For these two reasons it appears that the f group consists really of two groups, having a point of coincidence at λ 5306. Table VI gives the frequency difference of the doublets for the entire f group.

I have thus far been unable to find any other strong series lying within the heads of the f group. In the d and e groups, however, there are at least 15 series, distributed among the three heads. In most of these the second difference remains approximately constant for six or eight terms; in a few it forms more nearly an arithmetical progression, the third difference being constant. Such a relation can be satisfied by adding one more term to Deslandres' Law, so that it reads:

$$v = A + B(m+c)^2 + C(m+c)^3 . \qquad (3)$$

TABLE VI

	Designation						
	f 2	f 3	f 4	f 5	f 6	f 7	f 8
Difference (in $\frac{10^8}{\lambda}$)	1 173	1 193	1 167	1 157	1 161	1 176	1 123

	Designation						
	f 9	f 10	f 11	f 12	f 13	f 14	f 15
Difference	1 127	1 175	1 064	1 059	1 034	1 006	0 876

In the fifteen series the average difference of experimental and calculated values is slightly more than 0.01 A. In some cases it is over 0.02 A and evidently exceeds the experimental error of measurement. The lines forming the doublets in *f* are very difficult to measure correctly, because of their great intensity, and the nearness of adjacent lines. Yet they fit into series better than any other set of lines. Hence the deviations from formulae (3) or (2), in the case of other series are real, and not due to experimental errors.

The spacing arrangement in different series varies slightly, so that series often tend to cross one another, and this gives successive heads an entirely different appearance. This can best be shown by the five series in heads IV *d*. These five series include nearly two-thirds of all the lines present in these heads, and, with two exceptions, every strong line. Series δ and ϵ start from the same line and gradually diverge. Series δ, at the fifth term, breaks into a doublet, the components of which in turn diverge. The middle of the doublet is used for the last two terms. Such a sudden splitting of a line into a doublet is common in the series found in band spectra, and there are numerous examples of it in the spectrum under investigation. The five series are given in Table VII.

The foregoing portion of Section II has been concerned simply with the law followed by individual simple series, each being considered entirely independently. There are also relationships

TABLE VII

IV *d* α

λ (Air)	Frequency (vacuo)	First Difference	Second Difference
6758.054	14,793.111		
		184.265	
6674.908	14,977.376		1.460
		182.805	
6594.418	15,160.181		1.312
		181.493	
6516.403	15,341.674		1.340
		180.153	
6440.768	15,521.827		1.349
		178.804	
6367.416	15,700.631		1.251
		177.553	
6296.212	15,878.184		1.020
		176.533	
6226.078	16,054.717		1.165
		175.368	
6159.692	16,230.085		

IV *d* β

λ (Air)	Frequency (vacuo)	First Difference	Second Difference
6757.355	14,794.642		
		184.242	
6674.236	14,978.884		1.383
		182.859	
6593.730	15,161.743		1.411
		181.448	
6515.759	15,343.191		1.322
		180.126	
6440.150	15,523.317		1.311
		178.815	
6366.808	15,702.132		1.234
		177.581	
6295.606	15,879.713		1.127
		176.454	
6226.416	16,056.167		1.012
		175.442	
6159.114	16,231.609		

IV *d* γ

λ (Air)	Frequency (vacuo)	First Difference	Second Difference
6756.666	14,796.151		
		184.127	
6673.615	14,980.278		1.320
		182.807	
6593.155	15,163.085		1.340
		181.467	
6515.181	15,344.552		1.352
		180.115	
6439.500	15,524.667		1.272
		178.843	
6366.252	15,703.510		1.306
		177.537	
6295.077	15,881.047		

TABLE VII—*Continued*

IV d δ

λ (Air)	Frequency (*vacuo*)	First Difference	Second Difference
6755.948	14,797.724		
		184.040	
6672.954	14,981.764		1.368
		182.672	
6592.568	15,164.436		1.393
		181.279	
6514.687	15,345.715		1.435
		179.844	
6439.220	15,525.559		1.304
		178.540	
6366.010	15,704.099		

IV d ϵ

λ (Air)	Frequency (*vacuo*)	First Difference	Second Difference
6755.948	14,797.724		
		184.268	
6672.852	14,981.992		1.491
		182.777	
6592.423	15,164.769		1.293
		181.484	
6514.459	15,346.253		1.374
		180.110	
6438.887	15,526.363		1.273
		178.837	
6365.564	15,705.200		

between the spacing arrangement of simple series in different band groups. This can best be studied from the standpoint of the Cuthbertson arrangement.

In a two-parameter formula such as (1) there may be included one line from each band in the entire spectrum. It therefore comprises several simple series. The entire set of simple series, one for each band group, satisfying separately and collectively such a two-parameter formula I call a "complete" series. When the lines of any complete series are regrouped to form the p and n progressions, it appears that formula (1) is not the correct functional form. Table VIII shows this clearly. In this table I give only the average frequency intervals of the two progressions, using the data given in Table IV.

TABLE VIII

First Progression p = Constant			Second Progression n = Constant		
n	First Frequency Difference	Second Frequency Difference	p	First Frequency Difference	Second Frequency Difference
54			49		
	1615.0			1432.3	
53		25.6	48		27.3
	1589.4			1405.0	
52		29.9	47		28.0
	1559.5			1377.0	
51		28.9	46		27.8
	1530.6			1349.2	
50		29.0	45		27.8
	1501.6			1321.4	
49		29.6	44		28.0
	1472.0			1293.4	
48		29.6	43		28.6
	1442.4			1264.8	
47		29.7	42		28.5
	1412.7			1236.3	
46		29.9	41		28.9
	1382.8			1207.4	
45		30.2	40		29.4
	1352.6			1178.0	
44		30.3	39		29.7
	1322.3			1148.3	
43		30.8	38		30.3
	1291.5			1118.0	
42		31.2	37		31.0
	1260.3			1087.0	
41		31.6	36		31.0
	1228.7			1056.0	
40		32.4	35		33.4
	1196.3			1022.6	
39		31.3	34		
	1165.0				
38		34.5			
	1130.5				
37					

The second difference is an approximate arithmetical progression and requires a function of the type given in formula (3). Instead of formula (1) we must therefore use:

$$v = A + B(n+c_1)^2 + r(n+c_1)^3 + C(p+c_2)^2 + s(p+c_2)^3 . \qquad (4)$$

Since the variation of both n and p has the same functional form, it follows that the variation of both together, such as we find in a simple series, has also this same form. For that reason it is possible to combine two simple series in order to determine the constants of a complete series. The two conditions imposed upon such a pair of

simple series are: (1) each simple series must fit formula (3); (2) both simple series must have the same third difference.

In formula (3) this third difference equals $6C$; in (4) it is $6(r+s)$. It is therefore the same for both simple series. When the constants of a complete series are thus determined, all other simple series included in the complete series have definite predicted positions.

If we now choose the simple series given in Table IV, using only the band groups for which we have accurate measurements (groups *f*, *e*, and part of *d*), it appears that all three simple series satisfy condition (1), but no two of them satisfy condition (2). It is therefore impossible to group them together into a complete series satisfying formula (4), and so the first lines of the first heads of all bands do *not* satisfy the Cuthbertson arrangement. Another way of stating this is that the several First Progressions are not identical with one another. This was evident in compiling Table VIII. There are eight intervals in this table whose values can each be derived from two different First Progressions (and similarly for the Second Progressions), using only accurate data. For these eight intervals the average difference of the two values is 0.2 A, more than ten times the experimental error.

In the I heads of the *d* group there are three heavy lines in all. The two of shorter wave-length form a doublet of the same constant frequency difference as that in the *f* group. This suggested the combination of these two series of doublets into two complete series, which should differ from one another only by a constant value. It appears that the two simple series formed from the doublets in the *d* group are compatible with those of the *f* group, and so this rearrangement into complete series is possible.

The simple series in the *d* group, of shorter wave-length, is given in Table IX.

Using the two simple series given in Tables V and IX, we get the following constants for the complete series. The derivation is rather laborious, and the computations were not made by a strictly least-squares method:

$A = 22,108.476$	$r = +0.0245$
$B = -\ \ 18.0562$	$s = -\ \ .0254$
$C = +\ \ 17.2474$	$c_1 = +\ \ .3365$
	$c_2 = +\ \ .7222$

For p and n the derived values are respectively three and four units lower than Deslandres' values, which I have consistently used in designating the bands. This shows not only that the values of c_1 and c_2 (which define the "phase" of a series) are meaningless without more accurate data, but also that no deductions can be drawn from the exact value of $p-n$ for any band group.

TABLE IX

Designation	p n	λ (Air)	Frequency (vacuo)	First Difference	Second Difference
d 4	48–53	6787.712	14,728.479		
				184.174	
d 5	47–52	6703.879	14,912.653		1.291
				182.883	
d 6	46–51	6622.658	15,095.536		1.306
				181.577	
d 7	45–50	6543.942	15,277.113		1.335
				180.242	
d 8	44–49	6467.634	15,457.355		1.352
				178.890	
d 9	43–48	6393.636	15,636.245		1.210
				177.680	
d 10	42–47	6321.707	15,813.925		

For eight terms from the f group, and seven from the d group, the average difference (obs.—calc.) is 0.005 A. For the less refrangible member of the doublet we have

$$A = 22{,}107.315$$

The other constants remain the same. For 14 terms the average difference (obs.—calc.) is 0.01 A.

By means of the constants given above we can obtain the theoretical position of corresponding simple series in all other band groups. From the position of the component simple series, in the heads of the d and f groups, we should expect the predicted series in the b and c groups to lie just to the violet of the rough measurements of the first heads in those groups. This is found to be the case, within the limits of experimental error. In the e group, however, where we have accurate data, there is no series in the predicted position. All series in I e have a slightly different spacing arrangement, and one of them gradually crosses the predicted series.

Thus the only Cuthbertson arrangement I have been able to get is between alternate rather than adjacent groups. As already pointed out, $k\ \{=\frac{1}{2}(p+n)\}$ and $l\ \{=\frac{1}{2}(p-n)\}$ are the parameters in the von der Helm arrangement, corresponding to p and n in the Cuthbertson arrangement. In this latter arrangement the first heads of all the bands are represented by integral values of p and n. In the von der Helm arrangement integral values of k give a simple series. If, however, we keep k constant, and give l successive integral values, we get corresponding first heads only in every alternate band group. (See series k=constant on Fig. 1.) For the intermediate groups l has the value of an integer plus one-half, and cannot be satisfied by integral values of p and n. Therefore we might expect to find related simple series only in every alternate group. I have at present no other numerical evidence either for or against this view.

The previous discussion shows that many more lines can be fitted into series on the von der Helm arrangement than on the Cuthbertson. This naturally follows from the fact that each simple series involves only one parameter, while the Cuthbertson arrangement involves two. The individual series in different band groups should have related spacing arrangements, given implicitly by formula (4). The data show, however, that the relation is in general not accurate within the limits of experimental error.

One further point of interest is the continuity of successive band groups. The heads of the last band of one group practically coincide with those of the first band of the succeeding group. In this connection the band at λ 6186 is the most interesting in the entire spectrum. In this band we have at 6186.7 a head which agrees in its general position and appearance with the designation I *e* 1; similarly at 6185.2 a head I *d* 12. The entire appearance of the band is that of a *d* band, and it is doubtful whether the *e* group is represented save by I *e* 1, although I have recorded in Table I the lines in the vicinity of the theoretical position of II *e* 1 and IV *e* 1.

In the case of the *e* and *f* groups, the theoretical position of I *e* 16 is 5443.3, almost coinciding with the strong I *f* 6 head at 5442.3. Deslandres records I *e* 16 but there seem to be no lines

at this point resembling an *e* head. Again, however, the rough theoretical positions of these two heads almost coincide. For the other groups the coincidences are at 7059.6 and 7887. The data are so inaccurate here that the positions will fit equally well in either of the adjacent band groups. Considering that we have at least approximate coincidences at the four points mentioned above, several interesting relations follow.

The values of p and n at these points are:

	p	n		p	n		p	n		p	n
1	43	50	2	43	49	3	40	45	4	34	38
	50	56		51	56		49	53		44	47

Since the coincidence is between two heads of different band groups, the two values of $p-n$ at each point differ by unity. Two other unexpected facts, however, are: (1) that the discontinuity in p increases by unity at each succeeding point of coincidence; and (2) that the number of bands between points of coincidence increases by four, from group to group. This is also shown in Fig. 1. The coincident points are indicated by vertical dotted lines. The length of these lines gives the discontinuity in p. The number of bands between them is seen to increase by four, as one goes from red to violet. In group *c* there are six, in *d* ten, and in *e* fourteen bands.

If this rule were followed farther to the red we should expect only two bands in *b*, between the coincident bands 46–53 and 43–50. This would be the last group, the next one, by rule, having zero length. On the violet side we should expect nine more bands (including the coincident ones) in the *f* group, 25–28 coinciding with 36–38 of an unknown *g* group, and so on. In the next (*h*) group the last of the 26 predicted bands would have $p=-1$, $n=0$. Since the correct value of $p-n$ for any band group is indeterminate to at least one integer, it seems natural to suppose that all values of p should be raised by one integer.

We should then have a complete plan for the First Deslandres' Group. It would start, theoretically, at $p=0$, $n=0$, and would consist of seven groups of bands. The first head of some band near the end of each group would coincide approximately with the first head of a band in the next group. The number of bands

between coincidences would diminish by four, in each succeeding group. Some groups run past the points of coincidence and so overlap on each other.

Although groups *g* and *h* do not appear in the ordinary spectrum, Goldstein[1] believes he has seen the First Deslandres' Group, under certain low-temperature conditions, extending into the blue. Other investigators have been unable to verify this. Group *h* should start at λ 4430 and extend to λ 4530. Group *g* should extend from this latter point to λ 4890, and *f* from λ 4890 to λ 5442.8.

SECTION III

Under high dispersion successive bands have a very similar appearance, and this not only suggested to the author the formation of simple series, but also indicates the validity of the von der Helm arrangement of the bands. The general intensity of successive bands also varies continuously through a group of bands. All simple series were formed from lines of the same general appearance, and of a continuously varying intensity. The large number of possible series with approximately the same spacing is also good evidence of a connection between successive lines.

A few bands in group *c* have been measured and plotted beneath one another. It is these bands (λ 5900–λ 5700) that indicate the existence of some 50 simple series of lines, superimposed upon a much larger number of unrelated lines. Below λ 5700 all of the *c* series die out, save only those in the first heads. In the *d* group, however, the series extend to the last regular *d* band at λ 6185, and perhaps farther. In the case of the *f* group there seem to be no conspicuous series save the two mentioned in the first heads. This portion is the most irregular of the entire spectrum.

The only exceptions to the general rise and fall of intensity in the bands of one group are two very strong heads at λ 7072.8 and λ 6968.0. The latter lies at the predicted position of I *d* 2. The former lies somewhat to the red of I *c* 8. There is no apparent reason why either one should be strong.

On the other hand, all changes in appearance of the bands under changing physical conditions point to the Cuthbertson

[1] Goldstein, *Phys. Zeitschr.*, **6**, 14, 1905.

arrangement as the one indicating the actual physical connection between the sources of the radiation. Fowler[1] has shown that the spectrum of the active modification of nitrogen shows certain of the bands of the First Deslandres' Group greatly intensified, while the others are very faint or entirely lacking. The three strongest bands are those at λ 6253, λ 5804, and λ 5407 ($n=46$; $p=41$, 42, and 43), while the weaker bands on each side are those at λ 6323, λ 5854, λ 5442 ($n=47$; $p=42$, 43, 44) and at λ 6185, λ 5755, and λ 5373 ($n=45$, $p=40$, 41, and 42). Fowler has pointed out this evidence in favor of the Cuthbertson arrangement.

The fact that apparently the entire band is increased in intensity may point to a further relation, not included in the Cuthbertson. It would be very interesting to photograph this spectrum under high dispersion and to note whether *all* the lines of a band were intensified, or only those belonging in series.

Angerer[2] has made an exhaustive study of the First Deslandres' Group at low temperature. I have made no critical study of his results, and cannot well do so until I have my own measurements completed. Several points, however, are worth noting.

At low temperature the heads of a band are far more intense, relative to the rest of the band, than at ordinary temperature. This is especially true of the III heads which, at high temperature, escape detection in many bands—not having been measured at all by von der Helm. But they are particularly strong at low temperature. This would point to an independence between the series lying within the heads of the bands, and other series.

At low temperature the entire spectrum is relatively much fainter than at room temperature. Aside from two small groups of lines in the green, the only exceptions to this statement are the first heads of the three bands λ 6623, λ 6070, and λ 5593 ($n=51$; $p=46$, 47, and 48). The second of these is even more intense at low temperature, while the other two are fully as intense. Here again we have evidence in favor of the Cuthbertson arrangement.

There is one additional fact pointing to a general relationship between the heads of all the bands. The frequency difference of

[1] Fowler, *Proc. Roy. Soc.*, **85** A, 377, 1911.

[2] *Ann. d. Phys.*, **32**, 549, 1910.

the rough position of the I and IV heads is a constant for all bands from λ 5100 to λ 9100, although the length of the bands more than doubles within this range. The maximum variation of the difference is 7 units (from $v=68$ to 61). The frequency difference of the I and II heads, except in the *f* group, is also practically constant. I cannot recall having previously seen this fact explicitly stated.

This relation of the heads is what we should expect if the bands were composed of a number of identical series of lines. It seems evident that all possible series have *very closely* the same spacing, but it is also certain that the spacing is *not* identical.

Sections II and III may be summarized in the statement that numerical relationships among the lines of the First Deslandres' Group favor the von der Helm method of grouping, while changes in the bands under varying physical conditions of the source all point to the Cuthbertson method as the significant one.

CONCLUSIONS

1. The First Deslandres' Group of the positive band spectrum of nitrogen consists really of two spectra, one composed of a large number of superimposed series of lines, the other quite irregular.

2. The similarity in the spacing of all series gives the banded appearance of the spectrum, the length of a band being the distance between two successive lines of each series.

3. The so-called "heads" of the bands are formed by groups of particularly heavy lines, accompanied by more or less continuous radiation.

4. It is possible to fit a greater number of lines into the simple series of the von der Helm arrangement of bands than into the more complex two-parameter formula indicated by the Cuthbertson arrangement. All physical changes in the spectrum, however, favor the latter arrangement.

5. Simple series of lines, running through one band group of the von der Helm arrangement, obey Deslandres' Law for at least the first few bands, but later show a large and systematic deviation from it.

6. The First and Second Progressions of the Cuthbertson arrangement fit approximately into a formula containing both

the second and third powers of the parameter, but will *not* fit the simpler second-power formula of Deslandres' Law.

7. The successive band groups have certain heads which approximately coincide, and these points of coincidence show regularities which enable the entire set of bands of the First Deslandres' Group to be arranged so as to indicate a definite plan for the group.

The experimental part of the investigation is the resolving, for the first time, of the 39 bands between λ 5000 and λ 6800 into about 6400 lines, and the measurement of a portion of these lines with an average error of 0.01 A or less.

In conclusion the author wishes to express his thanks to Professor C. E. Mendenhall for the many helpful suggestions offered during the progress of this investigation.

Department of Physics
University of Wisconsin
August 1913

NOTE ON THE RELATIVE INTENSITY AT DIFFERENT WAVE-LENGTHS OF THE SPECTRA OF SOME STARS HAVING LARGE AND SMALL PROPER MOTIONS[1]

By WALTER S. ADAMS

One of the methods proposed by Professor Kapteyn for the investigation of the question of the absorption of light in space is the comparison of the intensity at several different wave-lengths of the spectra of stars which are known to be near the earth with such as are very distant. The effect of general absorption or scattering, so far as is known, always increases toward shorter wave-lengths, and if such absorption is present in space we should expect the spectrum of the more distant star to fall off more rapidly in intensity toward the violet end of the spectrum than would the spectrum of the nearer star.

It is clear that for the purpose of making such a comparison a marked advantage will be gained if the spectra of the two stars are obtained upon the same plate, since the two spectra are then treated precisely alike as regards development, and some of the difficulties connected with the possible effect of difference of development upon different portions of the spectrum are eliminated. Accordingly we have adopted this procedure in the case of a number of photographs obtained for this purpose with the Cassegrain spectrograph. In a few cases the spectra of the two stars to be compared are photographed side by side through the star window in the occulting bar, a comparison spectrum being added for convenience in the determination of wave-lengths. More recently, however, we have used no comparison spectrum and have photographed but one star through the central window. Two exposures of somewhat different lengths are made upon the second star, one through each portion of the comparison spectrum window. The spectra extend from

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 78.

about λ 4000 to λ 5000, and in securing the photographs the attempt has been made to obtain approximate equality of density for the two spectra at the less refrangible end.

Two conditions are essential to make a comparison of this sort of value: (1) The spectral types of the two stars must be practically identical. (2) The photographs must be taken with the stars at very nearly the same zenith distance, and with no appreciable variation in the transparency of the sky.

The investigation at this observatory of the radial velocities of a large number of stars with measured parallaxes and large proper motions has provided ample material for the selection of suitable spectra among the nearer stars. For the more distant stars it is necessary to depend upon proper motions alone, and the spectra of a considerable number of stars of small proper motion distant about 90° from the apex of the sun's motion, originally obtained for radial velocity determinations, are available for comparison with the parallax stars. The two spectra are compared side by side under a Hartmann spectro-comparator, and are rejected unless the agreement is essentially complete, line for line. Anyone who has classified stellar spectra is familiar with the rapid change in intensity at the violet end of the spectrum with even a comparatively small change of type.

Although the total number of photographs so far obtained is small, the results are of sufficient interest to deserve a few words of comment. Out of 20 pairs of stars investigated, two pairs are of type B8, one A0, one F4, one F7, two G5, two G6, one G8, seven K0, one K2, one K4, and one K6. Of these the pairs of stars of types B8, A0, and F4 show no appreciable relative difference between the two ends of the spectrum, and the same is true of one pair of type G6 and one of type K0. The remaining fourteen pairs all show a marked difference, which in some cases is very great. In every case the star which is relatively faint at the violet end of the spectrum is the star of small proper motion. The spectra of five of these fourteen pairs is shown in Plate IV, enlarged about eight times from the original negatives. The sections *a* and *c* show the two star spectra side by side with a comparison spectrum above and below. The other three sections show one star in the center and

two spectra of the second star, one on either side. The data for these stars are as follows:

		Star	Mag	Spectrum	μ	π
a.	Above	*Boss* 3542	5.7	F7	0.″009	
	Below	*Brad.* 1433	5.9	F7	0.19	+0.″11
b. . .	Outside	*Boss* 434	6.0	K0	0.11	
	Inside	*54 Piscium*	6.1	K0	0.59	+0.15
c.	Outside	*Boss* 430	6.1	K0	0.005	
	Inside	*Brad.* 3212	6.2	K0	0.42	+0.15
d	Outside	*Brad.* 3077	5.6	K4	2.10	+0.16
	Inside	*Boss* 6123	5.8	K4	0.020	
e	Above	*Boss* 3922	5.8	G5	0.027	
	Below	*Boss* 4032	4.8	G5	0.364	

The use of proper motion as a measure of distance, however valuable in the case of the average of a large number of stars, is of course by no means conclusive in the case of an individual star, and this consideration may well apply to some of the stars in the list. For the six pairs of stars which show no marked effect in their spectra only a single parallax determination is available, the relative distances of the remaining pairs being based upon proper motions alone. For three of these, those of types B8 and A0, the proper motion of the star supposed to be the nearer amounts to only three times that of the more distant star, and the results for these pairs accordingly are of little weight.

The evidence of this small amount of material is much too slight to warrant any extended discussion of its application to the problem of the absorption of light in space. The points of interest are: first, that two stars having the same type of spectrum may differ very greatly in the relative intensity of different portions of their continuous spectra; second, that in no case is the more distant star relatively stronger in the violet portion of the spectrum, but in a considerable majority of cases is weaker. It is clear, moreover, that the results so far found may be explained equally well as an effect of absolute brightness as of absorption of light in space. Since the stars compared are of nearly the same apparent magnitude

the more distant star must be intrinsically much the brighter, and probably the more massive as well, since the spectra, and hence no doubt the physical conditions in the stellar atmospheres, are very similar. If the atmosphere of the more massive star absorbs more strongly than that of the smaller star, which seems fairly probable from physical considerations, this would account for the differences observed. In order to separate the effect of absolute magnitude from that of absorption of light in space, Professor Kapteyn has prepared an observing-list of pairs of stars of the same type of spectrum but of greatly different absolute magnitude, which are at the same distance from the earth, as in the case of physically connected double stars, or for which the parallaxes are approximately equal. These stars would be affected identically by absorption of light in space, and any observed differences in relative intensity in different parts of their spectra would necessarily be due to the effect of absolute brightness. The results obtained from a very few photographs of such pairs of stars as yet show nothing conclusive, although they point to no very definite effect due to this source. As an illustration, a photograph of the spectra of the stars *α Aurigae* and *Groombridge 884* shows little or no difference in the relative intensity of the violet and red ends of the spectrum. According to the measured parallaxes these stars differ by 7.6 magnitudes in absolute brightness, and with every allowance made for uncertainties in the values of the parallaxes the difference must still be very great.

It is evident even from the results given in this brief communication that the method of comparing the actual spectra of stars has several advantages over most other methods when the stars are sufficiently bright to enable its use. The fact that stars of identical spectra may be selected, and that the comparison may be made over a wide range of wave-length, is of especial value where the difference to be investigated is small.

MOUNT WILSON SOLAR OBSERVATORY
December 15, 1913

SECONDARY STANDARDS OF WAVE-LENGTH, INTERNATIONAL SYSTEM, IN THE ARC SPECTRUM OF IRON ADOPTED BY THE SOLAR UNION, 1913

By H. KAYSER, J. S. AMES, H. BUISSON, F. PASCHEN

In continuation of the list of international standards of wave-length taken from the iron arc the lines contained in the accompanying list were adopted by the International Union for Co-operation in Solar Research, held in Bonn, August 1, 1913:

The following other propositions of the committee have been adopted by the Union: as some lines of the iron arc taken in atmospheric air are of poor quality, the committee recommends for the determination of tertiary standards the following conditions of the arc: (1) The length of the arc should be 6 mm. (2) For wave-lengths greater than 4000 A the current should be 6 amperes, and 4 amperes or less for the shorter wave-lengths. (3) A continuous current should be used with 220 volts, with iron rods 7 mm in diameter as electrodes. The arc should be vertical, the positive electrode being the upper one. (4) The middle part of the arc in its axis for a length of 2 mm should be used. (5) Only the lines belonging to the groups *a*, *b*, *c*, *d* of the Mount Wilson classification of the iron lines should be used, at least for that part of the spectrum for which this classification exists.

On Mount Wilson a slit perpendicular to the axis of the arc has given as good results as a slit parallel to the arc.

When the committee in its last report recommended that laboratories and observatories possessing concave gratings of first quality should be invited to measure tertiary standards from the iron arc, it was not the intention to exclude the use of plane gratings. On the contrary, the committee wishes to secure the collaboration of all those possessing plane or concave gratings or prisms with sufficient dispersion and resolution.

The committee recommends that many more secondary standards should be determined than are given in the published tables, in order to make the measurements by interpolation more easy and

more exact. It would be a great advantage to have some hundred secondary standards instead of a few.

International Standards	Fabry and Buisson	Pfund	Burns	Eversheim
6750.163	...	...	.164	162
6678.004	...	004	.000	.008
6592.928	...	.928	925	931
6546.250	...	.251	247	.252
5709.396	.396	396	.395	...
4233.615	.615	.615	...	616
4191.443	441	445	...	444
4147.676	.677	.677	...	.674
4134.685	.685	...	...	.685
4118.552	.552	.551	...	552
4076.642	.641	644	...	.642
4021.872	.872	871	...	872
3977.746	745	745	...	.747
3935.818	818	...	...	.818
3907.937	938	...	...	936
3906.482	481	.483	...	...
3865.527	526	...	...	.528
3850.820	820	...	...	.820
3843.261	261	261	...	...
3805.346	346	346	...	347
3753.615	615	616	...	614
3724.380	379	380	...	.380
3677.629	628	630	...	.630
3676.313	312		...	314
3640.392	391	392	.	392
3606.682	681	....	.	683
3556.881	879	883	.	882
3513.821	820	821		.822
3485.345	344	346		346
3445.154	155	154		154
3399.337	337	338		337
3370.789	789	788		791

NICKEL-LINES TAKEN BETWEEN RODS OF NICKEL

5892.882	.882	882	.	.881
5857.759	760	757	.	759

STUDIES OF THE NOCTURNAL RADIATION TO SPACE

SECOND PAPER

By ANDERS ÅNGSTRÖM

I. RADIATION TO DIFFERENT PARTS OF THE SKY

In a previous paper[1] an account was given of some measurements showing the influence of water-vapor on atmospheric radiation. In the historical survey of that paper, I referred to the important investigations of Homén and mentioned his measurements of the nocturnal radiation to different parts of the sky. As Homén's measurements afterward have been employed in extending observations of the radiation toward a limited part of the sky to the whole sky and as the question itself seems to be of interest for the knowledge of the atmospheric radiation in its dependence on other conditions, it was found valuable to investigate in what degree this distribution of radiation over the sky is subjected to variations. For this purpose the arrangement shown schematically in Fig. 1 seems to be a natural one.

To the Ångström nocturnal compensation instrument can be attached a half-spherical screen, *abcdef*, whose radius is 7.1 cm. From this screen can be removed a spherical cap *cd*, which leaves a hole of 32° solid angle open to the sky. The screen is brightly polished on the outside, but on the inside blackened in order to avoid multiple reflections.

The instrument with this screen attached to it was pointed to different parts of the sky, and the zenith angle was read on a circular scale as is shown in Fig. 1. The value of the radiation within the solid angle *csd* (32°) was obtained in the usual way by determining the compensation current through the black strip.

This arrangement has two obvious advantages over a bolometer arranged in a similar way. In the first place the instrument is very steady and quite independent of air currents, because both strips here are exposed in exactly the same way. The sensitiveness of

[1] *Astrophysical Journal*, **37**, 305, 1913.

the instrument is further quite independent of the position of the strips, it being possible to turn the instrument over at different angles, without change in the sensitiveness. Everyone who is familiar with bolometric work knows the difficulty that sometimes arises from the fact that the sensitiveness of the bolometer changes with its position, the conductivity of heat from the strips through the air being different for vertical and for horizontal positions.

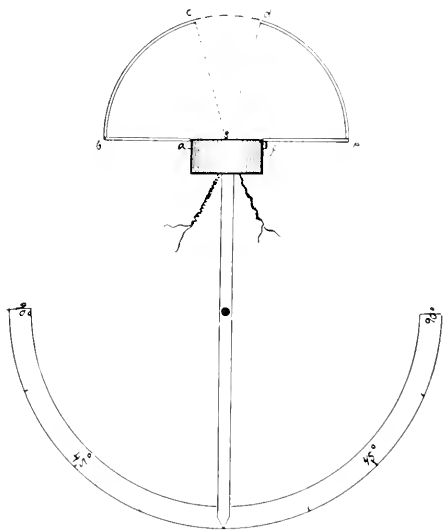

FIG. 1

In order to diminish the error that may arise from the fact that different parts of the strips radiate in somewhat different directions and under slightly different solid angles, the instrument was always turned over so that the strips were parallel to the earth's surface. In such a case, we may regard the above-mentioned influence of the dimensions of the strips as negligible, the strips being small in comparison with the radius of the hemispherical screen.

The results of these measurements for three different nights are contained in Table I. In order to obtain from these values a

TABLE I

Date	Humidity	Total Rad	0°	½ 45°	45°	1½ 45°	Temp	Curve
1912								
23:8	3.84	0.192	0.0173	0.0167	0.0164	0.0120	20.8	I
30:8	7.10	0.157	0.0158	0.0147	0.0127	0.0062	20.3	III
4:9	4.98	0.169	0.0168	0.0157	0.0140	0.0086	11.1	II
20:8	13.24	0.145	0.0153	0.0151	0.0126	0.0041*	18.9	—

* Doubtful

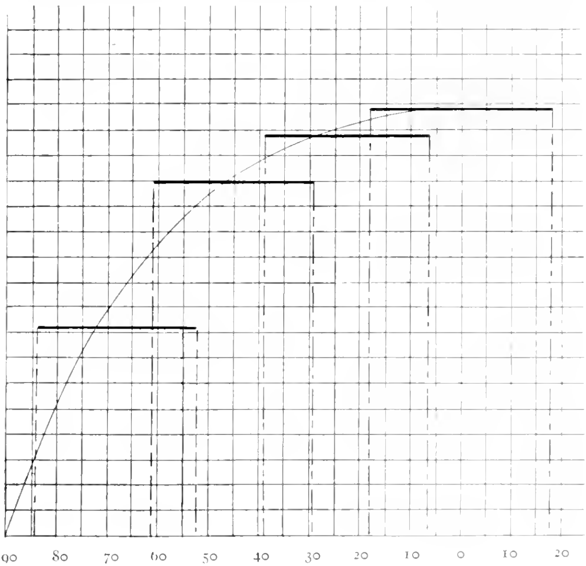

FIG. 2

more detailed idea about the effective radiation to different parts of the sky, I proceeded in the following way. In a system of co-ordinates, where the zenith angle is plotted along the x-axis,

the magnitude of the radiation along the y-axis, every measurement with the instrument corresponds evidently to an integral extending over 32° and limited by the x-axis and a certain curve, that is, the distribution curve of radiation. If the measurements are plotted as rectangular surfaces, whose widths are 32 and

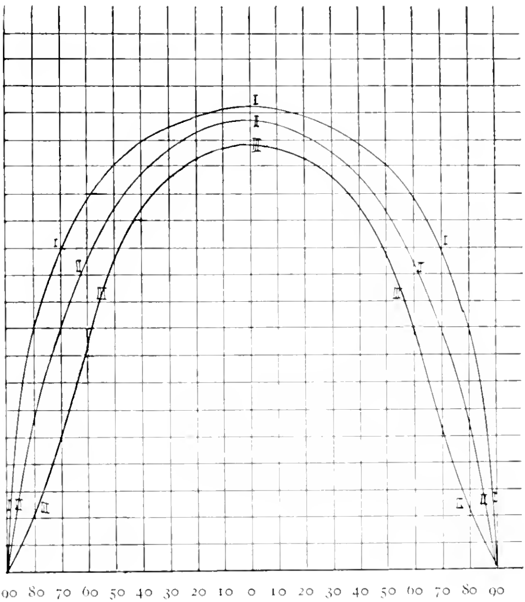

FIG. 3

whose heights are proportional to the magnitude of the radiation, we shall obtain from the observations a system of rectangles like those in Fig. 2. A curve drawn so that the integrals between the limits corresponding to the sides of the rectangles are equal to the areas of these rectangles will evidently be a curve representing the radiation as a function of the zenith angle.

Fig. 3 shows these curves for three different occasions, corresponding to different values of the total radiation and to a different pressure of the atmospheric water-vapor.

If we consider these curves in connection with the corresponding water-vapor pressure, we shall arrive at the following conclusions:

1. *An increase in the water-vapor pressure will cause a decrease in the effective radiation to every point of the sky.*

2. *This decrease is much larger for large zenith angles than for small ones.*

I have shown in my first paper on this subject how the total effective radiation can be written as a function of the water-vapor pressure at the earth's surface. An increase in the water-vapor pressure produces a decrease in the effective nocturnal radiation according to a logarithmic law.

The observations described in the present paper show in what way this decrease is produced. The nearer the observation approaches the horizon, the more effectively is the radiation influenced by the density of the radiating atmosphere. When the zenith angle approaches a value of 90° the effective radiation approaches zero in a way that indicates that the atmosphere for large zenith angles radiates almost like a black body.

If we regard the atmosphere as a plane parallel layer, having uniform density, ρ, and a temperature uniformly equal to the temperature of the earth's surface, the radiation of a certain wavelength in different directions, may be expressed by

$$I_\lambda = Ce^{-\gamma \cdot \frac{\rho}{\cos \phi}} \tag{1}$$

where C and γ are constants and ϕ is the zenith angle. For another density, ρ^1, of the radiating atmosphere we have:

$$I'_\lambda = Ce^{-\gamma \frac{\rho^1}{\cos \phi}} \tag{2}$$

and from (1) and (2)

$$\frac{I_\lambda}{I'_\lambda} = e^{-\gamma \left[\frac{\rho - \rho^1}{\cos \phi} \right]} \tag{3}$$

If ρ is bigger than ρ^1, I_λ will always be less than I'_λ. It is easy to see from the relation (3) that the ratio between I_λ and I'_λ diminishes as the zenith angle approaches 90°. If we have to deal with

the total radiation of many different wave-lengths, the equations (1) and (2) must be written as sums of several terms differing from another in respect to the constants C and γ. The expression for the ratio $\frac{I_\lambda}{I'_\lambda}$ will be more complicated, and it is only under special conditions that the ratio diminishes with an increase in the zenith angle. The observations show that these conditions are here fulfilled.

The curves in Fig. 3 represent the effective radiation within the unit of solid angle in different directions from a surface perpendicular to the radiated beam. From these curves we can compute the radiation from a horizontal surface like the earth's surface, to the different zones of the sky. We have therefore to multiply

TABLE II

Observer	0°–22°30′	22°30′–45°	45°–67°30′	67°30′–90°
Homén	1.00	0.93	0.87	0.61
Ångström I	1.00	0.94	0.86	0.60
" II	0.99	0.92	0.75	0.41
" III	0.97	0.91	0.65	0.23

every single value of $\sin\phi \cdot \cos\phi$ by a constant, whose value for the present does not interest us. In such a way the curves of Fig. 4 are obtained. Fig. 4 includes a dotted curve whose ordinates are everywhere proportional to a constant multiplied by $\sin\phi \cdot \cos\phi$. From the data of Fig. 4, Table II is calculated, which also gives a comparison between the values here obtained and the values found by Homén. In Table II are given the ratios between the values governed by the observations and the values obtained from the simple sin-cos law, that is, for a case where a horizontal surface radiates directly to a non-absorbing space, the radiation assumed to be *one* for zenith angle 0°.

All the observations just described were made at Bassour, Algeria, at a height of 1160 m above sea-level. The sky was clear and appeared perfectly uniform.

II. THE DIFFUSE RADIATION FROM THE SKY DURING THE DAYTIME

In the daytime, the radiation exchange between the sky and the earth is complicated by the diffuse sky radiation of short wave-

length that comes in addition to the temperature radiation of the sky. If this diffuse radiation is stronger than the effective temperature radiation to the sky, a black body like our instrument will receive heat. In the opposite case it will lose heat by radiation.

If one attempts to measure this positive (from sky to earth) or negative radiation with the Ångström nocturnal compensation

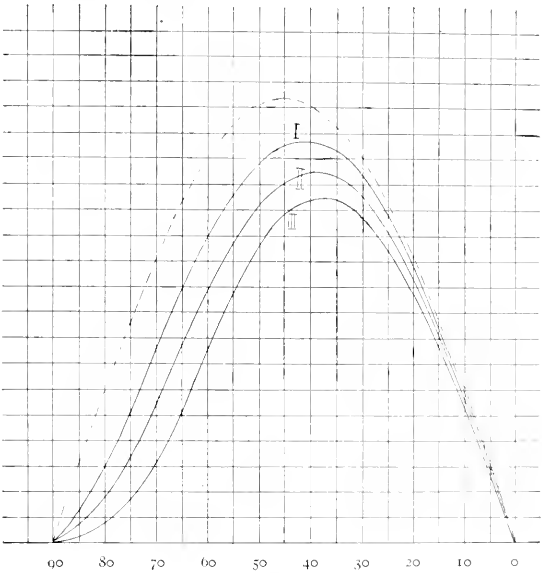

FIG. 4

instrument, the sun being carefully screened off, such an attempt meets with a systematic difficulty. The bright metal strip has namely a less reflecting power for the diffused radiation of short wave-length than for the longer heat waves and we can no longer apply the instrumental constant k, which only holds for long waves such as we have to deal with in the measurements of the nocturnal radiation. The reflecting power of the strips being about 92 per

cent for waves longer than 2μ, and only about 60 per cent for waves of 0.5μ length (a mean value of the wave-length of the diffused sky radiation), an application of the constant k to daylight measurements, will evidently give a value of the sky radiation that is about 30–35 per cent too low.

On several occasions during the summer of 1912, I had the opportunity to make sky-light measurements with the Ångström instrument and with an instrument founded on the same principle, but modified for the purpose of measuring diffuse sky radiation. This latter instrument is briefly described by Abbot and Fowle[1] in their interesting paper, "Volcanoes and Climate," where the effect of the diffusing power of the atmosphere on the climate is fully discussed. Both the strips are in this instrument blackened.

TABLE III

RADIATION OF THE SKY

	Sept 5	Sept 6	Sept 7	Mean
Before sunrise	−0.169	−0.205	−0.208	−0.194
Noon	+0.062	+0.092	+0.047	+0.067
After sunset	−0.208	−0.225	−0.220	−0.218
Total sky radiation	+0.250	+0.307	+0.261	+0.273

Instead of being side by side, the strips are here placed one above the other beneath a thin horizontal plate of brass. In the use of the instrument, a blackened screen was placed beneath it so that the lower strip was exchanging radiation only with this screen, which subtended a hemisphere. The upper strip was exchanging radiation with the whole sky. The radiation was computed from the current necessary to heat the upper strip to the same temperature as the lower one.

Even in the use of this instrument in its original form, it is difficult to avoid a systematic error. This error arises from the difficulty of protecting the screen, with which the lower strip exchanges radiation, from absorbing a small portion of the incoming radiation and in this way giving rise to a heating of the lower strip. I have reasons, however, to believe that the error arising from this

[1] *Smithsonian Miscellaneous Collections*, 60, 29, 1913.

cause is not larger than 10 to 15 per cent. As well in this instrument as in the original Ångström nocturnal instrument, the error, when we attempt to measure the sky radiation during the day, is in the direction to make this radiation appear weaker than it really is.

Table III gives some results of the measurements with the last named instrument. These measurements were made by C. G. Abbot with the author's assistance and are published in the paper "Volcanoes and Climate," already referred to. With Mr. Abbot's permission they are published here also. My measurements of the nocturnal radiation during preceding and following nights are given in the same place. The total diffuse sky radiation is calculated on the assumption that the effective temperature radiation during the daytime is a mean of the morning and evening values determined by the nocturnal apparatus. The sky was perfectly uniform during the observations but was overdrawn by a weak yellow-tinted haze, by Abbot ascribed to the eruption of Mount Katmai in Alaska. The energy of the direct solar beam at noon was for all three days 1.24–1.25 cal. From Table IV may be seen that, under the conditions described, there was always an income of radiation from the sky, indicating that the diffuse radiation from the sky was always stronger than the outgoing effective temperature radiation.

The measurements with the Ångström nocturnal instrument on two different occasions showed in one case no appreciable radiation in any direction and in the other case a weak positive radiation from the sky. If in order to correct for the reflecting power of the strips for short waves we multiply the corresponding nocturnal temperature radiation (0.17–0.20) by 1.5, we shall obtain a result that is in pretty close agreement with the determinations with the modified instrument.

The result is different from that obtained by Homén. Homén draws from his observations at Lojosee in Finland the conclusion that there is an effective radiation from earth to sky even in the daytime. Our result is consistent, however, with Lo Surdo's measurements at Neapel in 1909, where he found a positive incoming radiation from the sky in the daytime.

The surplus of the scattered sky radiation over the effective temperature radiation to the sky, is, however, not so great but that one may very well imagine cases where the direction of heat transfer may be reversed. Under conditions where the temperature radiation to the sky is extraordinarily large, that is, when the air is unusually dry, and at the same time the scattering power of the atmosphere is low, it may very well be possible that the outgoing temperature radiation becomes the larger.

The measurements described were made under conditions where the temperature radiation must have been more than usually strong, even if the scattering power of the atmosphere at the same time was somewhat greater than one generally finds at this place and in this climate (Algeria). All taken into account, I am inclined to think that the heat transfer in the middle of the day generally goes in the direction from sky to earth and that the observations of Homén were made under conditions that must be regarded as exceptional.

NOTE ON INSTRUMENTS AND METHOD OF OBSERVATION

The observations of the nocturnal radiation, *described in this and a previous paper*, were carried out with two Ångström compensation instruments No. 17 and No. 18, made by G. Rose, Upsala. The constants of these instruments, determined at the Physical Institute of the University of Upsala, were respectively 10.4 and 11.1. The readings of the instruments were compared on numerous occasions. The values of the radiation obtained with the two instruments never differed by more than $2\frac{1}{2}$ per cent. All the values given in the tables are generally means of three independent readings. *Special attention was always given to see that the instrument was at constant temperature, before the strips were exposed to the sky.* If the instrument is removed in the free air, it requires between 10 and 15 minutes before the state of equilibrium is reached.

The current used for compensation of the heat lost by radiation (0.10–0.15 amp.) was measured by a Siemens and Halske milliammeter.

The water-vapor pressure was computed from readings of wet and dry thermometers, graduated in Fahrenheit degrees and made by H. T. Green, Brooklyn (Slingpsychrometer 150*a*).

CORNELL UNIVERSITY
June 1913

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME XXXIX — MARCH 1914 — NUMBER 2

PRELIMINARY RESULTS OF MEASUREMENTS OF THE RIGIDITY OF THE EARTH

By A. A. MICHELSON

Installation and observations by H. G. GALE assisted by HAROLD ALDEN. Calculations by W. L. HART under the direction of F. R. MOULTON

Measurements of the temperature of the crust of the earth as found in the deepest mines show that it rises on the average about 0.02° C. for every meter below the surface; and as the rate of increase is even greater at greater depths it follows, as Lord Kelvin pointed out,[1] that the temperature in the interior must be high enough to melt the substances composing it, and for a long time it was considered that the vast bulk of the earth must be in a fluid, or at least semi-fluid, condition, a conclusion which was strongly supported by the fact of the ejection of molten lava from volcanoes.

The theoretical investigations of Lord Kelvin in 1863[2] indicated, however, that the earth must be considered a very rigid body, opposing an enormous resistance to changes of form such as tend to occur in consequence of the attractions of the sun and moon. It is evident from these investigations that the old idea (which is not entirely extinct) that we are living on a thin rock crust over an immense mass of molten lava must be abandoned.

[1] *Philosophical Transactions*, 1863. [2] *Ibid.*

The first attempt to measure the rigidity of the earth, that is, the resistance which it offers to change of shape, was made by G. H. and Horace Darwin in 1880.[1] They employed a horizontal pendulum by the use of which they hoped to measure the change in the direction of the gravitational vertical due to the attractions of the sun and moon; from which, by comparison with the values calculated on the basis of absolute rigidity, the effective rigidity could be determined. The results were so irregular and contradictory that no conclusion could be formed, and the Darwins express the belief that such experiments are not likely ever to furnish the desired results.

In the hands of Rebeur-Paschwitz this method did, however, give positive results confirming the deductions of Kelvin. Since then the method of the horizontal pendulum has been successfully employed by Ehlert, Kortazzi, Schweydar, Hecker, and Orloff, with essentially the same result, namely, that the coefficient of rigidity is found to be of the order 6×10^{11} c.g.s. (about the same as that of steel). The results deduced from Chandler's observations of the variation of the latitude give a value nearly twice as great.

But, in addition to the elastic yielding of any body ordinarily looked upon as solid there is a plastic yielding, characterized by a constant termed by Maxwell the "modulus of relaxation," and evidenced by a lag of the distortion of the earth relative to the forces producing it. Such experiments as these should be capable of determining the plasticity as well as the rigidity of the earth. To show what measure of reliability may be accorded to the observations mentioned, the following extract, Table I, is made of a discussion of these by Schweydar.[2]

Here $1-K$ represents the ratio of the observed amplitude of oscillation to that calculated on the assumption of absolute rigidity. κ represents the retardation (which should always be negative) of the phase of the observed motion relative to the phase of the disturbing forces.

[1] *B. A. A. S. Reports*, York meeting, 1880.

[2] Dr. Wilhelm Schweydar, *Untersuchungen uber die Gezeiten der Festen Erde*, Potsdam, 1912; Leipzig: B. G. Teubner.

The observations are divided into two classes, the latter being considerably more reliable than the former.

TABLE I

Experimenter		K	κ
v. Rebeur		0.362	$+ 7°30'$
Kortazzi		.608	0
"		.414	$- 2°54'$
Ehlert		.448	$+12° 1'$
Schweydar		.338	$- 8°31'$
"		.158	$+ 8°20'$
N.-S.	Hecker	.643	$-11°.9$
	"	.565	$- 0°.7$
	"	.544	$+13°.4$
	Orloff	.412	$+ 0°.8$
E.-W.	Hecker	.259	$- 7°.0$
	"	.382	$+ 5°.2$
	"	.468	$+20°.1$
	Orloff	0.326	$+ 3°.2$

While the numbers in the second column agree in showing that the earth's rigidity is of the order of that of steel, the differences are so considerable that it is hardly likely that they can be relied upon to within 20 per cent.

If accurate, the third column would give a measure of the plastic yielding of the body of the earth to the action of the distorting forces of the sun and moon. As mentioned before, these should all be negative, whereas the great preponderance is in the direction of positive lag, which is meaningless; so that further than showing that the lag is small (and therefore the viscosity high) these results are practically valueless.

It will be conceded that there is great need for more accurate determinations, if our knowledge of the properties of the matter constituting the earth's interior is to be increased; and it was in the hope of obtaining results of a higher order of accuracy, as well as such directness and simplicity of apparatus as practically to eliminate all the difficulties and uncertainties which seem to be unavoidable in the use of the horizontal pendulum, that the experiments recorded here were undertaken.

The prime object of the investigation is the determination of the direction of the gravitational vertical, or rather of the changes

to which it is subject in consequence of the attraction of the sun and moon and as modified by the resulting distortion of the body of the earth. But this may be furnished with any desired degree of accuracy by the changes in the position of the level of a liquid surface which is necessarily normal to the resultant of all the forces acting. If from these we can eliminate all but the gravitational forces the problem is solved.

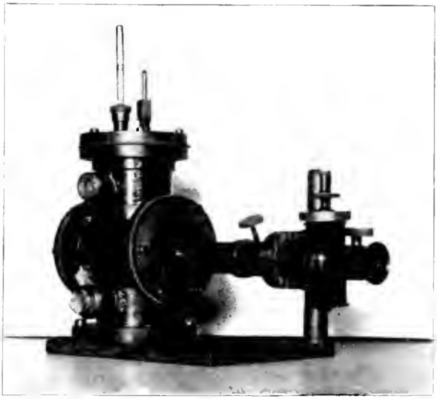

FIG. 1.—Microscope and gauge

A very sensitive method of measuring the changes in level is furnished by the interferometer; and a method of carrying this into practice was devised and the apparatus constructed[1] in 1910.

But before attempting to utilize so delicate an appliance it was deemed advisable to make these preliminary experiments with the microscope. Accordingly a 6-inch pipe, 500 feet long, was half filled with water,[2] the level of which could be read off through the glass sides of the end vessels, as shown in Fig. 1.

[1] An interference apparatus for this purpose was independently devised by Professor A. G. Webster.

[2] The vessels at the ends were at first connected by a pipe filled with water, but with this arrangement temperature changes produced enormous disturbances in the level.

This pipe was laid in a trench six feet deep, terminating in two pits, eight feet square and ten feet deep, walled with concrete, in a soil of sandy clay on the grounds of the Yerkes Observatory at Williams Bay, Wisconsin. The direction (E.–W.) of the pipe was laid out by measuring from the meridian line and is probably correct to within one foot in five hundred. The pipe was then carefully leveled, and after verifying the continuity of the water and

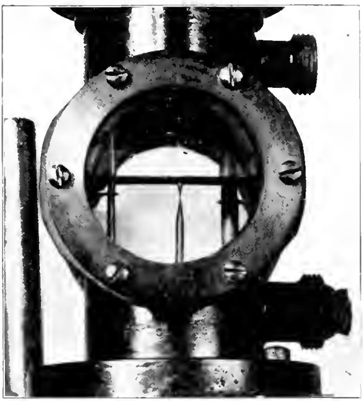

FIG. 2.—Point and totally reflected image

of the air space above it, the pipe was closed so that it was effectively air-tight, and the trench was filled with clay.

The distance between the total reflection image of a pointer, Fig. 2, and the direct image was read by micrometer microscopes of about 2-inch focus. These were calibrated at first rather roughly by measuring the diameter of a wire immersed in the water at the focus; and subsequently with much greater accuracy by measuring the known distance between two lines ruled on glass and placed in the focus, under water.

A preliminary series of observations was begun August 5 and continued until September 2 with such encouraging results that it

was decided to supplement the work by observing on a N.–S. line which was accordingly laid out in the same manner as the E.–W. line.

The final calibration of the microscopes gave the following results:

West	17.52 turns	=	1 mm
East	17.60 "	=	1 mm
North	17.00 "	=	1 mm
South	17.60 "	=	1 mm

The mean for the E.–W. line is 17.56, and for the N.–S. line, 17.30. Accordingly the factor by which the calculated difference in level in mm should be multiplied to reduce to micrometer divisions is

For the E.–W. line, 285
For the N.–S. line, 289

The factor actually used in the computation was 293, and accordingly these should be diminished by 2.8 per cent for the E.–W. line, and by 1.4 per cent for the N. S. line.

A continuous series of observations was conducted on both lines, beginning September 27 and ending November 29. The observations were made by setting the cross-hair of the micrometer on the pointer and taking a number of readings, the mean of which gave the fiducial reading to be subtracted from the readings of the reflected image. This difference was subtracted from a similar difference taken at the other end of the pipe, and the final differences from hour to hour (plus a constant) gave the observed curves. These readings were taken every hour from 6 A.M. until midnight, and every two hours from midnight until 6 A.M. The order of the observations was usually south end, north end, east end, west end. The time between readings at south end and north end, or between east end and west end, was about four minutes, and the mean between the two was taken to represent the time of observation. The mean time of the two observations gives the same result as though the observations had been simultaneous.

Occasionally the reflected image would be obscured by floating particles, and in clearing these away the value of the constant would be altered. The new constant was accordingly chosen so that the succeeding observations continued with the smallest discontinuity in the curve.

It was found that there was a gradual downward trend in the E.-W. curve during August, but little or none in the October-November series. The N.-S. line, however, on which readings were started the day after the pipe was covered, shows a considerable gradual drop. This was undoubtedly caused by uneven settling at the ends of the pipes. This was too slow to affect the result for the semi-diurnal period, but made doubtful the results for the fortnightly period.

The observations and their graphs are given in Tables III and IV and in Figs. 3 and 4.

In Figs. 5 to 12 these graphs are reproduced on a larger scale, together with those of the calculated values of the readings, multiplied by the factor 0.7 for the E.-W. line and 0.5 for the N.-S. line. The zero line of the observed readings is corrected to coincide with the calculated curve.

These curves were divided into periods corresponding to the semi-diurnal lunar tide, 12.42 hours, and the values at corresponding intervals of two hours tabulated and divided into groups of ten periods each.

The results of the comparison of the observed and calculated curves is given in Table II, in which the first column gives the ratio of the observed amplitudes to the calculated, on the assumption of an absolutely rigid earth, and the second the retardation of phase in hours.

TABLE II

Ratio of Amplitudes		Retardation in Phase	
E.-W.	N.-S.	E.-W.	N.-S.
0.69	0.50	-0^h05	-0^h06
79	54	+ 30	+ 30
65	53	− 14	− 08
71	50	− 01	− 03
82	53	− 08	+ 12
64	52	− 14	− 20
66	50	− 08	00
70	50	− 03	00
70	50	− 12	− 06
74	50	− 10	− 02
70	48	− 12	+ 08
0.71	0.52	−0.08	+0.04
Mean=0.709×1.028	0.510×1.014	−0.050	+0.0075

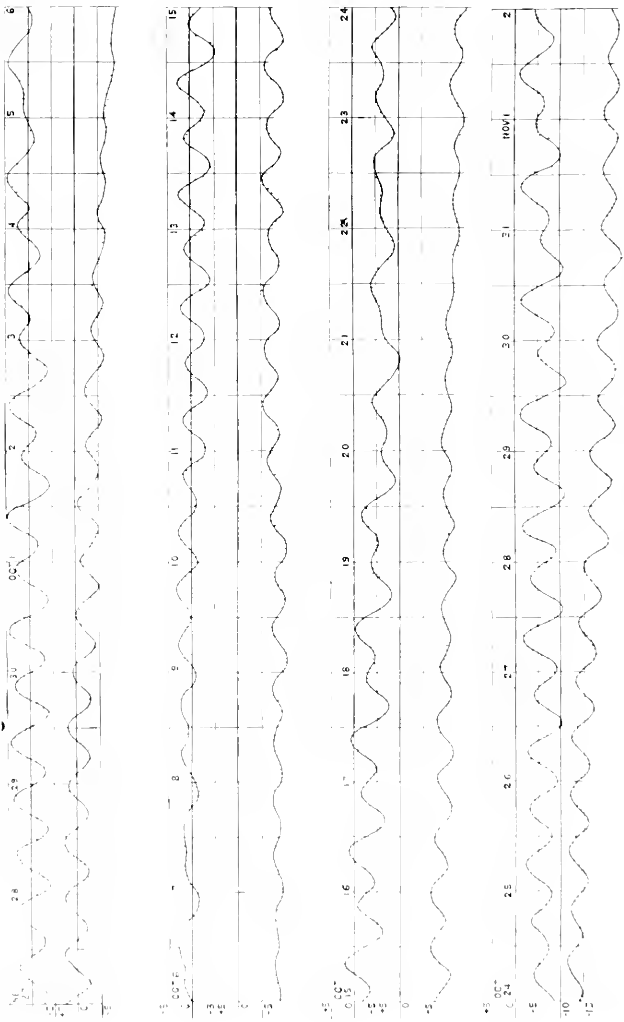

FIG. 3.—Upper curves, 10(E.—W.+C); lower curves, 10(S.—N.+C)

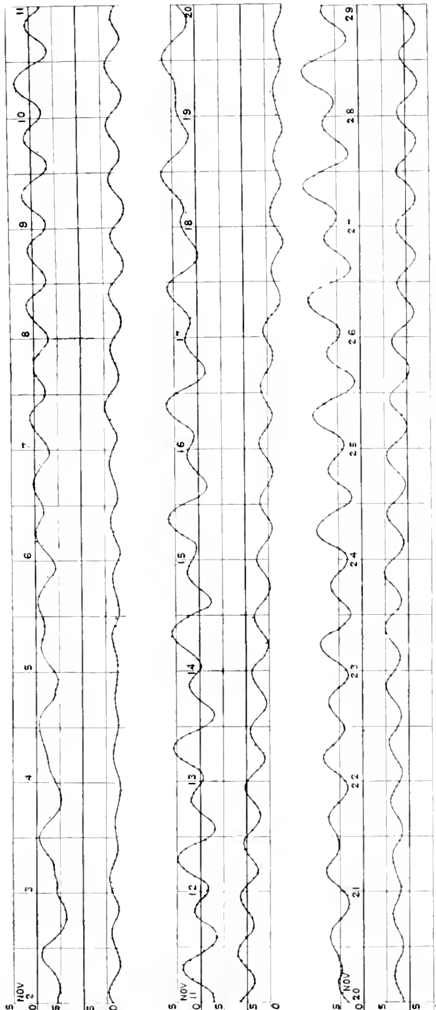

FIG. 4.—Upper curves, 10(E.—W.+C); lower curves, 10(S.—N.+C)

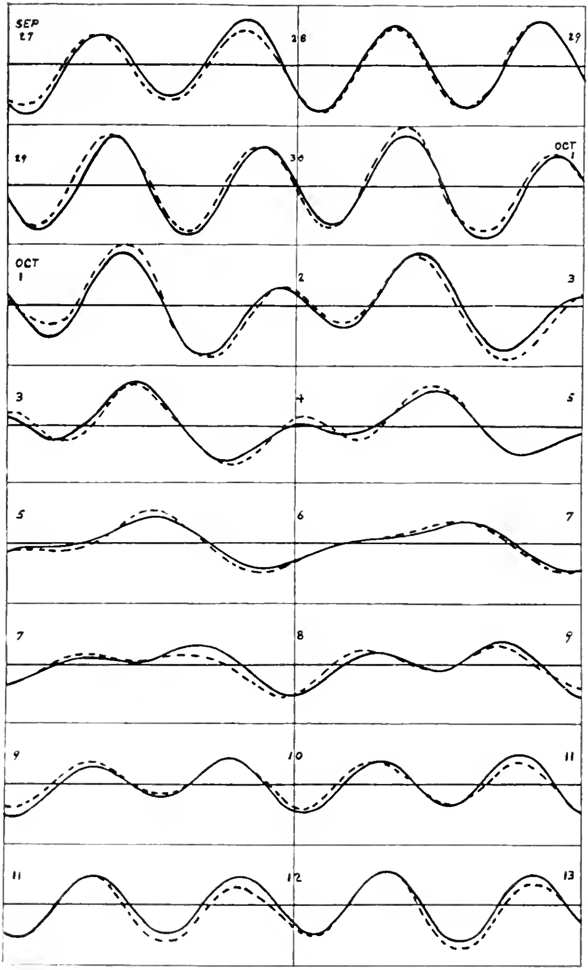

FIG. 5. E.-W. Dotted curve, observed values; full curve, 0.7 of calculated

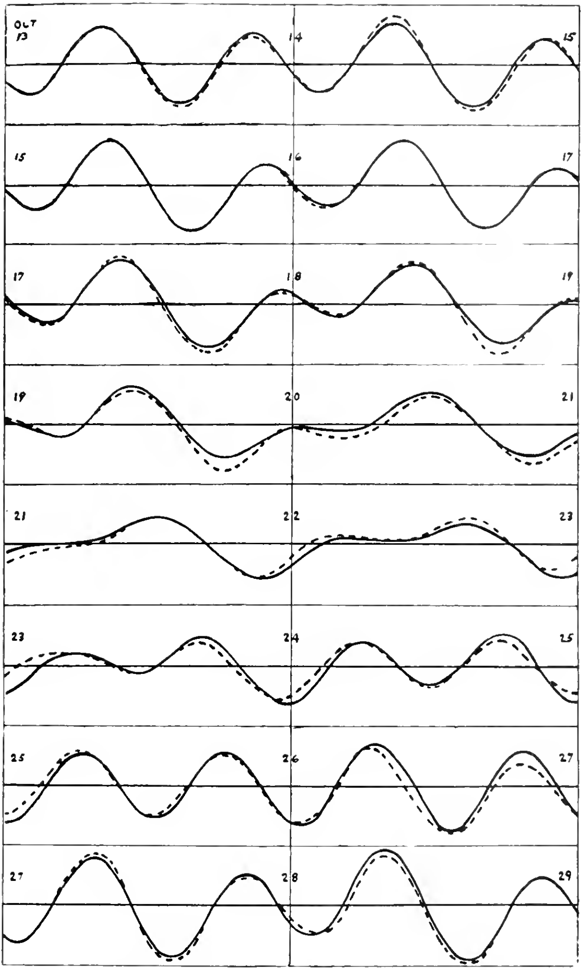

FIG. 6.—E.-W. Dotted curve, observed values; full curve, 0.7 of calculated

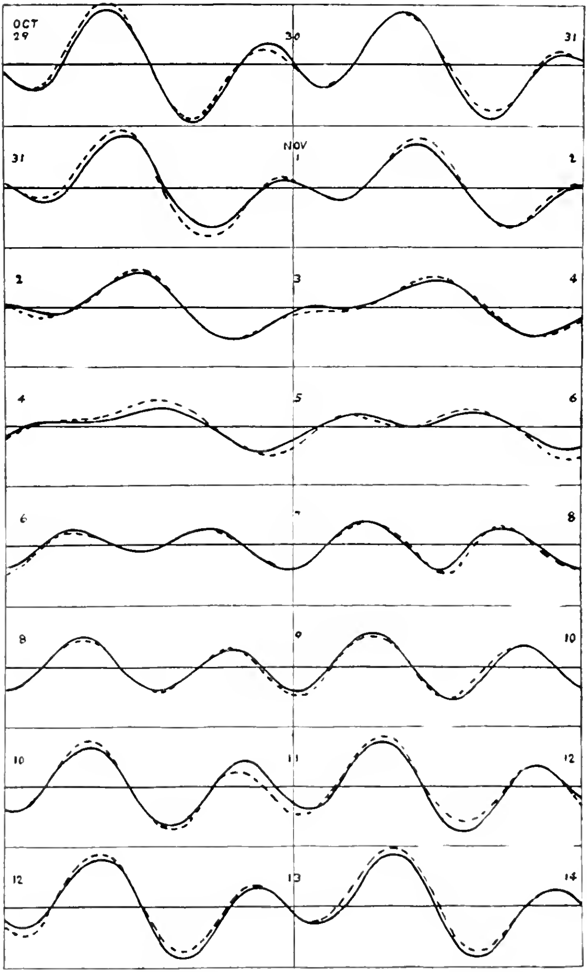

FIG. 7.—E.-W. Dotted curve, observed values; full curve, 0.7 of calculated

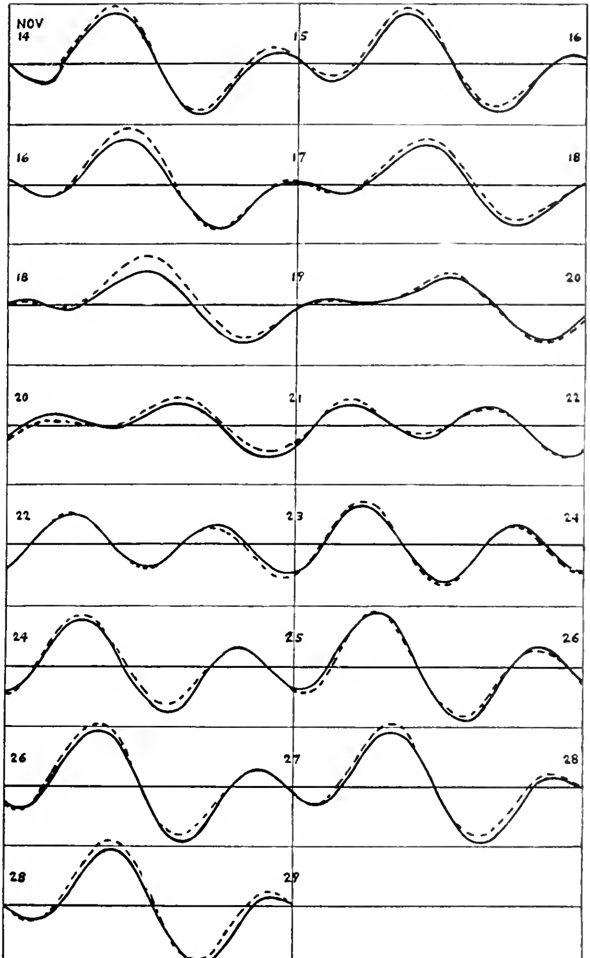

FIG. 8.—E.-W. Dotted curve, observed values; full curve, 0.7 of calculated

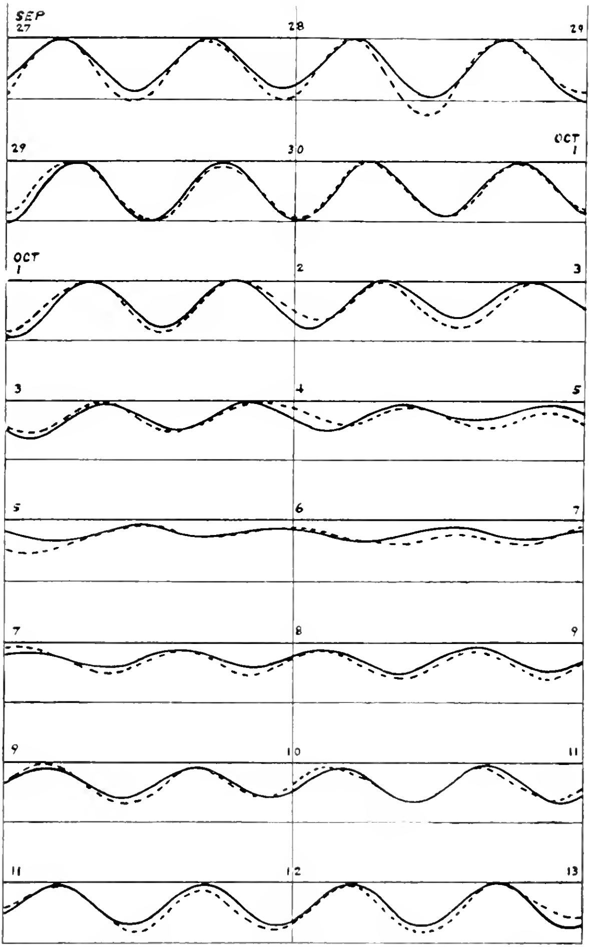

FIG. 9.—N.-S. Dotted curve, observed values; full curve, 0.5 of calculated

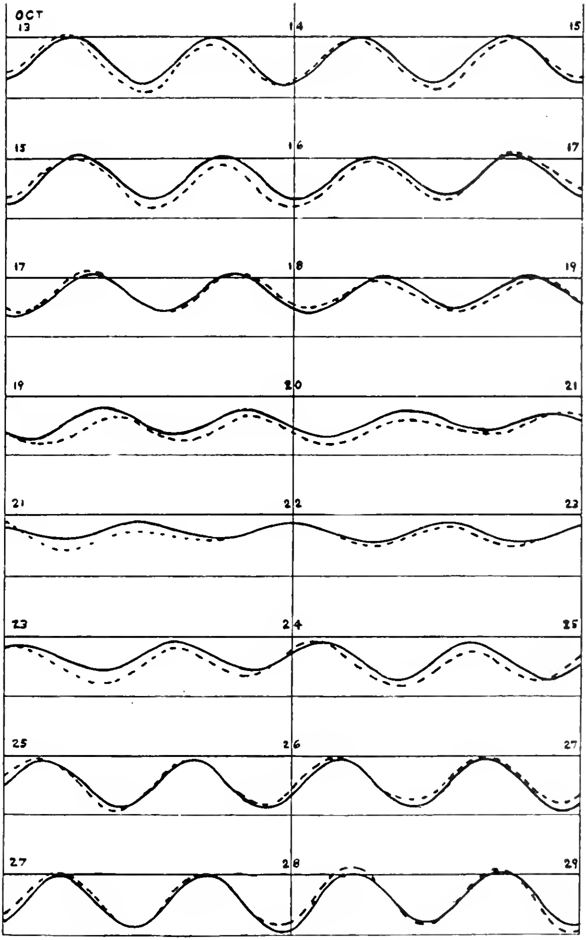

FIG. 10.—N.-S. Dotted curve, observed values; full curve, 0.5 of calculated

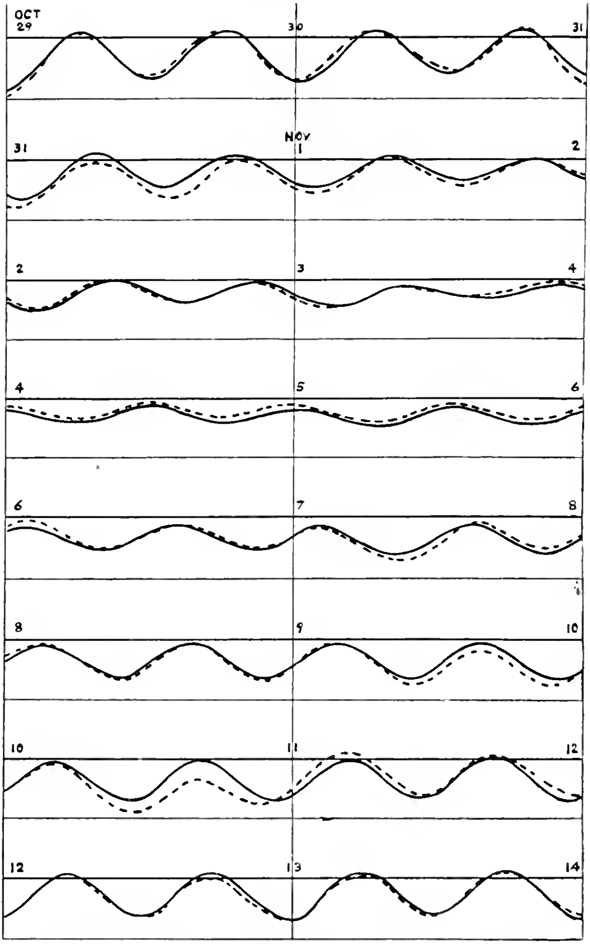

FIG. 11.—N.-S. Dotted curve, observed values; full curve, 0.5 of calculated

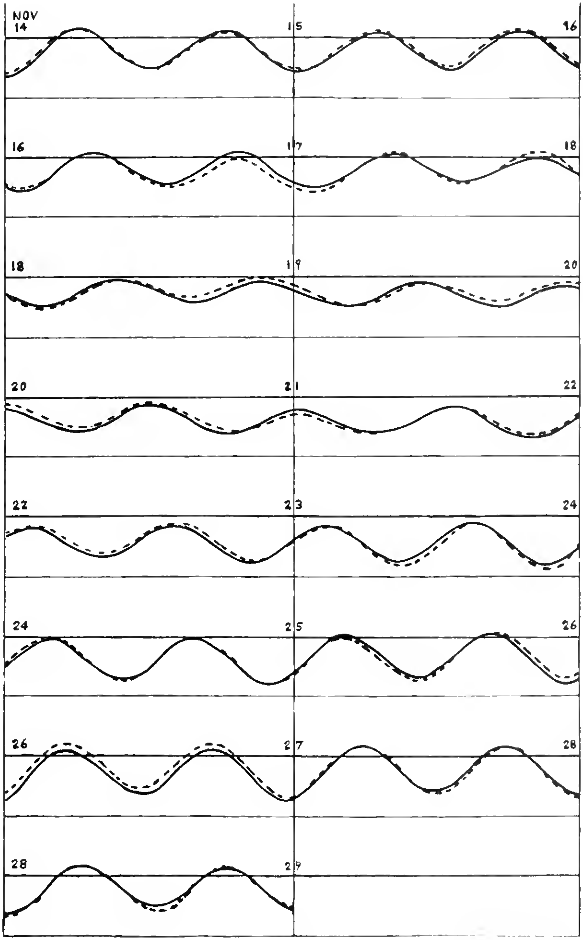

FIG. 12.—N.-S. Dotted curve, observed values; full curve, 0.5 of calculated

The resulting mean of all observations, for this method of grouping, is that the observed amplitude of the oscillation for the E.-W. line is 0.709 of that calculated on the assumption of absolute rigidity; while for the N.-S. line it is 0.510. The acceleration of phase of the observed oscillations relative to the calculated is $+0^{h}.059$ for the E.-W. line and $-0^{h}.007$ for the N.-S. line. The graphs of the final means of all the observations and the calculated values are reproduced in Figs. 13 and 14.

The curves correspond very closely with the following formulae:

$$\text{E.-W.}\quad y=-a\sin\frac{2\pi}{T}(t-\tau)+b$$

$$\text{N.-S.}\quad y=-a\cos\frac{2\pi}{T}(t-\tau)+b$$

with the following values for the constants:

E.-W.

Calculated × 0.7	Observed
$a=20.56$	$a=20.30$
$\tau=0.99$	$\tau=0.88$
$b=0.33$	$b=0.30$

N.-S.

Calculated × 0.5	Observed
$a=13.18$	$a=13.60$
$\tau=0.99$	$\tau=0.97$
$b=0.16$	$b=0.13$

Accordingly the ratio of the observed to the calculated amplitude for E.-W. is 0.691, and for N.-S. is 0.516, while the phase accelerations are $0^{h}.11$ and $0^{h}.02$ respectively.

These last results are slightly different from the preceding. The preference is for the latter as regards amplitude ratios, but these give relatively too much weight to the large oscillations in deducing the phase-difference, and for these the former results are preferred.

Multiplying the second set of values obtained for the amplitude ratios by the factors given above, 1.028 for E.-W. and 1.014 for N.-S., the final results are:

Amplitude Ratio	Phase Acceleration
E.-W. 0.710	**E.-W. . . . . . . +0.059 hour**
N.-S. 0.523	**N.-S. . . . . . . −0.007 hour**

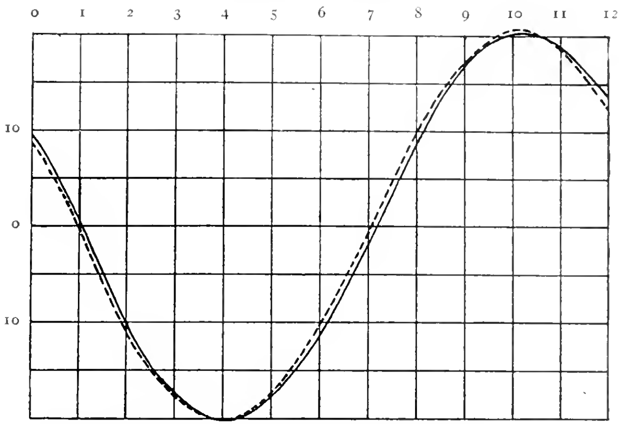

FIG. 13.—Mean of all observations, semi-diurnal Lunar tide. Dotted curve, observed values; full curve, 0.7 of calculated.

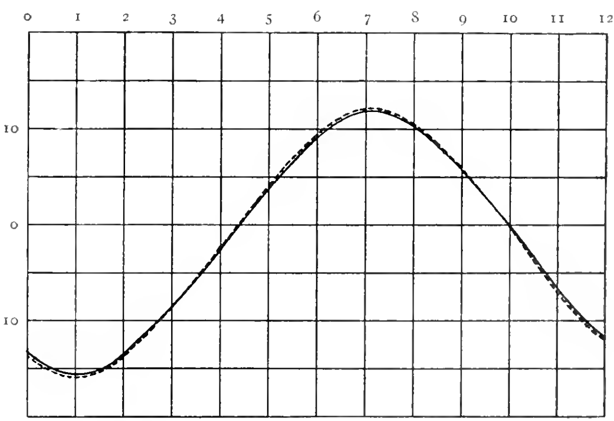

FIG. 14.—Mean of all observations, semi-diurnal Lunar tide. Dotted curve, observed values; full curve, 0.5 of calculated.

It is estimated that the errors in the amplitude ratios are under 1 per cent. The phase acceleration is probably correct to within $0^{h}.03$, but is so small as to leave some doubt as to whether or not it is real.

The disagreement between the E.-W. and N.-S. directions has been interpreted by Hecker, who found a similar difference, as

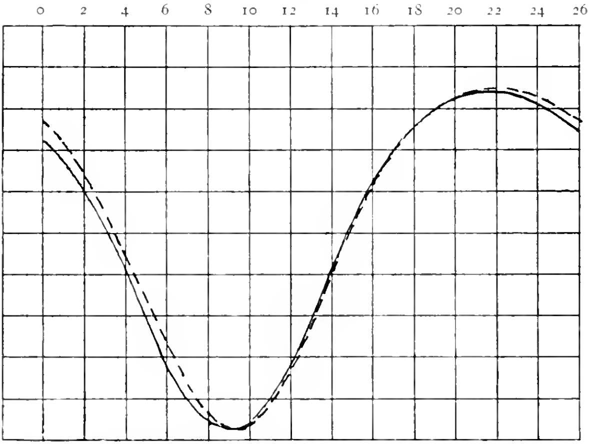

FIG. 15.—E.-W. Mean of observations of diurnal Lunar tide. Dotted curve, observed values; full curve, 0.7 of calculated.

indicating an actual difference in the earth's rigidity in the E.-W. and N.-S. directions.

Schweydar agrees with A. E. H. Love in attributing the difference to the effect of ocean tides, and shows on the assumption of an ocean covering the earth uniformly to a depth of 5000 meters that the tides have the effect of increasing the elastic earth tides, so that the ratio of the observed amplitudes to the theoretical is diminished by something like 40 per cent.

The mean values of the ratio adopted by Schweydar are 0.61 for E.-W. and 0.46 for N.-S., which should therefore be increased

to 0.85 and 0.64 respectively. A similar correction applied to the results of these experiments would give, instead of 0.71 and 0.52, the values 0.99 and 0.73. The E.-W. value would mean that the earth's rigidity is practically infinite, and it is undoubtedly too high.

The ocean is, however, anything but uniform in depth, and this and the irregularities in coast lines, make an accurate calculation of the disturbing effect of the ocean tides almost impossible. Accordingly Schweydar, considering that the results furnished by

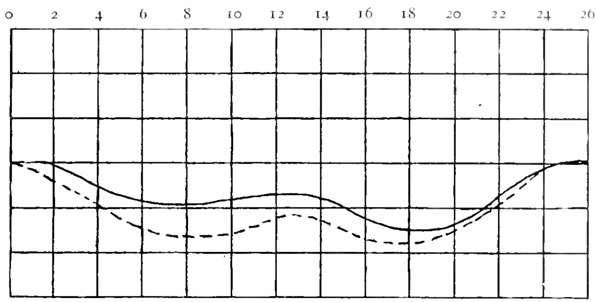

FIG. 16.—N.-S. Mean of observations of diurnal Lunar tide. Dotted curve, observed values; full curve, 0.5 of calculated.

the consideration of the semi-diurnal tides is not reliable, investigated the problem of the diurnal period, especially that corresponding to the "declination tide" whose period is $25^{h}812$. The dynamical theory shows that for an ocean entirely covering the earth, such tides should vanish; a result which is approximately true, at least for the Atlantic.

This analysis applied to Hecker's observations gives 0.85, as the value of the ratio, which, it will be noticed, agrees with the value for the semi-diurnal period when corrected for the calculated perturbation of the ocean tides.

The result of grouping the present observations into six groups of two periods each (of $25^{h}.81$) is reproduced in the graphs of Figs. 15 and 16.

These were analyzed by means of the harmonic analyzer, and

gave the following results, in which R is the ratio of the observed amplitude to the calculated, and ϕ the phase acceleration:

	R	ϕ
E.–W.	0.72	−0.4 hour.
N.–S.	0.66	+1.0 "

It appears therefore that the diurnal period gives for the amplitude ratios numbers which are decidedly in better agreement than those furnished by the semi-diurnal period.

It is to be noted that while the agreement between the E.–W. and the N.–S. results is considerably improved, the value of R for the E.–W. direction has not been altered, whereas according to Schweydar's investigation it should have been much larger (0.90 or more). It may be, however, that first, in consequence of the smaller number of periods entering the calculation, and secondly, on account of the smaller value of the resulting amplitude (41 : 100), these numbers have a considerably larger probable error than that of the semi-diurnal period.

Possibly a closer analysis of the actual ocean tides would show that the effect is small for the E.–W. direction, while in the N.–S. direction it may be considerable.

Regarding the acceleration of phase, it may be noted that the difference between the E.–W. and the N.–S. direction is much greater for the diurnal period than for the semi-diurnal, whereas, if the results of the latter were seriously affected by the ocean tides, the reverse should hold. The mean of the semi-diurnal accelerations is 0^h03, a quantity so small that it is within the probable error. Taking this value together with 0.70 as the ratio of the observed to the calculated amplitudes, the corresponding values of the earth's rigidity n and viscosity ϵ are:

$$n = 8.6 \times 10^{11} \text{ c.g.s.}$$
$$\epsilon = 10.9 \times 10^{16} \text{ c.g.s.}$$

This calculation is based on the assumption of uniform rigidity throughout the body of the earth, a condition which is certainly not fulfilled; and that as the time increases in arithmetical progression the stresses diminish in geometrical progression. It is clear, however, that the earth's rigidity is greater than that of steel. If the

ocean tides have the effect of diminishing the ratio R by from $\frac{1}{2}$ to $\frac{1}{3}$, as admitted by Schweydar, the rigidity is enormously greater.

The viscosity is also very great and probably of the same order of magnitude as that of steel.

The main object of this investigation was to demonstrate the feasibility of the method and to determine the order of magnitude of the earth's rigidity and viscosity. Evidently the method is capable of giving results of a high order of accuracy by recording a much longer series of observations. Such a series, in which the microscope will be replaced by the interferometer and in which the record is to be made automatic, is now in preparation. It is expected that the results will furnish a record of the earth tides which will be correct to within a tenth of 1 per cent.

Whether it may thereby be possible to obtain a more accurate value of the coefficients of rigidity and of viscosity will depend on the advance which may be made in the theory of the ocean tides and of their perturbing action. Doubtless it would be of importance to repeat the experiments at widely different stations, some in the southern hemisphere, some on islands in mid-ocean, and some on the continent as far as possible from the coast.

It may also be possible by a comparison of the moon and the sun tides to obtain an independent and perhaps more accurate value of the moon's mass.[1]

The conclusions from these and similar experiments and observations, including precession and variation of latitude, all agree substantially in refuting the old notion that the internal temperature, sufficiently high to melt most of the materials constituting the earth's crust, necessarily involves a fluid or semi-fluid earth supporting a relatively thin solid crust.

From the definitely ascertained result that the coefficient of rigidity and the coefficient of viscosity are both very large (of the order of, and perhaps exceeding, those of solid steel, whereas under normal pressure all substances at this temperature would be fluid), it follows that pressure increases the rigidity and the viscosity, at least of the substances which form the body of the earth.

[1] It would probably be necessary to make use of a tunnel sufficiently deep to eliminate the thermal effect, which even in the semi-diurnal period would be appreciable.

It would be interesting to confirm this important conclusion, even qualitatively, by experiments on the effect of such relatively small pressures as we are able to obtain in the laboratory.

Such experiments are now in progress; and while the highest pressures obtainable are a thousand times smaller than the pressure in the interior of the earth, it may be stated that there are distinct indications of an increase in the coefficient of rigidity, and a marked increase in the coefficient of viscosity of the few materials thus far investigated.

I would take this opportunity of expressing my appreciation of the interest taken in this work by Professor T. C. Chamberlin, at whose instigation the investigation was undertaken, and of tendering thanks to him and to President H. P. Judson for their efforts in securing the necessary funds. I would also gratefully acknowledge the friendly co-operation of Professor Frost and the members of the staff of the Yerkes Observatory.

TABLE III 10(E.−W.+C.)

A.M. September 27	A.M. September 30	P.M. October 2	A.M. October 5	A.M. October 8
C = +1.30	12:12 − 0.6	4:10 − 1.3	11:32 ± 0.0	6:29 + 0.7
	2:12 − 3.6	5:08 − 1.0	1:02 + 0.9	7:03 + 0.2
8:16 + 1.6	3:12 − 4.1	6:08 + 0.7	2:46 + 1.0	8:08 − 0.9
9:11 + 0.7	4:14 − 2.8	7:14 + 1.8	4:04 + 1.1	9:06 − 1.0
10:12 − 0.8	5:12 − 1.3	8:11 + 3.4	5:17 + 0.9	10:22 − 1.5
11:16 − 2.4	6:18 + 0.6	9:09 + 3.9	6:07 + 1.2	11:14 − 0.8
11:56 − 2.7	7:18 + 2.3	10:31 + 4.2	8:08 + 1.9	12:14 − 0.5
1:10 − 3.3	8:20 + 3.1	11:30 + 3.0	9:22 + 2.9	1:08 − 0.3
2:10 − 3.2	9:56 + 3.0	12:30 + 2.0	10:27 + 3.9	2:08 + 0.4
3:15 − 2.1	11:02 + 1.5	2:02 − 1.2	12:06 + 4.4	3:10 + 0.8
4:06 − 0.1	12:04 − 0.3	3:18 − 2.6	2:24 + 3.6	4:06 + 1.7
5:14 + 0.6	1:12 − 2.3	4:14 − 3.6	4:11 + 2.1	5:09 + 2.1
5:48 + 1.8	2:20 − 3.1	6:40 − 3.3	6:31 ± 0.0	5:53 + 2.2
7:08 + 2.5	3:20 − 3.0	7:58 − 1.8	8:34 − 1.1	7:05 + 2.7
8:16 + 2.5	4:14 − 2.2	9:10 ± 0.0	9:18 − 0.8	8:06 + 1.7
9:19 + 1.2	5:10 − 0.8	10:20 + 1.0	C = +3.00	9:22 + 1.1
10:26 − 0.4	5:52 + 0.8	11:10 + 1.5	10:18 − 0.7	10:35 + 1.1
11:12 − 1.8	7:11 + 3.4	12:06 + 2.2	11:22 − 0.1	12:23 + 0.8
12:14 − 2.8	7:53 + 4.2	1:18 + 1.8	12:15 + 0.3	1:54 + 1.7
2:04 − 2.9	10:32 + 3.3	2:14 + 1.1	1:23 + 0.9	4:19 + 2.4
4:04 − 0.8	1:46 − 3.0	3:14 + 0.2	2:16 + 1.2	6:10 + 2.5
6:09 + 2.8	3:31 − 3.9	4:15 − 0.2	3:18 + 1.6	7:10 + 1.9
9:30 + 1.8	4:21 − 3.7	5:06 − 0.2	4:24 + 1.7	8:02 + 1.2
10:26 + 0.4	5:28 − 2.6	5:45 + 0.2	5:02 + 1.7	9:12 + 0.1
11:15 − 1.1	6:30 − 0.8	7:09 + 1.7	5:54 + 1.9	10:06 − 0.5
12:22 − 3.0	8:39 + 2.4	8:22 + 3.1	7:04 + 1.5	11:12 − 0.9
1:17 − 4.1	9:48 + 2.7	9:48 + 4.2	8:24 + 2.1	12:08 − 0.7
2:40 − 3.7	11:12 + 2.8	11:14 + 4.4	9:18 + 2.5	1:08 − 0.3
3:33 − 2.4	11:59 + 1.5	12:06 + 3.5	10:14 + 2.5	2:15 + 0.7
4:12 − 1.8	1:38 − 0.9	2:38 + 0.8	11:10 + 2.7	3:06 + 0.7
5:04 + 0.1	3:20 − 1.8	5:42 − 2.1	12:05 + 3.1	4:16 + 2.0
6:12 + 2.1	4:10 − 1.7	8:02 − 1.6	2:07 + 3.1	5:14 + 2.8
7:08 + 2.3	5:13 − 0.6	9:09 − 0.6	4:18 + 2.1	5:54 + 2.9
8:06 + 3.0	5:51 + 0.5	10:10 + 0.8	6:19 + 0.6	7:00 + 2.8
9:08 + 3.0	7:17 + 3.1	11:19 + 1.5	7:10 − 0.2	8:10 + 2.4
10:20 + 1.0	8:05 + 4.3	12:16 + 2.5	8:22 − 0.8	9:22 + 1.5
11:28 − 1.4	9:22 + 5.0	1:22 + 2.2	9:12 − 1.2	10:06 + 1.0
12:20 − 2.5	C = −1.32	2:08 + 1.2	10:16 − 1.0	11:12 + 0.3
2:11 − 3.6	10:12 + 4.7	3:23 + 0.4	11:14 − 1.1	12:22 + 0.3
4:18 − 1.2	11:14 + 3.2	4:10 + 0.8?	12:01 − 1.1	2:10 + 0.8
8:20 + 3.8	12:10 + 1.2	5:01 + 0.3	1:04 − 0.6	4:25 + 2.5
9:14 + 3.3	2:10 − 2.4	5:58 + 0.7	2:08 + 0.3	6:16 + 3.3
10:08 + 3.1	4:16 − 4.6	7:16 + 2.2	3:06 + 0.9	7:20 + 3.1
11:18 − 0.2	6:28 − 3.4	8:02 + 2.5	4:10 + 1.2	8:13 + 2.0
11:58 − 1.2	7:15 − 1.8	9:06 + 3.5	5:08 + 1.7	9:08 + 1.3
1:19 − 3.0	8:22 − 1.4	10:16 + 4.6	6:02 + 1.8	10:08 + 0.5
2:06 − 3.3	9:10 + 0.6	11:08 + 4.7	7:17 + 1.5	11:10 − 0.7
3:10 − 3.0	10:05 + 1.5	12:12 + 4.4	8:08 + 1.4	11:58 − 1.4
4:20 − 1.2	11:12 + 1.6	2:09 + 2.4	9:05 + 1.3	1:10 − 0.8
5:21 + 0.8	12:00 + 1.1	4:20 + 0.3	10:12 + 1.8	2:10 − 0.5
6:49 + 2.9	1:08 + 0.4	6:39 − 1.1	11:11 + 1.7	3:12 + 0.4
8:32 + 4.4	2:21 − 1.0	7:40 − 1.2	12:05 + 1.7	4:10 + 1.3
9:50 + 3.5	3:09 − 1.4	9:10 − 0.5	2:16 + 1.9	5:12 + 2.3
11:15 + 1.2		10:33 ± 0.0		5:59 + 2.8

TABLE III—*Continued*

P.M. October 10	P.M. October 13	A.M. October 16	P.M. October 18	P.M. October 21
6:53 + 2.9	1:18 − 2.9	6:00 − 4.1	6:51 − 2.4	2:12 − 6.7
8:10 + 2.2	2:24 − 2.9	7:13 − 2.4	8:30 − 0.9	3:20 − 6.4
9:18 + 1.4	3:27 − 1.9	8:02 − 1.4	9:52 − 0.2	4:10 − 6.3
10:14 + 0.3	4:14 − 1.0	9:06 − 0.7	11:00 − 0.7	5:04 − 6.4
11:14 − 0.8	5:11 + 0.2	10:10 − 1.0	12:00 − 1.7	6:00 − 6.0
12:34 − 1.1	5:59 + 1.3	11:06 − 1.8	2:05 − 5.3	7:00 − 5.9
1:50 − 1.1	6:52 + 2.2	12:06 − 2.5	4:06 − 7.9	8:02 − 5.8
4:11 + 0.2	8:17 + 2.5	1:14 − 4.0	6:11 − 8.0	9:26 − 5.1
6:12 + 1.7	9:16 + 1.9	2:12 − 4.3	7:16 − 7:3	10:16 − 4.7
7:12 + 2.0	10:12 + 0.7	3:12 − 4.1	8:03 − 6.2	11:10 − 4.2
8:18 + 1.3	11:10 − 1.0	4:14 − 3.6	9:09 − 5.5	12:05 − 4.2
9:08 ± 0.0	12:07 − 2.5	5:18 − 2.6	10:08 − 4.1	1:08 − 3.9
10:18 − 1.0	2:13 − 4.2	6:52 − 0.3	11:03 − 4.0	2:00 − 4.4
11:14 − 2.2	4:16 − 3.4	8:14 + 0.9	2:03 − 4.3	4:11 − 5.5
12:02 − 2.6	6:14 − 0.5	9:09 + 1.1	3:02 − 4.7	6:14 − 7.6
1:10 − 2.8	7:09 + 0.8	10:09 + 0.9	4:04 − 5.2	7:12 − 8.6
2:04 − 2.8	8:21 + 1.3	11:09 − 0.6	5:04 − 5.2	8:13 − 9.0
3:18 − 1.9	9:20 + 0.9	12:12 − 2.3	6:12 − 4.9	9:14 − 9.1
4:06 − 0.7	10:10 + 0.4	2:14 − 5.4	7:58 − 3.1	10:10 − 8.6
5:01 + 0.4	11:04 − 0.8	4:26 − 6.6	9:15 − 1.9	11:08 − 8.5
5:51 + 1.3	12:01 − 2.1	6:06 − 5.0	10:08 − 1.8	12:00 − 7.8
6:56 + 1.8	1:00 − 2.7	7:14 − 3.4	11:02 − 1.8	1:09 − 7.2
8:11 + 1.6	2:17 − 3.3	8:10 − 2.5	12:06 − 2.7	2:11 − 6.6
9:26 + 0.4	3:09 − 2.3	9:04 − 1.7	2:10 − 5.0	3:05 − 6.3
10:15 − 0.7	4:08 − 1.7	10:00 − 1.6	4:12 − 7.9	4:04 − 6.2
11:10 − 2.1	5:02 − 0.6	11:05 − 2.0	6:55 − 8.9	5:07 − 6.1
12:11 − 3.1	5:41 + 0.3	12:00 − 2.8	8:12 − 8.0	6:44 − 6.3
2:12 − 3.2	7:03 + 2.0	1:08 − 3.5	9:10 − 7.4	8:01 − 6.3
4:05 − 1.5	8:06 + 2.7	1:54 − 4.1	10:16 − 6.5	9:03 − 6.5
6:10 + 0.7	9:05 + 2.4	3:20 − 4.9	11:10 − 6.3	10:08 − 6.3
7:40 + 1.1	10:06 + 0.9	4:10 − 4.4	12:02 − 6.0	12:09 − 5.3
8:46 + 0.6	11:02 − 0.5	5:10 − 3.5	1:07 − 6.5	1:56 − 4.9
9:46 − 1.0	12:05 − 2.6	5:40 − 2.7	2:08 − 6.8	4:08 − 4.9
10:52 − 1.3	3:31 − 5.2	7:14 − 0.9	3:17 − 7.1	6:06 − 6.5
11:53 − 2.3	5:10 − 4.5	8:10 + 0.2	4:07 − 7.1	7:21 − 7.9
12:51 − 2.8	6:30 − 1.3	9:08 + 0.7	5:06 − 7.3	8:34 − 8.7
2:10 − 2.8	8:18 ± 0.0	10:23 + 0.3	5:53 − 7.0	9:20 − 9.1
2:52 − 2.3	9:11 + 0.2	11:20 − 0.6	6:58 − 6.6	10:16 − 8.8
4:01 − 1.0	10:12 − 0.2	12:20 − 2.6	8:08 − 6.1	11:09 − 8.4
5:06 + 0.2	11:05 − 0.8	2:06 − 5.3	9:16 − 4.7	11:56 − 7.8
6:21 + 1.8	12:04 − 1.8	5:00 − 7.6	10:20 − 4.6	1:07 − 7.0
8:12 + 2.0	1:08 − 2.7	5:58 − 7.2	11:16 − 4.1	2:07 − 6.2
9:22 + 0.9	2:03 − 3.3	7:06 − 6.1	12:09 − 4.3	3:20 − 5.7
10:06 − 0.2	3:15 − 3.3	8:05 − 5.0	2:13 − 5.8	4:12 − 5.3
11:17 − 2.0	4:10 − 2.5	9:08 − 3.7	4:23 − 7.8	5:00 − 5.2
12:14 − 3.4	5:08 − 1.3	10:06 − 2.0	6:28 − 9.8	5:40 − 5.1
1:58 − 4.1	5:40 − 0.5	11:10 − 2.8	7:56 − 10.0	7:00 − 5.6
4:26 − 2.2	7:37 + 1.8	12:02 − 2.8	9:02 − 10.1	8:00 − 6.1
6:50 + 0.9	8:48 + 2.1	1:18 − 3.4	10:12 − 9.8	9:16 − 6.7
8:14 + 1.4	10:12 + 1.2	2:15 − 4.4	C = + 1.30	10:18 − 6.8
9:11 + 1.1	11:16 − 0.6	3:11 − 4.0		11:07 − 6.8
10:13 + 0.2	12:03 − 2.2	4:04 − 4.4	11:21 − 8.7	12:24 − 6.3
11:10 − 1.1	2:06 − 5.4	4:58 − 4.1	12:04 − 7.6	2:03 − 5.2
12:10 − 1.8	4:16 − 6.0	5:48 − 3.7	1:08 − 7.1	4:10 − 4.1

TABLE III—*Continued*

A.M. October 24	P.M. October 26	A.M. October 29	A.M. November 1	P.M. November 3
6:06 − 5.3	6:29 − 2.8	9:10 − 4.2	12:42 − 5.3	2:11 − 4.2
7:07 − 6.2	8:05 − 3.6	10:08 − 5.0	2:20 − 8.3	3:15 − 4.3
8:11 − 7.3	9:17 − 5.1	11:12 − 6.0	4:12 − 9.9	4:10 − 3.8
9:08 − 8.2	10:16 − 6.9	12:08 − 6.9	6:06 − 9.3	5:12 − 3.7
10:05 − 9.1	12:11 − 9.6	1:10 − 8.1	7:12 − 8.8	5:50 − 3.8
11:04 − 9.3	2:02 − 9.8	2:14 − 8.0	C = − 2.64	6:52 − 3.6
11:55 − 8.6	4:10 − 7.2	3:18 − 7.5	8:08 − 7.3	8:16 − 2.3
1:09 − 7.9	6:00 − 4.5	4:13 − 6.2	9:04 − 5.6	9:19 − 1.7
2:10 − 6.5	7:20 − 4.2	5:08 − 4.7	10:15 − 5.0	10:20 − 0.9
3:04 − 5.6	8:16 − 4.4	5:51 − 3.5	11:16 − 4.6	11:14 − 0.7
4:08 − 4.5	9:21 − 5.4	6:58 − 1.8	12:02 − 4.8	12:10 − 0.4
5:05 − 4.1	10:15 − 6.4	8:08 − 1.2	1:06 − 5.0	2:17 − 1.5
5:56 − 3.8	11:07 − 7.6	9:24 − 1.5	2:09 − 5.9	C = − 1.84
6:56 − 4.4	12:02 − 8.6	10:21 − 2.9	3:15 − 6.1	4:16 − 2.9
8:12 − 5.1	1:00 − 9.0	11:16 − 5.0	4:16 − 6.3	6:17 − 4.6
9:04 − 6.0	2:16 − 8.4	12:14 − 6.9	5:20 − 6.0	7:19 − 5.2
10:10 − 6.9	3:12 − 7.1	2:15 −10.7	5:53 − 5.3	8:18 − 5.2
11:16 − 7.5	4:16 − 5.5	4:16 −10.8	6:54 − 4.1	9:11 − 4.9
12:23 − 7.2	5:14 − 3.5	6:14 − 8.2	8:12 − 2.4	10:13 − 4.9
2:06 − 6.2	5:52 − 2.6	7:14 − 7.0	9:14 − 1.2	11:16 − 4.1
4:16 − 4.2	6:49 − 2.0	9:16 − 5.4	10:17 − 1.0	12:04 − 4.1
6:16 − 3.5	8:15 − 2.0	12:09 − 7.0	11:21 − 1.3	1:11 − 3.4
7:13 − 4.4	9:09 − 3.3	1:08 − 8.0	12:36 − 2.6	2:16 − 3.0
8:10 − 5.5	10:08 − 4.9	2:05 − 8.7	2:19 − 5.1	3:17 − 2.7
9:25 − 6.7	11:12 − 7.3	3:16 − 8.3	4:17 − 7.4	3:28 − 2.2
10:22 − 7.6	12:08 − 9.1	4:15 − 7.3	6:14 − 8.3	4:18 − 2.6
11:14 − 7.9	2:13 −10.5	5:08 − 6.2	8:17 − 6.1	5:07 − 2.6
12:01 − 7.9	4:12 − 8.3	5:56 − 4.6	9:19 − 5.5	5:54 − 2.3
1:08 − 7.2	6:49 − 4.2	6:58 − 3.3	10:17 − 4.8	6:50 − 1.8
2:10 − 6.1	7:30 − 3.8	8:13 − 1.6	11:10 − 4.3	8:05 − 1.7
3:20 − 4.5	8:18 − 3.5	9:18 − 1.6	12:15 − 4.2	9:14 − 1.5
4:19 − 3.6	9:06 − 3.6	10:18 − 2.3	1:18 − 4.2	10:09 − 1.1
5:09 − 2.7	10:06 − 4.4	11:20 − 3.7	2:25 − 5.0	11:10 − 0.6
5:53 − 2.4	11:07 − 5.4	12:33 − 6.0	3:15 − 4.7	12:23 − 0.3
7:08 − 2.7	12:02 − 7.0	2:16 − 9.4	4:14 − 4.7	2:12 − 0.6
8:02 − 3.7	1:14 − 7.6	4:16 −10.6	5:18 − 4.5	4:11 − 1.7
9:13 − 5.0	C = + 0.90	6:25 − 8.9	6:24 − 3.5	6:12 − 3.4
10:17 − 6.7	3:06 − 8.8	7:18 − 7.9	8:04 − 2.4	7:12 − 3.7
11:12 − 7.5	3:48 − 7.9	8:27 − 6.5	9:16 − 1.2	8:15 − 4.1
12:02 − 7.9	4:46 − 5.7	9:34 − 5.8	10:06 − 0.7	9:16 − 4.6
2:19 − 7.0	5:20 − 4.4	10:17 − 5.6	11:09 − 0.9	10:27 − 4.4
4:12 − 4.5	5:49 − 3.2	11:12 − 6.0	12:16 − 1.4	11:08 − 4.5
6:14 − 3.1	6:54 − 2.2	12:04 − 6.3	2:17 − 3.2	12:04 − 3.6
7:14 − 3.6	8:07 − 1.7	1:05 − 7.0	4:38 − 5.6	1:12 − 2.7
8:16 − 4.3	9:06 − 2.7	2:29 − 7.5	6:24 − 6.5	2:12 − 2.4
9:16 − 5.9	10:10 − 4.2	3:31 − 7.2	8:19 − 5.9	3:10 − 1.6
10:15 − 7.3	11:19 − 6.4	4:20 − 7.1	C = − 0.84	4:04 − 0.9
11:13 − 8.7	12:18 − 8.4	5:13 − 6.0	9:26 − 5.7	5:04 − 0.9
12:09 − 9.2	2:11 −10.5	5:56 − 4.9	10:10 − 5.0	5:52 − 1.4
1:14 − 8.9	4:08 −10.1	6:52 − 3.6	11:12 − 4.7	6:51 − 1.3
2:28 − 8.1	6:10 − 6.7	8:22 − 2.0	12:06 − 4.2	8:00 − 1.5
3:27 − 6.7	7:18 − 4.9	9:19 − 1.3	1:08 − 4.4	9:45 − 1.7
4:14 − 5.5	8:18 − 4.2	10:10 − 1.8		11:04 − 1.4
5:21 − 3.5		11:30 − 2.8		

TABLE III—*Continued*

A.M. November 6	P.M. November 8	A.M. November 11	P.M. November 13	P.M. November 16
1:04 − 0.9	12:06 − 3.0	8:27 + 0.4	8:00 + 5.8	1:17 + 1.5
2:19 − 0.1	1:05 − 2.4	9:23 − 0.6	9:10 + 5.4	2:30 + 1.1
4:19 − 0.7	2:13 − 1.4	10:14 − 1.6	10:07 + 4.0	3:17 + 1.0
6:17 − 1.8	3:14 − 0.5	11:10 − 2.4	11:08 + 2.2	4:06 + 1.2
7:12 − 1.7	4:18 + 0.7	12:04 − 2.8	12:08 + 0.3	5:12 + 2.3
8:04 − 2.7	5:10 + 0.8	1:04 − 2.7	2:10 − 2.5	6:24 + 4.2
9:05 − 3.5	5:55 + 1.5	1:55 − 2.6	4:16 − 2.6	8:00 + 5.7
10:07 − 4.3	6:55 + 1.6	3:08 − 1.0	6:38 + 0.1	9:17 + 6.4
11:07 − 3.9	8:03 + 0.9	4:18 + 0.7	8:06 + 1.7	10:16 + 6.7
12:07 − 3.5	9:11 − 0.3	5:11 + 1.9	9:13 + 2.3	11:06 + 6.1
1:19 − 2.8	10:10 − 1.2	5:56 + 3.1	10:10 + 2.0	12:19 + 4.5
2:12 − 1.8	11:18 − 2.3	6:59 + 3.8	11:09 + 1.5	2:07 + 1.8
3:08 − 1.5	12:20 − 2.6	8:10 + 3.8	12:07 + 0.8	4:38 − 1.4
4:06 − 0.2	2:10 − 1.8	9:11 + 2.3	1:08 − 0.4	7:22 − 0.1
5:06 ± 0.0	4:17 − 0.1	10:12 + 0.8	2:01 − 0.4	8:15 + 0.5
5:50 − 0.2	6:18 + 1.0	11:10 − 0.7	3:09 + 0.5	9:10 + 1.5
6:48 − 0.4	7:08 + 1.0	12:32 − 2.6	4:09 + 1.3	10:12 + 2.4
7:59 − 0.9	8:15 + 0.2	2:08 − 3.4	5:06 + 2.6	11:14 + 2.8
9:08 − 0.9	9:16 − 0.6	4:10 − 2.5	5:56 + 3.5	12:10 + 2.6
10:10 − 1.8	10:16 − 1.9	6:28 + 0.8	6:50 + 4.6	1:07 + 2.5
11:08 − 1.7	11:16 − 2.4	7:24 + 1.4	8:12 + 6.1	2:10 + 1.8
12:14 − 1.4	1:13 − 2.7	8:16 + 1.3	9:12 + 5.8	3:15 + 1.7
1:11 − 0.8	2:29 − 1.5	9:18 + 0.8	10:20 + 5.1	4:03 + 1.6
2:10 − 0.3	3:27 − 0.3	10:14 + 0.1	11:14 + 3.9	5:05 + 2.3
4:18 + 0.4	4:36 + 1.3	11:18 − 1.0	12:04 + 2.3	5:52 + 2.6
6:12 − 0.3	5:25 + 1.8	12:08 − 1.4	2:11 − 1.9	7:12 + 3.9
7:12 − 0.7	6:13 + 2.1	1:08 − 1.7	4:16 − 2.4	8:08 + 4.8
8:08 − 1.1	8:25 + 1.4	2:12 − 1.7	6:08 − 0.6	**C = +2.66**
9:13 − 2.4	9:18 − 0.1	3:10 − 0.7	7:58 + 1.8	9:16 + 5.7
10:09 − 2.5	10:18 − 1.1	4:04 + 1.1	9:06 + 2.6	10:09 + 6.3
11:05 − 3.3	11:20 − 2.6	5:05 + 2.8	10:18 + 2.5	11:09 + 6.4
12:04 − 3.0	12:20 − 3.3	6:00 + 3.7	11:08 + 2.3	12:12 + 5.7
1:07 − 2.2	2:04 − 3.2	7:05 + 4.7	12:04 + 1.7	2:08 + 3.4
2:22 − 1.3	4:08 − 0.3	8:08 + 4.9	1:12 + 1.1	4:12 + 0.4
3:12 − 0.7	6:04 + 1.1	9:17 + 3.9	2:16 + 0.6	6:13 − 0.2
4:17 + 0.3	8:20 + 1.0	10:08 + 3.0	3:16 + 0.7	7:30 ± 0.0
C = −2.83	9:32 − 0.2	11:08 + 1.0	4:11 + 0.8	8:23 + 0.9
5:12 + 0.5	10:51 − 1.5	12:15 − 1.0	5:16 + 2.4	9:19 + 1.7
5:56 + 1.0	12:07 − 2.4	2:27 − 3.2	5:57 + 3.5	10:14 + 2.1
6:52 + 1.1	1:10 − 2.2	4:18 − 2.0	6:51 + 4.7	11:10 + 2.4
7:48 + 0.2	2:08 − 1.4	6:12 − 0.1	8:17 + 6.4	12:03 + 3.1
9:16 − 0.4	3:08 − 0.2	7:30 + 1.6	9:13 + 6.4	1:12 + 3.2
10:10 − 1.7	4:20 + 1.3	8:26 + 2.2	10:08 + 6.3	2:08 + 3.0
11:13 − 2.0	5:16 + 2.9	9:23 + 2.0	11:05 + 5.2	3:05 + 2.0?
12:13 − 2.1	5:52 + 2.9	10:12 + 1.3	12:18 + 3.1	4:04 + 2.7
2:44 − 0.9	6:52 + 3.3	11:06 + 0.5	2:06 − 0.1	5:04 + 3.4
4:24 + 0.1	9:01 + 1.6	12:06 ± 0.0	4:22 − 1.9	5:53 + 3.1
6:13 + 0.2	10:10 + 0.4	1:03 − 0.7	6:14 − 0.8	6:49 + 4.0
7:22 − 0.3	11:13 − 1.3	2:04 − 0.6	7:12 + 0.2	8:08 + 5.3
8:16 − 1.1	12:26 − 3.4	3:12 + 0.3	8:12 + 0.9	9:24 + 6.2
9:12 − 1.6	2:10 − 3.7	4:10 + 1.3	9:09 + 1.7	10:11 + 6.9
10:10 − 2.3	4:18 − 1.8	5:15 + 2.8	10:13 + 2.5	11:10 + 7.4
11:09 − 3.1	6:14 + 0.5	5:58 + 4.0	11:10 + 2.4	12:11 + 7.3
	7:20 + 0.8	6:50 + 4.8	12:15 + 2.2	

TABLE III—*Continued*

A.M. November 19	A.M. November 21	A.M. November 23	A.M. November 25	A.M. November 27
2:14 + 5.8	4:12 + 6.8	6:12 + 7.0	10:06 + 4.8	11:06 + 6.3
4:15 + 3.7	6:04 + 4.8	7:11 + 6.1	11:04 + 3.8	12:08 + 5.5
6:16 + 1.7	7:15 + 4.0	8:10 + 5.1	12:06 + 3.4	12:48 + 5.2
7:16 + 1.3	8:12 + 3.4	9:10 + 3.9	1:05 + 3.6	2:05 + 4.9
8:08 + 1.3	9:09 + 3.1	10:12 + 3.2	2:05 + 4.4	3:17 + 6.2
9:21 + 2.0	10:08 + 3.2	11:08 + 2.9	3:09 + 5.9	4:17 + 7.4
10:12 + 2.4	11:12 + 3.5	12:08 + 3.4	4:10 + 7.6	5:12 + 9.1
11:08 + 3.0	12:06 + 4.3	1:12 + 4.4	5:08 + 9.0	6:17 +10.6
12:12 + 3.7	1:09 + 5.4	2:18 + 6.0	5:55 + 9.7	7:30 +11.9
1:23 + 4.0	2:10 + 6.3	3:08 + 7.1	6:54 +10.0	8:16 +11.8
2:25 + 4.1	3:03 + 7.4	4:07 + 8.4	8:10 + 8.8	9:28 +11.0
3:12 + 4.2	4:09 + 7.8	5:03 + 8.6	9:08 + 7.7	10:18 + 9.6
4:14 + 4.1	5:14 + 7.8	6:08 + 8.7	10:05 + 6.0	11:08 + 7.9
5:13 + 4.1	5:50 + 7.2	8:02 + 6.9	11:08 + 4.0	12:09 + 5.9
5:50 + 4.0	6:53 + 6.6	9:06 + 5.1	12:08 + 2.5	2:03 + 2.7
7:06 + 4.3	8:06 + 5.6	10:09 + 3.8	2:10 + 1.1	3:52 + 2.4
8:11 + 4.4	9:16 + 5.1	1:15 + 2.8	4:05 + 2.8	5:35 + 4.1
9:19 + 5.3	10:14 + 5.0	3:07 + 4.5	5:56 + 5.7	6:37 + 5.6
10:12 + 5.7	11:14 + 5.3	5:06 + 6.5	7:18 + 6.9	8:19 + 7.4
11:05 + 6.3	12:12 + 5.6	7:11 + 6.4	8:20 + 7.1	9:06 + 7.6
12:16 + 6.8	2:34 + 6.8	8:18 + 5.6	9:18 + 6.4	10:04 + 7.6
2:09 + 5.9	4:10 + 7.3	9:18 + 4.4	10:16 + 5.7	11:06 + 7.2
4:12 + 4.5	6:10 + 6.2	10:07 + 3.6	11:09 + 4.7	12:06 + 6.2
6:07 + 2.7	7:20 + 5.1	11:14 + 3.1	12:06 + 4.1	1:07 + 5.7
7:14 + 1.9	8:12 + 4.3	12:04 + 2.9	1:02 + 3.9	2:08 + 5.2
8:08 + 1.6	9:08 + 3.5	1:10 + 3.6	2:06 + 4.7	3:16 + 5.8
9:22 + 1.8	10:14 + 3.2	2:07 + 4.8	3:10 + 6.0	4:08 + 6.6
10:18 + 2.1	11:09 + 3.3	3:19 + 6.9	4:08 + 7.4	5:05 + 7.9
11:09 + 2.9	12:02 + 3.7	4:14 + 8.3	5:06 + 9.3	5:48 + 9.0
12:09 + 3.4	1:09 + 4.9	5:09 + 9.4	6:00 +10.3	6:56 +10.8
1:05 + 4.3	2:14 + 5.9	6:00 + 9.6	6:52 +11.0	7:54 +11.8
2:04 + 4.7	3:18 + 7.4	6:54 + 9.2	8:02 +11.0	8:44 +11.9
3:07 + 5.3	4:14 + 8.0	8:12 + 8.3	9:10 +10.0	10:03 +11.2
4:05 + 5.2	5:08 + 8.4	9:10 + 6.8	10:12 + 8.1	12:00 + 7.8
5:03 + 5.3	5:52 + 8.2	10:13 + 5.1	11:18 + 5.9	3:15 + 2.6
5:56 + 4.9	6:48 + 7.5	11:05 + 3.4	12:36 + 3.4	5:16 + 3.0
6:56 + 4.8	8:02 + 6.5	12:08 + 2.2	2:08 + 1.8	7:06 + 5.3
8:04 + 4.8	9:20 + 5.1	2:12 + 2.5	4:26 + 3.1	8:21 + 6.9
9:08 + 5.2	10:06 + 4.3	4:16 + 4.7	6:04 + 5.2	9:30 + 7.8
10:14 + 5.4	11:17 + 3.7	6:10 + 6.9	7:16 + 6.7	10:48 + 7.5
11:07 + 6.0	12:18 + 4.1	7:20 + 6.9	8:21 + 7.5	12:03 + 7.0
12:12 + 6.6	2:12 + 5.6	8:24 + 6.4	9:18 + 7.4	1:22 + 6.2
2:06 + 7.2	4:12 + 7.0	9:14 + 5.6	10:04 + 7.0	2:18 + 6.0

TABLE IV 10(S.−N.+C.)

A.M. September 27	P.M. September 29	P.M. October 2	A.M. October 5	P.M. October 7
C = +0.70	9:36 + 0.8	3:50 − 3.6	9:01 − 6.8	10:01 − 8.1
8:04 + 2.9	11:01 + 1.6	5:01 − 4.6	10:02 − 7.0	11:00 − 8.4
9:00 + 3.2	11:59 + 1.9	5:59 − 5.3	C = −2.30	11:55 − 8.5
10:00 + 3.3	2:00 + 0.9	7:02 − 5.8	11:51 − 6.3	2:06 − 9.0
11:04 + 3.2	3:00 − 0.9	8:00 − 5.5	12:52 − 6.1	6:20 − 7.9
12:06 + 1.9	4:02 − 2.2	8:58 − 4.9	2:37 − 6.8	6:54 − 7.6
12:58 + 0.6	5:02 − 2.8	10:20 − 3.6	3:56 − 7.3	7:56 − 7.6
2:00 − 0.9	6:06 − 2.8	11:17 − 3.2	5:08 − 6.9	8:58 − 7.7
2:58 − 1.8	7:10 − 2.9	12:19 − 2.3	6:18 − 7.4	10:12 − 7.9
3:56 − 3.0	8:09 − 1.9	1:51 − 2.5	7:55 − 8.2	11:06 − 8.5
5:03 − 2.2	9:43 ± 0.0	3:05 − 2.8	9:12 − 8.0	12:06 − 9.2
5:58 − 0.9	10:52 + 0.4	4:04 − 3.6	10:18 − 8.6	1:00 − 9.4
6:54 − 0.1	2:08 − 0.1	6:20 − 5.6	11:55 − 8.7	1:59 − 9.3
8:02 + 0.7	3:10 − 1.9	7:40 − 6.1	2:12 − 8.1	3:01 − 9.1
9:03 + 2.0	4:05 − 3.2	8:58 − 6.2	4:02 − 7.9	3:58 − 9.1
C = +0.80	5:02 − 3.9	10:12 − 5.8	6:20 − 8.2	5:01 − 8.6
10:13 + 2.8	6:04 − 4.2	10:56 − 4.9	8:24 − 8.2	6:00 − 8.4
11:01 + 2.7	6:57 − 3.8	12:16 − 4.1	9:08 − 8.4	6:56 − 7.9
12:02 + 1.9	8:06 − 2.7	1:10 − 3.6	10:09 − 8.8	7:58 − 7.7
1:52 − 0.1	10:20 − 1.0	2:04 − 3.4	11:12 − 8.5	9:15 − 7.6
3:48 − 2.0	1:31 − 0.7	3:04 − 3.5	12:24 − 9.0	10:20 − 7.9
5:56 − 1.2	3:14 − 1.4	4:05 − 4.0	1:15 − 9.1	12:12 − 8.5
9:18 + 1.5	4:06 − 3.2	4:54 − 4.7	2:08 − 9.0	1:45 − 9.1
10:18 + 2.6	5:13 − 4.0	5:58 − 5.2	3:09 − 8.9	4:08 − 9.3
11:04 + 2.5	6:18 − 5.0	6:50 − 5.7	4:04 − 8.5	6:00 − 8.7
12:13 + 1.9	7:24 − 4.7	C = +1.10	C = −1.76	6:58 − 7.6
1:26 + 0.7	8:25 − 3.6	8:08 − 6.0	4:53 − 8.5	7:54 − 7.5
2:32 − 1.3	10:48 − 1.7	9:37 − 5.0	6:04 − 8.0	9:04 − 7.4
3:23 − 2.3	12:10 − 0.9	11:04 − 4.4	6:52 − 8.1	9:56 − 7.9
4:03 − 2.3	1:22 − 1.5	11:56 − 3.8	8:14 − 7.9	11:02 − 8.6
4:54 − 2.5	3:08 − 3.2	2:28 − 3.9	9:08 − 8.1	12:16 − 10.0
6:00 − 1.6	4:00 − 3.5	5:33 − 5.1	10:05 − 7.9	12:58 − 10.1
6:56 − 0.7	5:04 − 4.3	7:52 − 5.7	10:58 − 8.8	2:23 − 10.4
7:54 + 0.6	6:02 − 4.5	8:58 − 6.0	11:55 − 8.9	2:57 − 10.5
8:56 + 2.0	7:04 − 5.7?	10:01 − 6.3	1:56 − 8.5	4:07 − 10.5
10:04 + 2.5	8:15 − 6.1?	11:11 − 6.7	4:07 − 8.2	5:05 − 9.9
11:12 + 3.4	9:08 − 3.8	12:06 − 5.9	6:09 − 8.0	6:02 − 9.2
12:04 + 2.6	C = +1.92	1:14 − 5.5	6:56 − 7.5	6:52 − 8.7
1:57 − 0.1	10:03 − 2.4	2:00 − 5.2	8:12 − 8.3	8:02 − 7.9
4:06 − 2.0	11:03 − 1.0	3:15 − 4.9	9:04 − 8.1	9:15 − 7.5
8:07 − 0.3	12:00 − 1.0	4:01 − 4.9	10:08 − 8.9	9:58 − 7.3
9:03 + 0.4	1:57 − 1.2	4:53 − 5.3	11:06 − 9.4	11:03 − 7.8
9:58 + 1.6	4:01 − 3.4	5:47 − 5.7	12:12 − 9.6	12:11 − 8.4
11:10 + 1.8	6:14 − 4.7	7:04 − 6.1	12:56 − 9.6	2:00 − 9.5
12:09 + 1.6	7:06 − 5.0	7:53 − 6.1	2:00 − 9.7	4:12 − 10.2
1:11 + 0.1	8:12 − 5.3	8:50 − 6.4	2:55 − 10.0	6:04 − 9.2
1:56 ± 0.0	9:01 − 5.0	10:04 − 6.7	4:01 − 8.5	7:09 − 8.8
2:58 − 1.4	9:57 − 4.4	10:56 − 6.6	4:56 − 8.6	8:02 − 8.2
4:05 − 2.2	11:06 − 3.7	12:02 − 6.1	5:51 − 8.0	8:58 − 8.0
5:07 − 2.8	12:10 − 2.6	2:00 − 5.7	7:07 − 7.7	10:00 − 8.0
5:52 − 2.3	1:00 − 2.4	4:09 − 5.6	7:58 − 7.5	10:59 − 8.3
7:04 − 2.5	2:12 − 2.3	6:28 − 6.5	8:56 − 7.7	12:11 − 9.3
8:19 − 0.8	3:10 − 3.2	7:30 − 5.9		12:58 − 10.0
				1:56 − 10.2

TABLE IV—*Continued*

P.M. October 10	A.M. October 13	P.M. October 15	P.M. October 18	P.M. October 21
3:02 −10.4	9:04 − 7.3	11:08 − 6.6	3:04 − 9.3	12:11 −11.6
4:03 −10.4	10:05 − 6.9	11:54 − 6.7	3:55 − 9.8	12:57 −11.1
5:02 −10.1	11:03 − 6.6	1:56 − 7.3	4:50 −10.1	2:03 −10.9
6:07 − 9.4	12:16 − 6.8	4:07 − 8.9	5:58 −10.6	3:10 −10.2
6:46 − 8.4	1:09 − 7.4	5:53 −10.1	6:42 −10.8	4:00 −10.1
8:01 − 8.0	2:16 − 8.4	7:05 − 9.5	8:10 −11.0	4:56 −10.4
9:10 − 7.1	3:04 − 9.2	7:55 − 9.2	9:42 −10.5	5:52 −10.5
10:06 − 7.1	4:06 −10.1	8:58 − 8.1	10:52 − 9.8	6:52 −10.7
11:02 − 7.7	5:02 − 9.5	10:02 − 7.1	11:52 − 9.7	7:53 −11.1
12:26 − 8.1	6:08 − 9.1	10:58 − 6.9	1:54 − 9.0	9:17 −11.6
2:00 − 8.8	6:44 − 8.9	12:12 − 6.8	3:54 − 9.9	10:06 −11.7
4:00 − 9.3	8:06 − 7.4	1:07 − 6.9	5:50 −10.1	11:01 −12.0
6:01 − 8.3	9:08 − 6.5	2:04 − 7.6	7:05 −11.4	11:54 −12.0
7:02 − 7.9	10:02 − 5.8	3:04 − 8.2	8:15 −11.8	12:58 −11.8
8:08 − 6.7	11:00 − 5.5	4:06 − 9.2	9:00 −12.3	1:50 −11.8
8:58 − 6.7	11:58 − 5.6	5:08 −10.2	9:59 −11.8	4:00 −11.7
10:09 − 6.3	2:02 − 7.6	6:38 −10.3	10:54 −10.9	6:05 −11.7
11:06 − 6.5	4:02 − 9.1	8:02 − 9.9	1:56 − 9.4	7:00 −11.8
12:12 − 7.1	6:03 − 9.2	8:56 − 9.4	2:54 − 9.4	8:04 −12.3
1:02 − 7.5	6:58 − 8.3	9:59 − 8.5	3:56 − 9.7	9:05 −12.0
1:54 − 8.0	8:13 − 7.5	10:58 − 7.7	4:56 −10.1	10:01 −13.0
3:10 − 8.8	9:12 − 7.1	12:00 − 7.6	6:22 −10.7	10:59 −13.4
4:00 − 9.1	10:02 − 6.4	2:02 − 8.4	7:50 −11.9	12:10 −13.2
4:55 − 8.8	10:56 − 6.4	4:16 −10.3	9:06 −11.8	1:00 −13.2
6:02 − 8.3	12:10 − 6.7	5:56 −11.1	9:59 −11.2	2:00 −12.7
6:49 − 7.7	12:58 − 7.2	7:08 −11.5	10:54 −10.8	2:57 −12.7
8:00 − 6.6	2:10 − 8.1	8:02 −10.9	11:58 −10.4	3:57 −12.3
9:18 − 5.7	3:00 − 9.3	8:55 −10.4	2:02 − 9.5	4:59 −12.0
10:08 − 5.5	4:00 −10.1	9:52 − 9.9	4:04 − 9.7	5:55 −11.8
11:03 − 5.4	4:53 −10.0	10:58 − 9.1	6:46 −10.9	6:54 −11.9
12:03 − 5.9	5:55 −10.0	12:09 − 8.5	8:03 −11.4	7:52 −11.9
2:00 − 7.3	6:53 − 9.6	12:58 − 8.2	9:04 −11.3	8:55 −12.2
3:55 − 8.6	7:58 − 8.9	2:03 − 8.3	10:10 −10.9	9:59 −12.6
6:00 − 8.1	8:55 − 7.7	3:07 − 9.3	11:02 −10.5	12:02 −13.3
7:28 − 6.8	9:55 − 6.8	4:02 −10.1	12:08 −10.0	1:47 −13.4
8:34 − 6.2	10:52 − 6.4	5:00 −10.8	12:54 − 9.5	4:00 −12.9
9:38 − 6.0	11:54 − 6.2	5:59 −11.4	2:00 − 9.1	5:56 −12.1
10:43 − 5.8	3:22 − 8.7	7:04 −11.4	3:10 − 9.2	7:12 −12.2
11:44 − 6.0	5:02 −10.2	8:00 −11.3	3:58 − 9.4	8:24 −12.5
12:44 − 6.6	6:31 −10.1	8:55 −10.6	4:56 − 9.7	9:11 −13.2
2:02 − 8.0	8:10 − 9.2	10:12 −10.0	6:02 −10.0	10:03 −13.4
2:46 − 8.8	9:03 − 8.6	11:07 − 8.9	6:48 −10.3	11:00 −14.1
3:54 − 9.1	10:04 − 7.9	12:11 − 8.5	7:58 −10.6	11:48 −14.2
4:58 − 8.9	10:58 − 7.6	1:54 − 8.1	9:04 −11.3	12:58 −14.1
6:14 − 8.2	12:12 − 7.4	4:50 −10.1	10:10 −11.2	1:58 −14.1
8:20 − 6.5	12:58 − 8.0	5:47 −10.7	11:05 −11.0	3:18 −12.8
9:14 − 6.2	1:54 − 8.3	6:58 −11.0	11:58 −10.0	4:02 −12.7
9:59 − 5.8	3:08 − 9.4	7:57 −11.2	2:02 −10.2	4:53 −12.2
11:10 − 5.4	4:03 −10.0	9:00 −11.1	4:09 −10.0	5:54 −11.6
12:06 − 5.7	5:02 −10.6	9:57 −10.3	6:14 −10.4	6:52 −11.4
1:51 − 7.3	5:58 −10.9	11:02 − 9.6	7:43 −11.1	7:54 −11.1
4:17 − 9.0	7:12 −10.3	12:10 − 9.1	8:52 −11.1	9:09 −11.2
6:50 − 8.4	8:36 − 8.6	1:12 − 8.6	10:01 −11.6	10:11 −11.9
8:07 − 7.0	10:02 − 7.4	2:07 − 8.8	11:06 −11.3	11:00 −12.5

TABLE IV—*Continued*

A.M. October 24		P.M. October 26		A.M. October 29		P.M. October 31		A.M. November 3	
12:16	−13.2	3:17	−16.0	9:01	−17.5	5:04	−21.1	11:03	−21.7
1:55	−13.9	4:07	−16.0	10:00	−16.7	5:47	−21.5	12:14	−21.3
4:00	−13.6	5:14	−15.7	11:04	−16.3	6:44	−22.1	12:58	−21.5
5:56	−12.5	6:35	−14.7	12:18	−16.3	8:13	−21.9	2:04	−21.0
6:58	−12.0	8:16	−13.1	12:56	−17.0	9:08	−21.1	3:07	−20.6
8:03	−11.9	9:10	−12.4	C = −1.60		10:00	−20.8	4:02	−20.9
9:00	−12.4	10:09	−12.4			11:19	−19.9	5:04	−21.1
9:56	−12.5	12:04	−13.5	2:03	−17.9	12:34	−19.4	5:58	−21.5
10:54	−13.3	1:54	−15.6	3:05	−19.6	2:10	−19.2	6:44	−21.8
12:04	−14.3	4:01	−16.9	4:01	−20.4	4:02	−20.3	8:07	−22.4
12:59	−14.7	5:52	−16.3	5:00	−21.4	5:57	−21.6	9:07	−22.4
2:00	−15.0	7:12	−14.9	5:42	−21.7	7:18	−22.2	10:11	−22.4
2:56	−14.7	8:08	−14.1	6:48	−21.4	7:58	−22.3	11:06	−22.1
4:00	−14.1	9:16	−13.3	7:56	−20.9	8:54	−22.3	12:00	−22.3
4:55	−13.3	10:07	−13.1	9:10	−19.7	10:04	−21.4	2:06	−21.9
6:05	−12.6	11:00	−13.2	10:10	−18.4	11:08	−20.5	4:07	−22.0
6:48	−12.4	12:10	−13.9	11:04	−18.1	12:10	−19.8	6:06	−22.2
8:00	−11.8	12:52	−14.6	12:02	−18.2	1:02	−19.4	7:08	−22.6
8:55	−11.5	2:07	−15.8	C = +0.60		2:00	−19.5	8:11	−22.8
10:02	−11.8	3:03	−16.8			3:02	−19.6	9:02	−22.9
11:08	−12.6	4:08	−17.5	2:04	−19.6	4:04	−20.4	10:03	−23.1
12:16	−13.6	5:07	−17.3	4:08	−21.2	5:09	−21.1	11:08	−22.9
1:57	−14.7	6:01	−16.9	6:04	−22.2	6:02	−22.0	11:56	−22.9
4:04	−14.1	6:42	−16.5	7:05	−21.9	6:45	−22.5	1:05	−22.6
6:00	−13.2	8:04	−15.4	8:04	−21.1	8:00	−22.4	2:08	−22.5
7:03	−12.5	8:58	−14.3	9:26	−19.8	9:04	−22.5	3:09	−22.1
8:00	−12.2	9:56	−13.7	10:19	−18.9	10:06	−21.8	3:19	−21.9
9:17	−12.5	10:58	−13.7	11:06	−18.4	11:10	−21.2	4:08	−21.6
10:14	−13.0	11:58	−14.0	12:17	−18.3	12:25	−20.9	4:58	−21.4
11:05	−13.5	2:00	−16.6	12:56	−18.5	2:06	−20.2	6:02	−21.5
11:53	−14.4	4:02	−18.3	1:58	−19.0	4:05	−21.1	6:41	−21.7
12:59	−15.0	6:37	−17.8	3:02	−20.2	6:01	−21.8	7:57	−21.9
2:01	−15.8	7:20	−16.9	4:05	−21.5	8:04	−22.4	9:07	−22.2
3:12	−15.7	8:10	−16.2	4:59	−22.2	9:04	−22.5	10:00	−22.4
4:11	−15.1	8:59	−15.4	5:47	−22.6	10:09	−22.1	11:02	−22.5
5:00	−14.5	9:58	−14.9	6:48	−22.7	11:01	−21.6	12:14	−22.6
6:00	−13.9	10:59	−14.8	8:00	−22.0	12:08	−21.4	2:03	−22.3
7:01	−13.0	12:09	−15.0	9:08	−21.5	1:09	−20.8	4:00	−22.0
8:08	−12.0	1:05	−15.9	10:08	−20.6	2:17	−20.5	6:01	−22.0
9:06	−11.7	4:05	−20.2	11:10	−20.1	3:06	−20.8	7:02	−21.9
10:10	−11.9	4:34	−20.7	12:21	−19.7	4:06	−21.0	8:00	−22.2
11:04	−12.5	5:10	−20.2	2:08	−20.1	5:07	−21.7	9:02	−22.4
11:54	−13.2	5:40	−20.2	4:07	−21.8	6:34	−22.4	10:15	−22.5
2:06	−15.4	6:44	−19.5	6:17	−23.5	7:56	−22.9	10:50	−22.5
4:00	−15.7	7:57	−18.5	7:10	−23.3	9:08	−22.5	11:56	−22.5
6:03	−14.2	8:56	−17.4	8:20	−22.7	9:58	−22.5	1:05	−22.7
7:04	−13.4	10:00	−16.5	9:27	−21.6	11:01	−22.1	2:02	−22.4
8:26	−12.2	11:10	−15.9	10:07	−21.0	12:06	−21.9	3:00	−22.3
9:08	−12.1	12:10	−16.4	11:04	−19.9	2:08	−21.3	3:56	−22.1
10:07	−12.0	2:00	−18.3	12:12	−19.1	4:30	−21.4	4:56	−21.8
11:06	−12.7	3:56	−20.3	12:58	−19.0	6:14	−22.0	6:00	−21.6
12:01	−13.1	5:56	−21.1	2:22	−19.5	8:10	−22.7	6:43	−21.4
1:06	−14.3	7:05	−19.6	3:20	−20.2	9:18	−22.6	8:08	−21.4
2:20	−15.8	8:00	−18.7	4:11	−20.8	10:02	−22.4	9:37	−22.2

TABLE IV—*Continued*

P.M. November 5	P.M. November 8	A.M. November 11	P.M. November 13	P.M. November 16
10:56 −22.3	12:57 −23.1	9:15 −22.5	9:03 −26.8	2:24 −28.4
12:56 −23.0	2:04 −23.5	10:06 −22.7	10:00 −26.1	3:09 −28.9
2:10 −22.7	3:06 −23.8	11:04 −23.2	11:00 −25.6	3:58 −29.4
4:08 −22.5	4:10 −23.5	12:12 −23.5	12:00 −25.7	5:01 −29.9
6:07 −21.8	5:01 −23.2	12:56 −24.2	2:00 −27.1	6:16 −30.7
7:03 −21.8	6:02 −22.6	1:48 −25.0	4:08 −28.8	7:52 −30.9
7:54 −21.8	6:48 −22.0	3:01 −26.0	6:29 −29.1	9:05 −30.4
8:57 −21.8	7:55 −21.4	4:10 −26.6	8:00 −28.4	10:09 −29.7
9:59 −22.2	9:02 −21.2	5:02 −26.5	9:06 −27.5	10:58 −29.2
11:00 −22.6	10:03 −21.2	5:48 −26.3	10:02 −26.9	12:12 −28.6
11:56 −22.9	11:10 −21.7	6:46 −25.5	11:02 −26.6	1:59 −28.5
1:10 −23.4	12:14 −22.6	8:00 −24.6	11:59 −26.0	4:30 −29.8
2:03 −23.4	2:00 −23.9	9:02 −23.7	12:58 −26.3	7:09 −31.0
2:58 −22.8	4:04 −24.2	10:04 −22.9	1:54 −26.9	**C = +4.54**
3:58 −22.6	6:04 −23.2	11:00 −23.0	3:00 −28.2	8:06 −31.2
4:57 −22.0	6:58 −22.5	12:24 −23.4	4:00 −28.9	9:04 −31.1
5:58 −21.9	8:04 −21.8	1:58 −24.9	4:56 −29.3	10:05 −30.8
6:38 −21.3	9:04 −21.5	4:01 −26.4	6:04 −29.6	11:06 −30.0
7:50 −21.3	10:05 −21.6	6:20 −26.0	6:42 −29.8	12:04 −29.4
9:00 −21.3	11:25 −22.4	7:14 −25.4	8:04 −29.0	1:00 −29.2
10:02 −21.7	1:07 −23.7	8:06 −24.8	9:03 −28.2	2:04 −28.9
11:00 −22.0	2:22 −24.3	9:08 −24.1	10:12 −27.3	3:07 −29.6
12:08 −22.6	3:33 −24.5	10:06 −23.6	11:06 −26.9	3:56 −30.0
1:03 −22.9	4:28 −24.5	11:10 −23.7	11:57 −26.6	4:57 −30.8
2:03 −22.9	5:18 −24.1	12:16 −24.0	2:02 −27.5	5:46 −31.8
4:06 −22.5	6:06 −23.6	12:58 −24.6	4:02 −29.0	7:02 −32.5
6:03 −21.6	8:10 −21.8	2:03 −25.6	5:59 −30.0	8:00 −32.6
7:04 −21.5	9:11 −21.4	3:01 −26.5	7:49 −29.8	9:08 −32.6
8:00 −21.1	10:10 −21.4	3:57 −27.3	8:58 −29.0	10:02 −32.5
9:05 −21.1	11:13 −21.5	4:57 −27.5	10:10 −28.5	11:00 −31.9
10:02 −21.2	12:12 −22.2	5:52 −27.3	11:00 −27.8	12:04 −31.3
10:57 −21.4	1:56 −23.6	6:58 −26.8	11:55 −27.2	2:00 −30.9
11:57 −22.0	3:57 −23.9	7:59 −25.9	1:03 −27.4	4:02 −31.2
12:59 −22.4	5:55 −22.9	9:03 −25.2	2:08 −28.0	6:03 −32.7
2:12 −22.8	8:12 −21.2	9:50 −24.3	3:08 −28.5	7:20 −33.0
3:04 −22.5	9:24 −21.2	11:00 −24.2	4:04 −29.5	8:14 −33.4
4:08 −21.9	10:44 −21.4	12:08 −24.2	5:08 −30:0	9:12 −33.3
5:02 −21.7	12:14 −22.5	2:18 −26.3	5:49 −30.5	10.04 −33.0
6:01 −21.3	1:01 −23.0	4:09 −27.6	6:42 −30.5	11:02 −32.6
6:45 −20.8	2:00 −23.6	6:02 −27.7	8:09 −30.5	11:56 −31.7
7:42 −20.2	3:00 −24.3	7:22 −27.1	9:06 −29.9	1:03 −31.3
9:04 −20.3	4:06 −24.5	8:18 −26.3	10:01 −29.2	2:00 −30.9
10:03 −20.3	5:08 −24.1	9:16 −25.4	10:58 −28.6	2:56 −30.8
11:05 −20.7	5:56 −23.7	10:04 −24.9	12:10 −28.1	3:55 −30.9
12:04 −21.7	6:48 −22.5	11:00 −24.4	1:58 −28.3	4:56 −31.0
2:36 −23.0	8:54 −21.3	11:58 −24.6	4:11 −30.0	5:46 −31.6
4:11 −23.0	9:59 −20.9	12:55 −25.2	6:06 −30.8	6:40 −32.0
5:59 −22.0	11:03 −21.0	1:58 −26.0	7:04 −31.0	8:02 −32.7
7:09 −21.5	12:04 −21.4	3:06 −27.1	8:02 −31.0	9:14 −33.0
8:04 −21.4	2:01 −23.0	4:02 −28.0	9:02 −30.5	10:00 −33.0
9:00 −21.2	4:05 −24.3	5:07 −28.6	10:04 −29.7	11:02 −32.9
10:02 −21.4	6:03 −23.6	5:50 −28.9	11:01 −28.9	12:02 −32.3
11:00 −21.6	7:10 −22.8	6:42 −28.4	12:08 −28.3	2:01 −31.7
11:56 −22.1	8:20 −22.3	7:54 −27.7	1:10 −27.9	

TABLE IV—*Continued*

A.M. November 19	A.M. November 21	A.M. November 23	P.M. November 25	P.M. November 27
4:05 −32.0	4:03 −32.3	7:00 −30.8	12:13 −31.7	2:01 −34.4
6:00 −32.2	5:54 −31.7	8:00 −30.2	12:58 −32.7	3:09 −35.8
7:08 −33.0	7:08 −31.2	9:03 −30.5	1:58 −33.4	4:09 −36.7
8:00 −33.5	8:01 −31.6	10:02 −30.6	3:02 −34.4	5:03 −36.9
9:12 −33.6	9:00 −31.8	11:00 −31.1	4:02 −34.8	6:10 −36.7
10:02 −33.3	9:59 −32.3	12:00 −32.2	5:00 −34.6	7:22 −36.2
11:00 −33.0	11:02 −32.5	1:05 −32.6	5:48 −34.1	8:09 −35.5
12:02 −32.6	11:57 −32.8	2:12 −33.2	6:47 −33.6	9:20 −34.7
1:16 −32.5	1:00 −33.1	3:01 −33.2	8:00 −32.4	10:09 −33.8
1:52 −32.3	2:00 −32.7	4:00 −33.1	9:00 −31.6	11:00 −33.3
C = −1.25	2:56 −32.5	4:55 −32.4	9:57 −31.2	12:00 −33.3
	4:01 −32.0	6:02 −31.5	11:00 −31.3	1:54 −34.4
2:58 −32.3	4:55 −31.6	7:55 −30.2	12:00 −32.0	3:42 −36.1
4:04 −31.6	5:56 −31.6	8:59 −30.1	2:02 −34.0	5:28 −37.2
5:04 −31.2	6:46 −31.3	10:01 −30.1	3:57 −35.2	6:29 −37.1
5:42 −32.0	7:58 −31.3	1:07 −32.7	5:48 −35.2	8:11 −35.9
6:57 −32.0	9:10 −31.5	2:59 −33.3	7:00 −34.2	8:58 −35.1
8:03 −32.4	10:07 −32.3	4:58 −32.7	8:10 −33.1	9:58 −34.3
9:12 −33.3	11:06 −32.7	7:03 −31.0	9:10 −32.2	11:00 −33.7
10:04 −33.1	12:05 −33.2	8:10 −29.9	10:08 −31.7	12:10 −33.2
10:58 −33.6	2:25 −32.4	9:12 −29.7	11:03 −31.7	1:00 −33.4
12:06 −33.3	4:00 −32.5	10:00 −29.9	12:12 −32.4	2:01 −34.3
2:00 −32.6	6:00 −31.4	11:08 −30.9	12:56 −33.2	3:10 −35.5
4:03 −32.2	7:06 −30.9	11:58 −31.8	1:58 −34.2	4:00 −36.3
5:55 −32.1	8:00 −31.0	1:04 −32.9	3:02 −35.3	4:58 −37.0
7:04 −32.4	9:00 −31.3	2:00 −33.7	4:00 −36.0	5:54 −37.5
8:00 −32.6	10:07 −31.7	3:12 −34.1	4:57 −36.6	6:49 −37.4
9:16 −33.3	11:00 −32.1	4:07 −33.9	5:51 −36.4	8:00 −37.0
10:10 −33.3	11:54 −32.8	5:01 −33.5	6:44 −35.8	8:39 −36.4
11:00 −33.4	1:00 −33.4	5:55 −32.8	7:54 −34.5	10:10 −35.1
12:02 −33.6	2:04 −33.2	6:44 −32.1	9:00 −34.1	11:52 −34.1
12:57 −33.4	3:11 −33.3	8:04 −30.8	10:04 −33.0	3:07 −35.4
1:55 −32.9	4:06 −32.7	9:02 −30.5	11:08 −32.7	5:09 −36.0
3:00 −32.4	5:01 −32.1	10:05 −30.6	12:27 −33.4	6:58 −37.2
3:58 −32.0	5:48 −31.8	10:56 −30.9	1:58 −34.6	8:14 −36.5
4:56 −31.8	6:40 −31.2	12:00 −31.8	4:17 −36.6	9:22 −35.6
5:40 −31.6	7:54 −30.8	2:02 −33.6	5:57 −36.6	10:41 −34.7
6:46 −31.9	9:09 −31.0	4:08 −34.4	7:07 −35.6	11:56 −33.7
7:56 −32.0	9:58 −31.4	6:00 −32.9	8:14 −34.8	1:15 −33.8
9:00 −32.5	11:10 −32.1	7:11 −32.0	9:10 −33.7	2:10 −34.3
10:06 −33.0	12:12 −32.9	8:16 −31.1	9:58 −32.9	
10:50 −33.3	2:01 −33.3	9:06 −30.6	10:58 −32.8	
12:04 −33.3	4:01 −33.0	9:50 −30.3	12:01 −32.7	
1:57 −33.1	6:02 −31.4	10:57 −30.7	12:42 −33.2	

RYERSON PHYSICAL LABORATORY
February 15, 1914

THE VARIATION WITH TEMPERATURE OF THE ELECTRIC FURNACE SPECTRUM OF TITANIUM[1]

By ARTHUR S. KING

Certain features of the spectrum of titanium, when this metal is vaporized in the electric furnace, have been treated in previous papers by the writer. In the first[2] of these it was shown that a rich titanium spectrum may be produced by the furnace, and its leading characteristics were compared with those of the arc spectrum. The second paper[3] dealt with the pressure-shifts of titanium lines in the furnace; while a more recent one[4] takes up the conditions for the production of enhanced lines in this source, the discussion leading to a consideration of the "tube-arc," the spectrum of which is very different from that of the furnace when operated in the regular way.

In the present paper, the treatment of the titanium spectrum will follow the general plan of a recent paper[5] on the spectrum of iron. The purpose is to examine the spectra given by three stages of furnace temperatures in the range from 2000° C. to above 2600° C. The data thus obtained show the approximate temperature at which a line appears and the rate of growth in intensity as the temperature rises. The strength of the line in the arc spectrum is also noted and a classification of the lines is obtained similar to that given for the iron spectrum. The range of wave-length studied is approximately that of the visible spectrum, from λ 3888 to λ 7364.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 76.

[2] *Contributions from the Mount Wilson Solar Observotory*, No. 28; *Astrophysical Journal*, **28**, 300, 1908.

[3] *Contributions from the Mount Wilson Solar Observatory*, No. 60; *Astrophysical Journal*, **35**, 183, 1912.

[4] *Contributions from the Mount Wilson Solar Observatory*, No. 65; *Astrophysical Journal*, **37**, 119, 1913.

[5] *Contributions from the Mount Wilson Solar Observatory*, No. 66; *Astrophysical Journal*, **37**, 239, 1913.

APPARATUS AND METHODS

The tube resistance furnace and the two grating spectographs used in this work have been described in previous papers, the experimental method being very similar to that employed for the iron spectrum. The metal, "cast titanium," supplied by Eimer & Amend, was placed in the tube in a finely powdered condition. On account of the high melting-point of titanium (in the neighborhood of 1900° C.), a temperature close to 2100° C. as measured by a Wanner pyrometer was required before any considerable number of lines appeared. As nearly as I have been able to determine, even the lowest temperature lines do not appear below 2000° C. At about 2300° C. many lines have developed which are absent at 2100°. At 2300° to 2400° almost all lines have appeared which show in the arc spectrum, the absent ones being weak arc lines and the enhanced lines. The change from 2300° to 2600° has as its most interesting feature the differences in the rate of development of the lines which are present at the lower temperature. The highest temperatures employed may have been considerably above 2600° in the hottest part of the tube, but the relative intensities of the titanium lines were not greatly different from the condition at 2500°, except that, as was shown in a former paper,[1] at about 2600° we reach the stage when enhanced lines can be made to appear faintly in the furnace spectrum. The strong vaporization of carbon at these higher temperatures is disturbing, not only on account of the band spectrum in certain regions, but by reason of the strong continuous spectrum emitted, probably at least in part from the reflection of light from the incandescent wall of the tube by the vapor particles. This vaporization of the carbon causes the gain in temperature to be slow in proportion to the electrical energy expended, and with the size of tubes thus far used, superheating much above 2600° C. has not yielded photographs satisfactory for comparison with those taken at lower temperatures.

A set of photographs for the range from λ 3880 to λ 5700 was taken three years ago, the first order of the vertical plane-grating spectrograph being used with objective of 30 ft. (9.1 m) focal

[1] *Contributions from the Mount Wilson Solar Observatory*, No 65; *Astrophysical Journal*, **37**, 119, 1913.

length, the scale being about 2.05 A per mm. The requirements as to scale for this spectrum in the blue and violet are somewhat exacting, however, on account of the numerous close lines, and a decided gain was obtained in a set of plates taken during the past year in which the bright second order of a new grating was used for the spectrum to the violet of λ 4600, the scale being approximately 0.9 A per mm. Cramer "Crown" plates were used for this region. To supplement the former set, the first order of the same grating was employed for lines to the red of λ 4600, with Seed "Gilt-Edge 27" plates bathed with pinacyanol, pinaverdol, and homocol according to the formula of Wallace. A fairly uniform intensity of the spectrum was thus obtained from λ 4600 to about λ 6800, from which point the decreasing sensitiveness of the plates made furnace photographs with the long-focus instrument very difficult. For the few lines beyond λ 6800, I have relied upon bathed films used with a concave grating of 1 meter radius, the brightness of whose spectra makes it possible to photograph the furnace radiation as far into the red as titanium lines have been measured in the arc spectrum.

A number of photographs were made with this concave grating, covering with one exposure the whole visible spectrum, frequently simultaneously with exposures made with the large-scale instrument. While the estimates of intensities were based almost entirely on plates taken with the plane grating, the films taken with the concave were useful as supplementary material, especially for the low and medium temperatures, and showed the general variation of the spectrum with wave-length for the several temperatures and for the arc. They were especially useful by reason of the strong spectrum which could be obtained at low temperature, the large spectrograph giving the spectrum very faint for this condition; so that if a line did not appear on the film taken with the concave grating, it was reasonably certain that the temperature employed would not produce such a line.

The method of estimating line intensities was discussed in some detail in connection with the iron spectrum. The same system was used for the titanium lines. The effort has been to obtain correct relative intensities among the lines for each temperature,

a line distinctly outlined on the negative being graded "1," a fainter appearance being indicated by "trace." The difficult gap between reversed and unreversed lines could usually be bridged on a given plate by means of partially reversed lines, the relation of whose intensity both to unreversed and to completely reversed lines was fairly clear. A considerable degree of consistency can thus be attained for the intensity scale of lines at a given temperature. When different temperatures are compared, however, each temperature condition involves a certain adjustment of emission and absorption between the vapor at the center and that toward the ends of the tube, the effect being to give a line less density in proportion to its width at the higher temperatures. While this affects the photometric accuracy attainable in comparing lines for different temperatures, the differences on which the classification of lines is based are so decided that this grouping, which shows the stage for the initial appearance of a line and its rate of growth relatively to other lines, may be considered as governed by the actions attendant on temperature change.

The detection of possible blends with impurity lines has been assisted by the fact that furnace spectra were available for several of the elements most likely to be found with titanium. By this means it could be seen whether a line of one of these elements, even if weak in its own arc spectrum, was likely to occur at a given furnace temperature. The use of tubes of specially purified Acheson graphite, and the large scale employed in the blue and violet where the lines are most numerous, aided in the treatment of impurity lines. Vanadium was the most disturbing element in this regard, the stronger lines being given by the graphite tube as well as by impurities in the titanium. In a few cases, vanadium lines which might be present agreed so closely in position with titanium lines that it was necessary to ascertain the probable intensity of the vanadium line from the regular vanadium furnace spectrum and deduct this from the blend appearing on the titanium plate. Such cases are noted in the "Remarks" column of Table I.

EXPLANATION OF THE TABLE

Wave-lengths.—It was desirable in this work to use a table of wave-lengths covering the whole visible spectrum and including

the fainter lines visible in a strong arc spectrum of titanium. These requirements were filled satisfactorily by the tables of Exner and Haschek,[1] supplemented for the extreme red by the measurements of Fiebig[2] on the international system. The wave-lengths given by Rowland for the solar spectrum are not as complete for titanium as they are for iron, owing to the general faintness of the titanium spectrum in the sun. Many such lines are not identified by Rowland as titanium. However, for convenience in comparing with the solar spectrum, the solar wave-length most closely agreeing with the titanium line in the arc is entered in the second column, whether given by Rowland as titanium or not. In nearly all cases there is little question that the solar line is due at least in part to titanium. The wave-lengths of Hasselberg[3] are used in a few cases when close doublets were not resolved by Exner and Haschek. As far as the measurements of Hasselberg extend (to λ 5900), they agree, except in rare cases, with those of Exner and Haschek within 0.05 A.

Arc intensities.—These have been estimated by the writer from arc spectra taken on the same plates with some of the best furnace spectra. A carbon arc containing titanium in the lower (positive) terminal was used, with currents of 6 to 8 amperes, the central portion of the arc being projected on the slit. Usually several arc spectra with varying exposures were photographed on the same plate. It was necessary to select a strong arc spectrum for comparison with the furnace, since many titanium lines which are faint in the arc were found to show with considerable strength in the furnace. Many lines in the arc spectrum are somewhat diffuse, shading off from the center without well-defined edges. These nebulous lines are indicated by *n* after the intensity. There is some difficulty in rating their intensities on the same scale as is used for the sharper lines. The letters *R* and *r*, both for arc and for furnace lines, indicate complete and partial self-reversal, respectively. These symbols point out the lines most subject to reversal, but the degree of reversal depends upon the supply of vapor and whether the arc is quiet or explosive in its action.

[1] *Spektren der Elemente bei normalem Druck*, Leipzig, 1911.

[2] *Zeitschr. f. wiss. Phot.*, **8**, 73, 1910.

[3] *Astrophysical Journal*, **4**, 116, 212, 1896.

Furnace intensities.—The furnace spectrum was photographed at three temperatures, the centigrade values of which were about 2100°, 2300°–2400°, and 2600°–2700°. These are designated as "low," "medium," and "high," respectively. It will be noted that in Table I the intensities scale downward in general, for the three furnace temperatures. This results from taking the spectra just as they appear on the plates and using as the unit of intensity for each a line distinctly outlined on the plate. With the large-scale spectrograph, exposures of an hour or more at 2100° did not suffice to give a general intensity of the spectrum such as could be obtained with 2600° in two to three minutes. Even the most pronounced low-temperature lines did not come to the same absolute strength as on the high-temperature photographs. This does not interfere with the classification, however, since the important point is the relative change of different lines with increase of temperature.

Classification.—The class to which a line belongs is given in the seventh column. The method of forming classes, which was described in detail for the iron spectrum, has been adhered to as far as possible and may be given briefly here. *Class I* lines are relatively strong at low temperature and strengthen slowly at higher temperatures. For the iron spectrum this class was divided into I A and I B according to the strength of the line in the arc spectrum. A few Class I A lines occur with titanium, being faint in the arc, but it has been found convenient to use "A" with other classes which show this peculiarity of relative weakness in the arc as compared with the furnace, especially for Class III; so that lines corresponding to Class I B for iron, which are strong in the arc as well as in the furnace, will be given here as belonging to Class I. *Class II* lines appear at the lowest temperature but strengthen rather rapidly as the tube becomes hotter, and are strong in the arc, except in the case of a few lines placed in Class II A. The lines of *Class III* are absent or faint at low temperature, appear at medium temperature, and are usually considerably stronger at high temperature. Many titanium lines of this kind are relatively weak in the arc, so that for this spectrum Class III A becomes important. *Class IV*

TABLE I

TEMPERATURE CLASSIFICATION OF TITANIUM LINES

λ (Exner and Haschek)	Nearest Solar Line (Rowland)	Arc	Furnace			Class	Remarks
			High Temp.	Medium Temp.	Low Temp.		
3888.16.....	.179	4 *n*	2	Tr.	...	IV	
3890.12.....	.069	6	6	6	Tr.	III	
3895.42.....	.377	30 *n*	8	2	1	III	
3897.45.....	.482	1	1	.	.	IV	
3897.72.....	.785	1	1	....	.	IV	
3898.64.....	.645	8	6	6	1	III	
3900.72.....	.681	50		.	.	V E	
3901.14.....	.114	12	10*r*	6	2	II	
3904.99.....	.926	40*n*	25*R*	8	3	II	
3911.36.....	.316	8*n*	4	1	.	III	
3912.75.....	.732	2	1	....	..	IV	
3913.68.....	.609	40	...	....	...	V E	
3914.50.....	.477	35	18*R*	7	4	II	
3914.86.....	.880	5	5	5		III	
3916.00.....	5.951	3	6	4	..	III A	
3916.21.....	.207	3	Tr.	....	....	IV	
3919.95.....	.956	5	5	2	...	III	
3921.61.....	.563	30	15*R*	8	3	II	
3924.71.....	.673	50	30*R*	12	5	II	
3926.45.....	.465	10	4	Tr.	......	IV	
3930.04.....	.022	40	20*R*	10	6	II	
3934.35.....	.508	9	8	6	Tr.	III	
3938.13.....	.116	2*n*	1	......	....	IV	
3947.98.....	.918	40	30*R*	12	6	II	
3948.87.....	.818	60	50*R*	20*r*	8	II	
3956.50.....	.470	60	40*R*	18	10	II	
3958.39.....	.355	80	60*R*	25*r*	12	II	
3963.05.....	2.995	35	20*R*	12	6	II	
3964.48.....	.416	35	20*R*	12	6	II	
3980.98....	.963	1	1	....	.....	IV	
3981.95.....	.917	70*r*	50*R*	25*r*	10	II	
3982.63.....	.630	30	15*R*	10	6	II	
3984.46.....	.479	3	3	1	...	III	
3985.40.....	.385	3	1	Tr.	....	III	
3985.70.....	.745	3	2	Tr.	....	III	
3989.94.....	.912	80*r*	60*R*	30*r*	10	II	
3992.55.....	.538	2*n*	..		...	V	
3993.94.....	.978	1	Tr.			IV	
3994.84.....	.828	4*n*	4	1		III	
3998.80.....	.790	100*R*	70*R*	40*R*	12	II	
3999.47.....	.495	7*n*	3	1	..	III	
4002.62.....	.652	9*n*	5	2		III	
4003.95.....	.912	10*n*	5	2		III	
4006.10.....	.114	6*n*	4	1		III	
4007.31....	.310	3*n*	2	Tr.		IV	
4008.21.....	.215	9*n*	5	2	.	III	
4009.12.....	.079	35	25*R*	12	8	II	
4009.82.....	.807	15	12*r*	8	3	II	
4012.55.....	.541	10	.....	......		V E	
4012.95.....	.941	3	1	Tr.	..	III	

TABLE I—*Continued*

λ (Exner and Haschek)	Nearest Solar Line (Rowland)	Arc	Furnace: High Temp.	Furnace: Medium Temp.	Furnace: Low Temp.	Class	Remarks
4013.72	.729	12*n*	6	3		III	
4015.51	532	12*n*	6	3		III	
4016.40	434	6	3	1		III	
4017.10	114	3	1	Tr.		III	
4017.00	025	15*n*	7	4		III	
4022.00	018	25*n*	8	4		III	
4024.76	.726	35	25*R*	15	8	II	
4025.30	.286	4				V E	
4026.71	691	25*n*	8	4		III	
4027.64	623	4	Tr.			IV	
4028.48	497	5				V E	
4030.66	646	25*n*	10	4		III	
4031.90	942	3*n*	3	1		III	
4032.76	789	3*n*				V	
4034.05	052	6	4	1		III	
4035.04	018	5	2	Tr.		III	
4035.96	976	10	6	2		III	
4040.45	464	4*n*	4	1		III	
4043.91	956	2	1			IV	
4049.51	482	2*n*	2	Tr.		III	
4053.10	001	2	2	Tr.		III	
4053.96	981	4				V E	
4055.17	.189	20	10	8	Tr.	III	
4057.76	817	5	3	Tr.		III	
4058.29	372	7	5	1		IV	
4060.42	415	20	10	10	Tr.	III	
4064.39	361	15	9	6		III	
4065.24	239	15	9	8	Tr.	III	
4068.29	268	3	1			IV	
4069.15	115	4*n*	1			IV	
4071.41	501	2	1			IV	
4071.67	680	2	1			IV	
4074.55	488	3	1			IV	λ by Hasselberg, not given by Exner and Haschek
4076.53	516	4	8	8		III λ	
4077.20	221	4	1	Tr.		IV	
4078.62	631	30	18	15	2	III	
4079.86	863	6	3	1		III	
4082.65	589	20	12	12	Tr.	III	
4099.32	327	8	5	2		III	
4112.87	869	20	20*r*	15	5	II	
4120.16	202	2	1	Tr.		III	
4121.79	805	4	6	4		III	
4122.30	306	10	3	1		III	
4123.43	430	5*n*	1			IV	
4123.70	713	10	5?	2?		III?	Coincides with V. Probable intensity of V line subtracted
4127.67	689	15	4	2		III	
4129.30	337	4	2	Tr.		III	
4131.42	419	4	2	Tr		III	
4137.46	428	10*n*	5	3		III	
4140.03	089	2	1			IV	
4142.68	629	2	1			IV	

TABLE I—*Continued*

λ Exner and Haschek	Nearest Solar Line (Rowland)	Arc	Furnace High Temp.	Furnace Medium Temp.	Furnace Low Temp.	Class	Remarks
4143.22	.200	7*n*	4	1		III	
4143.41	430	3	2	Tr.		III	
4150.73	706	3	2	Tr.		III	
4151.12	.129	10	5	2		III	
4155.00	4.976	2	1			IV	
4159.80	.805	9	5?	2?		III?	Slightly affected in furnace by V blend
4161.70	.682	2				V E	
4163.83	.818	8				V E	
4164.31	.313	4	3	1		III	
4166.49	511	6	4	2		III	
4169.51	.409	7	5	2		III	
4171.20	213	8	5	2		III	
4172.10	.066	5				V E	
4173.71	710	3				V E	
4174.27	240	2				V E	
4174.65	647	3	2	Tr.		III	
4183.46	.480	4	2	Tr.		III	
4186.29	280	25	10	8	Tr.	III	
4188.84	894	5	3	Tr.		III	
4200.90	946	6	4	1		III	
4203.59	620	8	6	1		IV	
4211.88	.899	4	4	3		III	
4224.97	5.020	5	2			IV	
4227.81	822	5	3	Tr.		III	
4238.05	050	7	4	1		III	
4245.65	.671	2	2	Tr.		III	
4249.28	272	5*n*	4	1		III	
4251.74	.783	3	4	1		III	
4251.90	.905	2*n*	2	Tr.		III	
4256.19	177	8*n*	7	2		III	
4258.70	.774	4*n*	5	1		IV	
4260.91	.888	2	2			IV	
4261.79	.748	5*n*	6	2		III	
4263.31	.290	15	12	5	Tr.	III	
4265.44	418	3*n*	3	Tr.		III	
4265.88	.832	4	5	1		IV	
4266.39	.374	3*n*	4	Tr.		IV	
4270.30	329	7*n*	7	1		IV	
4272.58	590	8	12	6	1	III A	
4273.46	482	2	2			IV	
4274.75	746	15	15	9	Tr.	III	
4276.60	.587	8	10	3		III	
4276.80	836	2	3	Tr.		III	
4278.39	.390	7	6	1		IV	
4278.95	9.009	3*n*	4	1		III	
4280.25	194	2*n*	3	Tr.		III	
4280.49	494	1*n*	Tr.			IV	
4281.54	.530	10	10	8	2	III	
4282.87	.860	12	10	4		III	
4285.15	164	8	8	3		III	
4286.19	.168	25	25*r*	20	6	II	

TABLE I—*Continued*

λ (Exner and Haschek)	Nearest Solar Line (Rowland)	Arc	Furnace High Temp.	Furnace Medium Temp.	Furnace Low Temp.	Class	Remarks
4287.59	.566	22	25*r*	20	5	II	
4288.04	038	2	...	...	..	V E	
4288.30	.310	3	12	5	.	III A	
4289.26	.237	25	25*r*	22	8	II	
4290.09	080	3	3	Tr.	...	III	
4290.40	.377	8	Tr.		...	V E	
4291.19	174	22	25*r*	20	6	II	
4291.34	.375	5*n*	10	1		IV	Shaded toward red in furnace and arc. May be double
4292.84	827	1	4	1		III A	
4294.29	301	8		.		V E	
4295.93	914	22	25*r*	20	6	II	
4298.89	.828	40	40*R*	25	10	II	
4299.40	410	15	20	5	1	III	
4299.81	803	15	20	12	2	III	
4300.20	211	12	Tr.		.	V E	
4300.73	732	50	50*R*	30	15	II	
4301.24	262	50	50*R*	30	15	II	
4302.06	085	5				V E	
4303.13	072	1*n*	3	1		III A	
4305.65	614	2	4	1		III A	
4306.09	078	60	60*R*	40	20	II	
4307.10	017	1	5	2		III A	
4308.05	081	12	Tr.			V E	
4308.65	601	2	6	2		III A	
4309.23	198	1	1			IV	
4310.55	540	1	4	1		III A	
4311.81	880	2	2			IV	
4313.04	034	7				V E	
4314.52	479	5	10	6		III A	
4314.96	964	25	25	20	6	II	
4315.12	138	5	.			V E	
4318.85	817	10*n*	8?	3?		III	Slightly affected by blend with V and *Ca*
4321.15	119	1				V E	
4321.89	813	8*n*	6	2		III	
4323.63	670	1	5	2		III A	
4325.36	300	9*n*	7	3	.	III	
4326.54	.520	9	10	8	2	II	
4327.15	082	2	7	3		III A	
4330.91	.866	1	....	.		V E	
4335.03	4.965	2	8	4	Tr.	III A	
4338.09	.084	10	Tr.	..	.	V E	
4338.65	.616	1	1?			IV?	Diffuse in furnace. May belong to carbon band
4340.19	192	1	Tr.?			IV?	
4341.50	.530	2	.			V E	
4343.06	4.015	1	1		.	IV	
4344.44	451	3	..			V E	
4346.28	.278	5	5	1		IV	
4346.75	.725	1	1	..		IV	
4354.25	.228	3	3	1		III	
4355.48	500	1	2			IV A	

TABLE I—*Continued*

λ (Exner and Haschek)	Nearest Solar Line (Rowland)	Arc	Furnace High Temp.	Furnace Medium Temp.	Furnace Low Temp.	Class	Remarks
4360.68	.644	4	3?	1?	......	III?	Blend with V. Probable intensity of V line subtracted
4361.30	.219	1	1	....	....	IV	
4367.89	.882	1	......	......	......	V E	
4369.10	.052	2	2	......	.....	IV	
4369.87	.873	5n	4	Tr.	......	IV	
4372.57	.498	3	4	Tr.	....	IV	
4375.00	.643	1	1	....	......	IV	
4388.21	.260	3	4	1	.....	III	
4388.65	.571	1	1	......	......	IV	
4394.11	.093	8	8	2	......	III	
4394.27	.225	2	....	.....	.....	V E	
4395.19	.201	25	Tr.	.....	.....	V E	
4396.00	.008	1	.. .	....	.....	V E	
4399.94	.935	6	.	.....	.....	V E	
4404.42	.433	10	8	2	......	III	λ's by Hasselberg, given by Exner and Haschek as 4404.47
4404.57	.563	5	10	4	......	III A	
4405.04	.082	5	6	2	.....	III	
4405.83	.896	2	9	3	....	III A	
4412.60	.581	1	6	2	......	III A	
4416.70	.636	4	3?	1?	....	III?	Blend with V. Probable intensity of V line subtracted
4417.47	.450	15	10	5	Tr.	III	
4417.86	.884	8	....	....	......	V E	
4418.52	.499	1	....	......	......	V E	
4421.67	.616	1	1	.....	.....	IV	
4421.98	.928	6	6	2	......	III	Not enhanced, as given by Lockyer. Enhanced line is λ 4422.12
4423.01	2.985	10	12	7	2	II	
4424.56	.531	2	3	Tr.	.....	III	
4426.01	5.931	3	10	2	...	III A	
4426.20	.201	10	6?	2?	.	II?	Blend with V. Probable intensity of V line subtracted
4427.28	.266	40	25	15	2	III	
4430.21	.221	3	2	Tr.	.....	III	
4430.52	.524	7	10	4	....	III A	
4431.45	.453	4	4	1	.. .	III	
4432.76	.736	2	2	..	.....	IV	
4433.76	.742	3	2	Tr.	...	III	
4434.16	.168	15	20	8	Tr.	III A	
4434.54	.504	1	1	.	...	IV	λ by Hasselberg. Not given by Exner and Haschek
4436.70	.750	4	5	2	. .	III	
4438.39	.359	2	2?	.....	.	IV	May be partly *Sr*
4440.53	.515	10	10	3	Tr.	III	
4441.45	.433	4	5	1	....	IV	
4444.00	3.976	25	1	....	.....	V E	
4444.74	.728	1	......	......	.....	V E	
4449.35	.313	30	15	12	2	III	
4450.20	.267	1	1	......	....	IV	
4450.70	.654	4	....	...	.	V E	
4451.13	.087	25	15	10	2	III	
4453.52	.486	30	25	18	5	II	

TABLE I—*Continued*

λ Exner and Haschek	Nearest Solar Line (Rowland)	Arc	Furnace			Class	Remarks
			High Temp.	Medium Temp.	Low Temp.		
4453.91	.870	20	12	8	1	III	
4455.50	.485	30	25	18	5	II	
4457.61	.600	40	35	25	7	II	
4463.52	.569	8	7	2		III	
4463.70	.698	8	7	2		III	λ's by Hassel-
4464.62	.617	2				V E	berg. Given
4465.97	.975	20	12	8	1	III	by Exner and
4468.65	.663	25	1			V E	Haschek as
4469.38	.316	1				V	λ 4463.62
4471.04	.017	2	1			IV	
4471.43	.408	20	12	8	1	III	
4475.02	.026	8	10	5	Tr.	III	
4475.68 ...	.633	1	1			IV	
4479.88 ...	.879	9	9	3		III	
4480.78	.752	5	7	2		III A	
4481.40 ...	.438	30	18	12	2	III	
4482.87	.994	10	10	5		III	
4485.24 ...	.244	1	2			IV A	
4488.45 ...	.493	2				V E	
4489.25 ...	.262	20	15	8	1	III	
4492.73 ...	.790	3	4	1		III	
4495.18	.182	4	3	1		III	
4496.35	.318	20	15	5		III	
4496.44	.409	2	15	10	2	III A	Measured by
4497.91	.842	3	4	1		III	writer. In
4501.42	.445	25	1			V E	arc appears
4503.94 ...	.926	4n	5	1		IV	as satellite of
4505.89 .	.959	1	1			IV	λ 4496.35
4506.50 .	.497	2n	3			IV	
4508.20 .	.177	2	2			IV	
4508.44 ...	.455	1	1			IV	
4511.31 ...	.345	3	2			IV	
4512.90 ...	.996	40	40r	20	8	II	
4513.85 ...	.886	1	3			IV A	
4515.74 ...	.763	1	2			IV A	
4518.19	.198	50	50r	20	12	II	
4518.83 ...	.866	8	10	4		III	
4522.98 ...	.974	40	40r	20	12	II	
4526.52 ...	.579	1	4	1		III A	
4527.47 .	.490	35	35r	18	10	II	
4529.64	.656	1				V E	
4532.30	.306	1	1			IV	
4533.40	.419	80	80R	45	35	II	
4534.14 ...	.139	20	Tr.			V E	
4534.95 ...	.953	60	60R	30	25	II	
4535.71	.741	50	50R	25	20	II	
4536.12	.994	40	40r	20	15	II	λ's by Hassel-
4536.25	.222	40	40r	18	12	II	berg. Given
4537.40 ...	.389	2	2			IV	by Exner and
4539.26 ...	.263	3	2			IV	Haschek as
							λ 4536.10

TABLE I—*Continued*

λ (Exner and Haschek)	Nearest Solar Line (Rowland)	Arc	Furnace			Class	Remarks
			High Temp.	Medium Temp.	Low Temp.		
4540.66.....	.672	1	10	5	.....	III A	May be slightly
4541.05.....	.043	1	3	1	.....	III A	affected by *Cr*
4544.88.....	.864	30	30*r*	15	8	II	λ 4540 64
4547.51.....	.587	1	Tr.	.....	.....	IV	
4548.01.....	.024	2	2	Tr.	.....	III	
4548.30.....	.301	2	2	Tr.	.....	III	
4548.93.....	.938	35	35*r*	18	12	II	
4549.80.....	.808	25	Tr.	.....	.....	V E	
4552.70.....	.725	35	35*r*	18	12	II	
4555.27.....	.264	3	3	Tr.	....	III	
4555.70.....	.662	30	30*r*	15	10	II	
4558.03.....	.060	2	2		...	IV	
4558.30.....	.285	2	2	.	..	IV	
4560.10.....	.102	6	8	3	.....	III	
4562.80.....	.814	6	12	10	3	II A	
4563.61.....	.599	5	4	1	.	III	
4563.93.....	.939	15	.....	...	.....	V E	
4564.36.....	.352	1	3	1	.....	III A	
4571.07.....	.095	3*n*	3	Tr.	.....	III	
4572.15.....	.156	15	.....	.....	.....	V E	
4585.97.....	6.047	2	Tr.	.....	.....	IV	
4590.12.....	.126	3	.....	.....	.....	V E	
4599.40.....	.408	5	1	.....	.....	IV	
4609.54.....	.540	2	.....	.....	.....	V	
4614.45.....	.388	1	.....	.....	.....	V	
4617.40.....	.452	30	15	8	2	II	
4619.06.....	.607	3	*	.....	.....	IV?	
4623.25.....	.279	25	12	5	1	III	
4629.47.....	.521	15	8	3	Tr.	III	
4635.67.....	.736	3	*	...	.....	IV?	
4637.36.....	.352	2	*	.....	.	IV?	
4638.00.....	.050	8	3?	.....	.....	IV?	Furnace line may
4639.49.....	.538	18	10	3	1	III	be due to *C*
4639.79.....	.846	15	10	3	1	III	
4640.10.....	.119	15	10	3	1	III	
4640.55.....	.468	2	2?	.....	.....	IV?	Furnace line may
4645.36.....	.368	12	6	2	Tr.	III	be due to *C*
4650.18.....	.193	10	5	2	Tr.	III	
4655.84.....	.832	3	3?	.....	.....	IV?	Furnace line may
4656.20.....	.228	6	5	2	.....	III	be due to *C*
4656.63.....	.644	25	20	12	10	I	These lines are
4667.77.....	.766	25	18	12	10	I	given very
4668.55.....	.550	2	*	.....	.....	IV?	strongly by
4675.30.....	.294	10	10	3	Tr.	III	vapor near
4677.15.....	.096	1	*	.....	.....	IV?	ends of fur-
4677.68.....	.604	2	*	.....	.....	IV?	nace tube
4682.10.....	.088	30	20	15	12	I	Similar to λλ4657
4684.71.....	.702	2	*	.....	.	IV?	and 4668
4687.10.....	.114	4	*	.....	..	IV?	
4688.56.....	.554	3	*	.....	.....	IV?	

*Occurrence uncertain on account of strong carbon spectrum at high temperature, but *Ti* lines are weak in furnace, if present at all. Provisionally placed in Class IV.

TABLE I—*Continued*

λ (Exner and Haschek)	Nearest Solar Line (Rowland)	Arc	Furnace			Class	Remarks
			High Temp.	Medium Temp.	Low Temp.		
4691.02.....	0.977	3	*	....	......	IV?	
4691.50.....	.523	20	12	8	2	II	
4693.88.....	852	5	10	6	2	II A	
4697.13.....	.101	4	*	..	.....	IV?	
4698.97.....	.946	20	12	7	2	II	
4709.15.....	.153	1	*	..	...	IV?	
4710.38.....	368	18	15	4	Tr.	III	
4715.48.....	474	4	12	6	3	II A	
4722.79.....	.797	10	10	4	Tr.	III	
4723.35.....	.359	10	10	4	Tr.	III	
4731.35.....	.356	9	4	1	..	III	
4733.57.....	.604	6	*	1	.....	III?	
4734.86.....	.847	3	*			IV?	
4742.32.....	307	3	1	.	...	IV	
4742.98.....	979	20	8	3	....	III	
4747.46.....	469	1	4	1	.	III A	
4747.87.....	.868	3	2	1	..	III	
4758.30.....	308	25	12	3	Tr.	III	
4759.08.....	.107	4	9	3	Tr.	III A	
4759.43.....	.403	25	12	3	Tr.	III	
4764.10.....	108	1	...	.	......	V E	
4766.51.....	.621	4	2	Tr.	.....	III	
4769.98.....	991	4	3	Tr.	...	III	
4771.30....	.279	3	8	2	...	III A	
4778.43....	441	10	5	2	..	III	
4780.17....	169	2				V E	
4781.90....	913	6	12	4	Tr.	III A	
4792.70....	.702	10	6	2	.	III	
4796.41.....	373	6	4	Tr.	....	III	
4798.17.....	.169	5	2	Tr.	..	III	May be slightly affected by F blend
4800.00.....	0.084	12	8	2	.	III	
4805.24....	.285	4	...			V E	
4805.60.....	.606	12	9	2	...	III	
4808.70.....	.733	5	3		.	IV	
4811.27.....	235	4	2	.		IV	
4812.43.....	.427	5	4	1		III	
4819.25.....	.205	2	1		.	IV	
4820.50.....	.503	20	15	8	2	II	
4825.65.....	666	3	2		..	IV	
4827.76.....	.804	2	?			IV?	Blend with strong F line
4836.29.....	.313	6	4	1		III	
4841.08....	074	25	20	15	12	I	
4844.16....	.210	2	1			IV	
4848.60.. .	605	8	5	1		IV	
4856.20.....	203	20	12	3		III	
4864.39....	362	4	3	1		III	
4868.42....	451	18	10	3		III	
4870.31 ...	323	20	12	4		III	
4881.13.....	128	3	2			IV	
4882.54....	518	2	1			IV	
4885.25....	264	20	15	8	2	II	

TABLE I—*Continued*

λ (Exner and Haschek)	Nearest Solar Line (Rowland)	Arc	Furnace High Temp.	Furnace Medium Temp.	Furnace Low Temp.	Class	Remarks
4892.03.....	.047	1	Tr.	....	...	IV	λ by Hasselberg.
4893.26....	.228	2	1	...	..	IV	Not given by
4893.64.....	.606	2	Tr.	...		IV	Exner and
4900.10.....	095	20	15	7	1	III	Haschek
4901.15.....	152	1	Tr.	...	...	IV	
4909.28.....	.283	2	6	2	.	III A	
4913.80.....	.803	20	12	7	1	III	
4915.40.....	414	5	4	1		III	
4920.05.....	.047	10	8	2	.	III	
4921.96.....	963	12	9	2		III	
4925.59.....	.594	5	5	1	...	IV	
4926.34.....	.334	4	10	3		III A	
4928.50.....	.511	12	10	2	.	III	
4937.93.....	.902	4	12	3	..	III A	
4938.47.....	467	8	4	Tr.	.	IV	
4941.76.....	.752	3	1		..	IV	
4948.18.....	.120	1	4	2	....	III A	
4948.39.....	.368	3	1	.		IV	
4958.49.....	.431	2	5	1	....	III A	
4964.94.....	.903	5	5	1	..	III	
4968.74.....	.769	6	6	2	...	III	
4973.21.....	.281	6	5	1	.	III	
4975.50.....	.530	10	4	1	..	III	
4976.99.....	7.056	1	Tr.	...		IV	
4977.95.....	.891	5	4	Tr.	...	IV	
4978.39.....	372	10	8	2	.	III	
4981.93.....	.912	60	50r	30	20	II	
4989.30.....	.325	10	9	3	...	III	
4991.24.....	.247	50	40r	25	18	II	
4997.27.....	.283	8	12	12	6	I A	
4999.68.....	.689	45	40r	25	18	II	
5001.16.....	.165	10	15	5	.....	III A	
5007.35.....	.398	40	35	20	15	II	
5009.79.....	.829	7	15	12	5	I A	
5013.47.....	.479	18	10	3	..	III	
5014.39.....	.369	50	50r	30	25	II	
5016.29.....	.340	20	15	12	6	II	
5020.20.....	.208	25	20	15	8	II	
5023.02.....	.052	25	20	15	8	II	
5025.01.....	.027	20	18	12	7	II	
5025.75.....	.749	18	8	4	Tr.	III	
5036.08.....	.189	25	15	12	4	II	
5036.65.....	.645	25	15	12	4	II	
5038.59.....	.579	25	15	10	3	II	
5040.14.....	.138	22	30	20	15	I	Given strongly
5040.79.....	.787	6	9	4	1	III A	by vapor near
5043.75.....	.761	7	12	5	1	III A	ends of furnace
5044.43.....	.394	2	1	....	...	IV	tube.
5045.58.....	.582	5	10	4	1	III A	
5053.06.....	.056	8	*	*		IV?	
5054.29.....	.261	3	*	......	..	IV?	

TABLE I—*Continued*

λ (Exner and Haschek)	Nearest Solar Line (Rowland)	Arc	Furnace High Temp.	Furnace Medium Temp.	Furnace Low Temp.	Class	Remarks
5062.27.....	.285	7	3	1	.....	III	
5064.25.....	.244	4	Tr.	.....	.....	IV	
5064.79.....	.836	25	30	20	18	I	Similar to λ 5040.14
5066.15.....	.174	7	8	3	.....	III	
5068.48.....	.485	3	Tr.	.....	.....	IV	
5069.50.....	592	5	2	.....	.....	IV	
5071.63.....	.666	7	6	3	.....	III	
5085.50.....	.513	4	3	1	.....	III	
5087.22.....	.239	8	8	4	1	III	
5098.56.....	.492	1	*	.....	.....	IV?	
5103.32.....	.297	2	*	.....	.....	IV?	
5109.60.....	.601	4	*	1	.....	III?	
5111.23.....	.138	1	*	.....	.....	IV?	
5113.60.....	.617	10	8	5	1	III	
5120.59.....	.592	12	10	3	.....	III	
5145.61.....	.636	12	12	6	2	II	
5147.63.....	.652	10	20	15	15	I A	
5152.35.....	.361	10	20	15	15	I A	
5173.92.....	.917	30	30*R*	20	20	I	
5186.49.....	.497	3	4	1	.....	III	
5188.80.....	.863	4	.....	.....	.....	V E	
5193.12.....	.139	35	35*R*	20	20	I	
5194.21.....	.216	4	6	1	.....	IV A	
5201.29.....	.260	4	4	1	.....	III	
5206.37.....	.215	5	3?	2?	?	III?	Blend with *Cr*. Probable intensity of *Cr* line subtracted
5208.04.....	.038	3	3	1	.....	III	
5210.59.....	.555	40	40*R*	30	20	I	
5212.47.....	.503	3	2	1	.....	III	
5218.24.....	.369	1	Tr.	.....	.....	IV	
5219.86.....	.875	8	15	12	6	I A	
5222.87.....	.849	6	4	2	.....	III	
5223.82.....	.791	6	4	2	.....	III	
5224.50.....	.471	15	8	6	1	III	
5224.75.....	.712	6	4	3	.....	III	
5225.12.....	.101	8	6	4	.....	III	
5226.75.....	.707	3	.....	.....	.....	V E	
5238.76.....	.742	6	10	8	1	III A	
5246.28.....	.310	2	.....	.....	.....	V	
5246.72.....	.733	3	8	4	.....	III A	
5247.46.....	.466	5	4	1	.....	III	
5251.10.....	.085	2	6	2	.....	III A	
5252.23.....	.276	8	12	10	8	I A	
5255.98.....	.973	5	4	1	.....	III	
5260.15.....	.142	3	1	.....	.....	IV	
5263.68.....	.669	3	3	1	.....	III	
5266.10.....	.141	10	7	4	Tr.	III	
5282.51.....	.576	3	5	4	Tr.	III A	
5283.60.....	.613	8	5	4	Tr.	III	
5284.52.....	.453	2	4	2	.....	III A	
5295.96.....	.955	4	5	4	1	III	
5297.40.....	.407	6	4	3	.....	III	

TABLE I—*Continued*

λ (Exner and Haschek)	Nearest Solar Line (Rowland)	Arc	Furnace			Class	Remarks
			High Temp.	Medium Temp.	Low Temp.		
5298.58.....	.672	4	2?	1?	......	III	Slightly affected
5300.20.....	.152	1	5	2	......	III A	by *Cr* in fur-
5336.94.....	.974	2	......	......	......	V E	nace
5338.48.....	.517	1	5	3	......	III A	
5341.67.....	.670	1	*	......	......	IV?	
5351.31.....	.261	4	3	1	......	III	
5366.83.....	.827	2	8	5	Tr.	III A	
5369.85.....	.782	4	2	1	......	III	
5389.35.....	.371	2	6	5	1	III A	
5390.20.....	.203	3	3	2	......	III	
5396.80.....	.778	1	8	8	4	I A	
5397.28.....	.344	4	4?	3?	......	III	Blend with *Fe*.
5404.24.....	.357	2	1	......	......	IV	Probable in-
5409.81.....	.823	6	8	5	2	II	tensity of *Fe*
5418.99.....	.979	1	......	......	......	V E	line subtracted
5419.41.....	.421	1	4?	3	......	III A	Blend with *C* at
5426.48.....	.474	3	20	15	8	I A	high tempera-
5429.35.....	.349	6	6	2	......	III	ture
5436.94.....	.938	1	4	3	1	III A	
5438.53.....	.507	1	3	2	......	III A	
5446.85.....	.797	2	12	10	4	II A	
5449.38.....	.369	1	3	2	......	III A	
5453.90.....	.860	3	6	4	Tr.	III A	
5460.74.....	.721	4	25	20	15	I A	
5471.41.....	.414	5	6	5	1	III	
5472.92.....	.916	2	7	6	1	III A	
5474.42.....	.436	6	8	6	2	II	
5477.92.....	.901	8	6?	4	1	III	Blend with *C* at
5481.66.....	.652	6	3	1	......	III	high tempera-
5482.10.....	.078	5	9	6	3	I A	ture
5488.42.....	.374	5	3?	2	......	III	Blend with *C* at
5490.37.....	.367	12	10	8	4	II	high tempera-
5504.10.....	.117	8	3	1	......	III	ture
5511.99.....	.872	2	4	2	Tr.	III A	
5512.72.....	.741	25	20	15	8	II	
5514.55.....	.563	20	20	12	7	II	
5514.72.....	.753	25	20	15	8	II	
5565.70.....	.700	9	7	4	1	III	
5644.31.....	.365	18	12	5	......	III	
5648.80.....	.796	5	6	1	......	IV	
5662.35.....	.374	12	12	3	......	III	
5663.11.....	.155	4	4	1	......	III	
5675.59.....	.647	9	12	3	......	III A	
5680.15.....	.149	2	3	1	......	III	
5689.66.....	.694	10	8	3	......	III	
5702.86.....	.876	6	5	1	......	III	
5708.41.....	.317	3	3	Tr.	......	IV	
5712.08.....	.098	4	4	1	......	IV	
5714.09.....	.120	3	3	Tr.	......	IV	
5715.30.....	.308	9	6	2	......	III	
5716.64.....	.671	4	3	1	......	III	

TABLE I—*Continued*

λ (Exner and Haschek)	Nearest Solar Line (Rowland)	Arc	Furnace			Class	Remarks
			High Temp.	Medium Temp.	Low Temp.		
5720.04	.666	3	3	1	......	III	
5739.66	.698	9	5	1	.	III	
5740.10	.195	4	2	Tr.	...	IV	
5741.43	.432	1	1	..	.	IV	
5753.10	.105	1	1	...	...	IV	
5757.10	.037	1	Tr.	.		IV	
5762.50	.479	4*n*	2*n*			IV	
5766.53	.550	4*n*	2*n*			IV	
5774.23	.250	5*n*	2*n*			IV	
5781.01	.024	2	2	Tr.		III	
5786.19	.193	5*n*	3*n*	Tr.		IV	
5797.70	.715	1	Tr.			IV	
5804.61	.681	5*n*	4*n*	Tr.		IV	
5823.97	.910	3	3	1		III	
5866.69	.675	35	20	30	10	II	Relatively weak at high temperature, though slightly wider than at medium temperature
5880.55	.490	5	10	5	1	III A	
5899.53	.518	25	15	20	6	II	Similar to λ 5867
5903.56	.555	5	10	4	1	III A	
5918.81	.773	10	12	10	3	II	
5922.40	.334	18	15	15	5	II	Similar to λ 5867
5938.05	.035	6	12	6	1	III A	
5942.00	1.985	12	18	12	4	II A	
5953.41	.386	30	15	8	2	II	
5966.06	.055	30	15	8	2	II	
5978.72	.768	25	15	8	2	II	
5988.80	.785	2	3	1		III	
5995.90	.916	2				V	
5996.24	.247	2	2	1		III	
5999.25	.204	4*n*	2*n*			IV	May be close double
5999.90	.920	8	5	2		III	
6064.83	.853	9	15	8	2	II A	
6085.49	.470	20	15	12	4	II	
6091.43	.395	20	9	4		III	
6093.05	.030	4	5	2		III	
6098.92	.870	7	2	Tr.		III	
6121.26	.215	3	4	1		III	
6126.40	.435	20	15	15	5	II	
6138.62	.725	1	1			IV	
6146.44	.445	3	4	2		III	
6149.94	.950	2	2	Tr.		III	
6186.34	.424	3	3	1		III	
6215.48	.360	20	8	2		III	
6220.72	.700	12	7	2		III	
6221.60	.552	8	6	2		III	
6258.34	.322	40	20	20	12	II	
6258.93	.927	50	25	25	15	II	

TABLE I—*Continued*

λ (Exner and Haschek)	Nearest Solar Line (Rowland)	Arc	Furnace High Temp.	Furnace Medium Temp.	Furnace Low Temp.	Class	Remarks
6261.32	.316	35	20	20	12	II	
6303.93	.985	10	12	6	1	III	
6312.42	.456	10	12	6	1	III	
6318.20	.329	5	9	3	Tr.	III A	
6336.26	.329	8	10	4	1	III	
6366.60	.564	8	10	4	1	III	
6497.95	.840	3	5	3		III A	
6508.37	.380	3	5	2		III A	
6546.52	.479	20	20	8		III	
6554.49	.470	20	20	12	1	III	
6556.31	.308	25	25	15	1	III	
6565.90	783	4	.....	.....		V	
6575.39	.270	3	..	...		V	
6599.35	.353	12	25	15	1	III A	
6743.37	.381	10	30	18	1	III A	
6861.76	770	6	4	1		III	
7039.060*	.040	6	2	.....	.....	IV	The photographic sensitiveness decreases rapidly in this region. The relative faintness of the medium temperature spectrum and the absence of low-temperature lines may be in part due to this condition
7069.344*	.350	2	1	....		IV	
7100.208*	150	2	Tr.?	...		IV?	
7209.786*	.780	20	20	4	..	III	
7216.590*	482	5	5	1		III	
7245.218*	.152	10	10	2	.	III	
7252.070*	.032	8	8	2	....	III	
7345.068*		4	4	.....		IV	
7358.028*		3	3		..	IV	
7364.465*		2	2			IV	

* λ according to Fiebig (International System).

lines for titanium are among the weaker arc lines, being usually below intensity 4, and appear only at the highest furnace temperature, sometimes faintly at medium. Their intensity at high temperature, however, is comparable with that in the arc. *Class V*, which for the iron spectrum embraces a large number of lines, frequently very strong in the arc but faint or absent at the highest furnace temperature, is limited for titanium almost entirely to the enhanced lines. The latter are indicated for brevity as V E. This difference is based on an important feature of the titanium spectrum, in that almost all lines of the arc spectrum, excepting enhanced lines, can be produced with fair strength in the furnace.

GENERAL CHARACTERISTICS OF THE FURNACE SPECTRUM

1. *Lines divided according to initial appearance.*—The titanium lines may readily be classed as low-, medium-, and high-temperature lines, according to the stage at which they first appear, by an inspection of the three columns of Table I devoted to the furnace intensities. On this basis, lines of Classes I and II belong to low temperature, those of Class III to medium, and those of Class IV to high temperature. Lines of Class V for titanium, these being chiefly enhanced lines, are produced with great difficulty if at all by the furnace, requiring the discharge conditions of an electrical source to give them strongly. This basis of division leaves out of account the rate of strengthening with increase of temperature, which is perhaps the most important feature of the classification here used.

2. *Important types of furnace lines.*—Titanium shows comparatively few lines for which a low temperature is especially favorable. Nineteen lines are placed in Class I, ten of which, owing to their relative weakness in the arc, belong to the sub-class I A. When the spectrum was photographed with the 1-meter concave grating, a lens of much shorter focal length was used to focus the light from the tube on the slit than was employed with the large spectrograph. By suitable adjustment of this lens, the slit could be illuminated mainly by light near the end of the furnace tube rather than from the middle of its length. The resulting spectrum was essentially the same as that photographed by the plane grating at the same temperature, but the concave showed an exaggerated intensity for the lines of Class I, as if there were enough difference between the temperatures at the middle and end of the tube to give an unusual strength to these lines. This gave additional evidence of their low-temperature origin.

A large proportion of the strongest lines in the spectrum, both for the arc and for the furnace, belong to Class II. In addition to isolated lines and small groups throughout the spectrum, three notable collections of Class II lines are found near λ 4300, λ 4530, and λ 5000. About the same rate of intensity increase from low to high temperature is observed for most of the lines of this class, and the scale adopted usually places the line given by the high-

temperature furnace as about equal in strength to that of the arc. Multiplying the intensity at medium temperature by 2 and that at low temperature by 3 will give approximately equal values for the arc and the three furnace intensities in the case of most of the Class II lines.

The lines of Class III, which appear distinctly at about 2300° C., are very numerous for titanium. A part of the lines of this class are strong also in the arc, but one of the most striking features of this spectrum is the large number of Class III A lines, especially to the red of λ 4300. The greater part of these are not more than half as strong in the arc as in the high-temperature furnace, while many would not show at all if a strongly exposed arc spectrum had not been used for this comparison. The total number (64) of these Class III A lines is so large as to be accounted for only as showing a real physical condition whereby only a narrow range of temperature excitation is favorable for their production. They do not show at the lowest temperature, and they weaken under the conditions of the arc discharge. The less numerous lines in Classes I A and II A, the former of which is important in the iron spectrum, are similar to those of Class III A in their arc behavior but appear at lower furnace temperatures. If the arc spectrum be considered as resulting mainly from a higher temperature stage than that of the furnace, the light-vibration emitting Class III A lines is given strongly by a degree of this excitation embraced between narrow limits. An alternative view is that there is a fundamental difference in kind between the arc and the furnace in the method of causing the light-vibration. When extensive material for the furnace spectra of a number of elements is available, a comparison with the spectrum of the arc under various conditions may justify a more definite conclusion than it is at present possible to draw.

The nebulous lines, indicated by *n* in the intensity column, are distinctly favored by the arc conditions. Unlike the nebulous lines of iron, the furnace at high temperature gives those of titanium with considerable intensity, but their strength falls off rapidly at medium temperature, where in most cases they are faint. With one exception (λ 3904.99), these lines belong in Classes III and IV. Their diffuseness is usually less marked in the furnace than in the

arc. This probably results from the furnace being operated in a partial vacuum, which should result in a sharper appearance for lines of this kind. An important group in the yellow, however, consisting of λλ 5763, 5767, 5774, 5786, 5804, retains some diffuseness at the high temperature of the furnace.

3. *Change of general intensity with wave-length.*—The data on this point have been obtained from films taken with the 1-meter concave grating, covering at one exposure the visible spectrum. As a salicylic acid filter was generally used, absorbing light of wave-length shorter than λ 3500, there are as yet but few data on the extension into the ultra-violet with changing temperature. From several films taken without absorbing filter, however, the condition appears to be similar to that observed for iron, namely, that lines farther to the violet appear as the temperature rises.

The distribution of line intensities in the visible spectrum also resembles that for iron, and does not correspond with what would be expected if the metallic vapor followed closely the intensity gradation shown by the spectrum of an incandescent solid. With spectra at different temperatures taken on the same film, the red end suffered a greater weakening at low temperature than the violet, lines to the red of λ 5300 being faint at lower temperatures even with the bright spectra of this instrument. It is somewhat difficult to say what should be expected in this regard, on account of the lack of known series and the tendency of lines of certain classes to collect in groups, but it is clear that the strong low-temperature lines do not extend beyond the green, the red end requiring a temperature of about 2400° to show any considerable number of lines.

A comparison of the arc and the furnace in this regard shows a more distinct relation. When an arc spectrum was taken on the same film with that of the furnace, the latter being hot enough to show lines throughout the visible region, the general intensity of the arc spectrum increased toward the violet more rapidly than the furnace spectrum. As the comparison arc could not conveniently be placed in the same position as the furnace tube, a different optical system was required in focusing the image on the slit. Quartz windows and lenses were used throughout for work with the

concave grating, and when the arc was used in front of the furnace and with a lens of shorter focus than that used for the furnace photograph, approximately the same degree of absorption was attained for each source. The steady strengthening of the arc spectrum toward shorter waves in comparison with the furnace spectrum was then very decided. The effect could be varied in an interesting way by placing the arc off to one side and reflecting its light from a silvered mirror through the lens used for the furnace. The decrease toward the violet in the reflecting power of the mirror then served almost to equalize the general intensity of the arc and furnace spectra, though a slight strengthening toward the violet remained for the arc. The comparison of the arc and furnace thus shows a different relation from that which seems to hold for different temperatures of the furnace, the evidence at present being that the movement of higher intensities toward the violet for the arc does not result from temperature difference alone. Evidence from the spectra of other elements is needed on this point.

4. *Absence of the band spectrum.*—The numerous flutings in the titanium spectrum, extending from the green into the red and always shaded toward greater wave-length, are well known. They are given by the metal in the carbon arc when burning in open air, and observers have generally followed Fowler in ascribing them to the oxide, though Kayser[1] considers this as not fully proved, since the oxide, which gives them most strongly, may always be dissociated in the arc or spark, in which case the band spectrum might come from the metal. The furnace, as used in this investigation, does not show the band spectrum, which makes it improbable that this comes from the metal itself. This fact was very clear on many plates and films taken with both the large and small spectrographs, in which strong bands were given by the arc used for comparison, but no trace of these appeared in the furnace spectra, sometimes taken for three different temperatures on the same plate. This shows clearly that vaporization at lower temperature than the arc, at least in the range employed here, does not give the bands for the metal itself. I have not made tests at atmospheric pressure or with any titanium compounds to see under what conditions the

[1] *Handbuch der Spectroscopie*, 6, 704.

bands will appear in the furnace. The small amount of oxygen remaining when the furnace is pumped out is probably taken up rapidly by the carbon tube, so that the furnace is a favorable source for tests as to whether a pure metal gives a banded spectrum.

COMPARISON WITH THE ARC SPECTRUM

One of the most striking features of the titanium furnace spectrum is its richness in lines as compared with the arc spectrum. The preceding table contains 625 lines. Of these only 9 are placed in Class V. These appear in the arc (all of low intensity), but could not be identified with certainty in the furnace spectrum. Forty-four other lines occur in the arc tables of Exner and Haschek and, strictly speaking, should go into Class V; but they are so very faint, some of them not appearing at all on my arc photographs, that little would be gained by entering them. This means that, aside from the enhanced lines, almost the entire arc spectrum, up to a certain low minimum of intensity, is given by the high-temperature furnace with an intensity so nearly comparable with that of the arc as to justify placing the lines at least in Class IV. The capacity of the furnace to produce difficult lines has not been fully tested. It was shown in a former paper[1] that the stronger enhanced lines between λ 4300 and λ 4600 were given faintly at the highest temperature of the furnace, and longer exposures under these conditions would doubtless add to the list. The furnace, however, is at a decided disadvantage in producing the enhanced lines, which have been shown, especially by the latest experiments with the tube-arc,[2] to require physical conditions distinctly different from those favorable to the arc lines.

In the relative richness of the arc and furnace spectra, titanium offers a strong contrast to the conditions observed for the iron spectrum, where a large number of lines, frequently strong in the core of the iron arc and not to be classed as enhanced lines, required as extreme conditions of furnace temperature as have been needed

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 65; *Astrophysical Journal*, **37**, 119, 1913.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 73; *Astrophysical Journal*, **38**, 315, 1913.

for the titanium enhanced lines. In the study of the iron spectrum,[1] it was shown that different regions of the arc, from the core to the outer envelope of vapor, affected the relative intensities of various groups of lines in much the same way that they are affected by different temperatures of the furnace. A horizontal section of the arc projected upon the slit of a plane-grating spectrograph thus served to give a classification of lines similar in its main features to that given by the furnace. The titanium arc has also been examined in this way, both photographically and visually. Currents of 5 to 20 amperes were employed with either loaded carbon terminals or electrodes of the metal. The arc under these conditions consists of a brilliant column of vapor of rather uniform luminosity throughout its width. It would be difficult to pick out the furnace classes in the spectrum of this arc, as may be done to some extent with the iron arc. Most of the arc lines show about the same gradual diminution of intensity from the core to the outside of the luminous column, where the lines end rather abruptly, probably as a result of the titanium spectrum requiring a fairly high temperature to show any of its lines. The most decided "core lines" of titanium are the enhanced lines. These are much strengthened by the innermost arc vapor, especially toward the poles. It has been noted that the enhanced lines are given faintly at the highest furnace temperatures. Taken as a whole, a certain measure of agreement is to be observed between the intensities of titanium lines in different parts of the arc and their behavior in the furnace at various temperatures, but the arc is not a favorable source for the division of the lines into classes.

EXPLANATION OF PLATE V

The two sections of spectra in Plate V cover the range from λ 5867 to λ 6366, the spectra being that of the arc (*a* and *e*) and of the furnace at 2600°, 2400°, and 2150° C., respectively. The five spectra were photographed on the same plate and are in many ways typical of the appearance of titanium lines in arc and furnace. The exposure times for the arc were 4^m and 1^m and for the three

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 66, 32; *Astrophysical Journal*, **37**, 270, 1913.

furnace temperatures 3^m, 10^m, and 70^m, respectively. It is thus seen that in this region of the spectrum the absolute brightness of the high-temperature furnace is of the same order as that of the arc. The intensity at medium temperature can be made closely comparable with that at high temperature by trebling the exposure time, while the low temperature brings out these orange and red lines with great difficulty. An exposure of at least 5 hours would have been required to give the low-temperature spectrum the same average strength as the others. The reason for this is not yet clear, but the condition plainly exists for this spectrum and for iron. The fainter arc spectrum (*c*) shows the stronger lines of about the same intensity as the high-temperature furnace, but a number of lines which are distinct in the furnace, such as λλ 5881, 5904, 5919, 5938, 6065, 6304, 6312, 6336, 6366, require prolonged exposure of the arc to give them a strength equal to that in the furnace. The condition holds throughout the spectrum that an arc photograph strong enough to give the majority of the lines of about equal strength in arc and furnace shows only faintly many lines which are strong in the furnace spectrum. The titanium-oxide bands shaded toward the red are visible in the arc spectrum (*a*) but not in that of the furnace. The flutings given by the furnace (*b*) are due to carbon.

SUMMARY

1. The foregoing study of the titanium spectrum covers the range of wave-length from λ 3888 to λ 7364, the spectra given by three furnace temperatures being compared among themselves and with that of the arc with reference to the temperature at which a line appears and its rate of strengthening with increase of temperature.

2. A considerable number of the strongest lines appear at about 2100° C., some of them showing a relatively high intensity at this temperature.

3. A large proportion of the titanium lines appear first at about 2300° C. and usually show a rapid intensification at 2600°.

4. A furnace temperature of 2600° C. brings out almost all lines

appearing in the arc with the exception of the enhanced lines. Traces of the latter can also be obtained under these conditions.

5. The relative weakness in the arc of many furnace lines, observed also for some lines of iron, is an important feature of the titanium spectrum. Most of the lines of this kind are moderately strong in the furnace at medium and high temperatures.

6. Lines of nebulous appearance are high-temperature lines and are usually much stronger in the arc.

7. The strong low-temperature lines occur in the blue end of the spectrum, higher temperatures being required to give distinctly the lines in the yellow and red. The arc spectrum shows a gradual increase toward shorter wave-length in the average intensity of lines as compared with the furnace spectrum.

8. The band spectrum does not appear when titanium metal is vaporized in the vacuum furnace at temperatures ranging from 2000° C. to 2600° C. It is usually ascribed to the oxide.

Mount Wilson Solar Observatory
August 30, 1913

THE DEPTH OF THE REVERSING LAYER

By S. A. MITCHELL

In investigating the wave-lengths[1] of the chromosphere from spectra obtained at the total eclipse of 1905, it became necessary to compare the intensities of the lines of the chromosphere with the corresponding lines from Rowland's tables. To make identification of lines more certain, comparisons were made of the intensities of the arc and spark spectra. For each of the 2841 lines measured in the flash spectrum, there are thus given the intensities of chromosphere, Rowland, arc, and spark. In addition to these intensities, measures were made of the heights above the sun's surface of the vapors forming each of the chromospheric lines. Such measures of eclipse spectra give practically the only means available for determining the depth of the reversing layer.

On account of St. John's discussion[2] of "The Distribution of the Elements in the Solar Atmosphere" the depths of the reversing layer and the relative intensities of the spectrum lines assume a rather unusual importance.

St. John's article gave the data for testing Abbot's interesting theory of the formation of the Fraunhofer lines as outlined by him in his book, *The Sun*, pp. 251–252. In brief, Abbot's idea is that the level at which a Fraunhofer line originates is not sharply bounded, but that some portion of the whole depth of the gas is more effective than all the rest in the production of the line.

> From the point of view that no light from the continuous spectrum background appears in the Fraunhofer lines, and that their relative intensities depend upon the light in the lines emitted by the gases, it is evident that light of the wave-length considered, coming from the lowest depths from which it can reach the surface, emerges greatly weakened by absorption and scattering; that light from the lesser depths is greatly weakened because of lower temperature; and that it is the layer between these temperatures that may be considered the effective layer.

The data available to St. John, particularly in regard to the depths of the reversing layer, were, unfortunately, too meager to adequately test Abbot's theory.

[1] *Astrophysical Journal*, 38, 407, 1913. [2] *Ibid.*, 38, 157, 1913.

Since the flash spectrum was photographed by a grating without a slit, it was easy to obtain the heights of the vapors forming each line of the spectrum by measuring the lengths of the cusps forming the lines of the flash spectrum. As a matter of fact, this was done[1] by laying a glass protractor on top of a sixfold enlargement of the flash spectrum, and reading off the angular length of the chromospheric cusps. A small table calculated with the sun's semi-diameter equal to $15'50''.7$, and the moon's augmented semi-diameter equal to $16'35''.7$, gave the corresponding heights in kilometers. It was found that H and K extended 14,000 km above the sun's surface.

Among the results derived from a discussion of the 1905 eclipse spectra (*op. cit.*, p. 494) are the following:

1. The flash spectrum is a reversal of the Fraunhofer spectrum.

2. The flash is not an instantaneous appearance, but the chromospheric lines appear gradually. At the beginning of totality, those of greatest elevation appear first, and at the end of totality remain the last. The "reversing layer" which contains the majority of the low-level lines of the chromosphere is about 600 km in height.

3. Wave-lengths in chromospheric and solar spectra are practically identical.

4. The chromospheric spectrum differs greatly from the solar spectrum in the intensities of the lines.

5. Those differences in intensity find a ready explanation in the heights to which the vapors ascend.

6. Especially prominent in the chromosphere are the enhanced lines which become stronger, mainly because at the heights to which they ascend the vapors are mixed with hydrogen at reduced pressures.

In order to compare the relative behavior of the various elements, a statistical study was made of intensities and heights. Each line of the 2841 lines was taken up in succession, and there were put down in four different tables the intensities arranged according to elements. Each hundred angstroms of wave-lengths were tabulated separately. There were thus given, for each hundred angstroms, the total number of lines for each element, the various

[1] *Ibid.*, **38**, 423, 1913.

intensities of these various lines, from which were readily obtained the total intensities of the lines for each element, and also the average intensities. Four such tables were made, for the flash spectrum, for the lines in Rowland with which these lines were identified, for the arc spectrum according to Exner and Haschek, and also for the spark spectrum from the same authors. With single lines in the flash spectrum identified with single lines in Rowland having single sources, this matter was very simple. The question was to know how to treat blended lines, and lines in Rowland of more than one source. For the case of one line in the flash spectrum corresponding to two lines in Rowland blended together, it was thought that more satisfactory values would be obtained if it was imagined that such a blended line in reality consisted of two lines in the chromospheric spectrum. The intensity of the one actual chromospheric line was divided between the two supposed lines, equally or unequally, as experience dictated. This method gave the total numbers for the various elements slightly different from those obtained in the first discussion.

Instead of giving here the intensities for each hundred angstroms, there are given the results for the whole spectrum from λ 3318 to λ 6191. In the first column is given the element, in the second the total number of lines. In the next four columns are given the average intensities in Rowland's tables, in the chromosphere, and in the arc and spark spectra as given in Exner and Haschek's tables. In the following columns are given various ratios of these four intensities as indicated. Table I does not give a complete list of all the elements found in the flash spectrum. *H*, *He*, and *C* are omitted, and also all elements identified by few lines. On account of similar properties among the elements, which become apparent from the numbers hereby tabulated, the elements are divided into four groups as follows:

Group I contains *Fe*, *Ni*, *Co*, and *Mn*. This may be called the *Fe*-group.

Group II contains *Cr*, *V*, *Ti*, *Zr*, *Sr*, *Ba*, and may be called the *Ti*-group.

Group III contains the rare earths *Sc*, *Y*, *La*, *Ce*, *Nd*, *Sa*, *Er*.

Group IV contains *Ca*, *Mg*, and *Al*.

In taking the average of the intensities of Rowland and the chromosphere it was difficult to know how to treat intensities of 0, 00, 000, etc. The rule was made to treat intensity 0 as if it were $\frac{1}{2}$, to treat 00 as $\frac{1}{4}$, 000 as $\frac{1}{8}$, etc.

TABLE I

INTENSITIES AND RATIOS CORRESPONDING TO THE CHROMOSPHERIC LINES IN THE 1905 ECLIPSE SPECTRUM

Element			Intensities				Ratios of Intensities			
Group	Element	Number of Lines	Rowland	Chromosphere	Arc	Spark	Chromosphere / Rowland	Spark / Arc	Arc / Rowland	Spark / Chromosphere
I	*Fe*	727	3.96	2.14	5.20	1.74	0.54	0.33	1.31	0.82
	Ni	154	2.72	1.09	6.63	2.18	0.40	0.33	2.44	2.00
	Co	101	2.38	1.21	6.54	4.72	0.51	0.72	2.76	3.94
	Mn	89	2.28	2.11	6.63	5.13	0.93	0.77	2.89	2.43
II	*Cr*	198	1.95	1.96	4.48	4.62	1.00	1.03	2.30	2.36
	V	186	1.75	1.72	4.02	5.95	0.92	1.21	2.24	3.46
	Ti	331	2.10	3.36	4.48	4.74	1.60	1.06	2.13	1.41
	Zr	123	1.40	1.80	3.21	5.37	1.29	1.68	2.29	2.98
	Sr	8	2.46	11.2	265.1	126.5	4.56		.	
	Ba	12	3.64	7.07	136.2	161.8	1.94	1.20	37.4	22.8
III	*Sc*	60	2.28	3.73	1.83	13.4	1.64	7.33	0.80	3.60
	Y	52	1.36	2.33	6.93	15.0	1.72	2.17	5.10	6.44
	La	48	1.08	1.72	5.89	10.4	1.60	1.77	5.46	6.05
	Ce	87	1.26	1.61	4.62	3.78	1.28	0.82	3.67	2.36
	Nd	80	1.28	1.67	4.46	4.02	1.31	0.90	3.48	2.42
	Sa	33	0.73	1.17	7.12	3.94	1.61	0.56	9.76	3.37
	Er	24	1.48	1.01	5.78	3.47	0.68	0.60	3.91	3.44
IV	*Ca*	42	44.5	7.82	54.0	46.1	0.18		.	.
	Mg	11	13.0	9.20	32.7	115.6	0.72		.	..
	Al	3	11.8	5.5	...	...	0.47		.	...

The reasons for dividing the elements into four groups will be evident by referring to Table I. Group IV, comprising *Ca*, *Mg*, and *Al*, contains elements which have very strong lines in the Fraunhofer spectrum and also in the chromospheric spectrum. This is Group I (*op. cit.*, p. 486), obtained by a discussion of the total numbers of lines in the two spectra.

Reference to the column which gives the ratio of the intensity of the chromosphere to that of Rowland shows that Groups II and III of Table I, comprising *Cr*, *V*, *Ti*, *Zr*, *Sr*, *Ba*, and the rare earths

Sc, Y, La, Ce, Nd, Sa, and *Er*, are evidently elements whose lines are relatively stronger in the chromospheric spectrum than in the solar spectrum. Together with *H, He*, and *C*, these elements form Group II of the previous discussion (*op. cit.*, p. 486).

Fe, Ni, Co, and *Mn* form a group by themselves where these elements are represented in the solar spectra by stronger lines than are found in the chromosphere.

Before taking up the meaning of this division of the elements into groups, it might be well to speak again of the method of determining intensities in the chromospheric spectrum (*op. cit.*, p. 418). Intensities in Rowland and the chromosphere are both arbitrary. In Rowland the strongest line, K, has an intensity 1000, in the chromospheric spectrum, K has an intensity 100. Necessarily then, the two scales are not the same. Intensities in the chromosphere were actually obtained from negatives sixfold enlarged, as given in Plates XIII to XVIII (*op. cit.*). Such enlargements were at all times compared with a reduced photograph of Rowland's *Atlas* as given in Plate XIV. This comparison insured that on the average the intensities for chromospheric and solar spectra were equal when the intensities were less than 10. Above this, the increased scale of Rowland was manifest. As there are comparatively very few very strong lines, it is felt where so many lines are considered as in Table I that the average will give results free from systematic error. As a matter of fact, 96 per cent of the lines of the chromosphere have an intensity of 8 or less.

Where there are few lines as for *Sr, Ba, Mg*, and *Al*, or where there are very strong lines as in *Sr, Ca, Mg*, and *Al*, the averages become unreliable. With these limitations, therefore, it is felt that the numbers in the column giving the ratio of the intensities of the chromospheric spectrum to Rowland have a real signification, and that the division of the elements into groups is a rational one.

Interesting figures are seen in the column giving the ratio of the intensities of the spark to the arc. Intensities for both spark and arc are taken from Exner and Haschek's tables. The great care with which this splendid work of theirs was done, guarantees that on the average, the intensity of the arc for each element would be very approximately equal to the intensity of the spark. That this

is not so for the lines considered in the chromosphere, points to some peculiarity in behavior of the elements and lines considered. Lines which are stronger in the spark than in the arc are called "enhanced" lines. Lockyer's well known list of enhanced lines, though they play an important rôle in the discussion of eclipse spectra (*op. cit.*, p. 487), are in themselves not sufficiently numerous to explain the systematic increases in intensities in Groups II and III in Table I, and they would not at all explain the systematic decrease for the elements of Group I whose lines total over one thousand.

The numbers giving the ratio of the chromospheric intensities to those of Rowland, and the numbers giving the ratio of the intensities of the spark to the arc might be called more or less equal to each other for each element. What is the meaning of the fact that both the columns increase together in Groups II and III over Group I? (On account of the strong lines present in the elements *Ca*, *Mg*, *Al*, and *Sr*, such as H and K, λ 4226, the *b*-group, the *Sr* line λ 4077, etc., the numbers representing these four elements are unreliable, and they are therefore omitted since they do not fairly represent the averages.)

The first meaning that may be drawn from the similarity of the ratios $\frac{\text{Chromosphere}}{\text{Rowland}}$ and $\frac{\text{Spark}}{\text{Arc}}$ is that while the intensities of the Fraunhofer lines as given in Rowland's tables correspond to arc intensities, the intensities of the chromospheric lines correspond more closely with spark intensities than they do with arc intensities. For this reason, there are added to Table I two additional ratios, which give, respectively, the ratios of the intensities of the lines in the arc to those in Rowland, and also the ratios of the intensities of spark and chromosphere. These two last columns might be said to give the intensity of an arc line which would produce a line in the Fraunhofer spectrum of intensity 1, and also the intensity of the line in the spark corresponding to a line in the chromospheric spectrum of intensity 1. For the various elements, these two last columns increase and decrease together.

There are very great differences among the elements. The mean between the two last columns would smooth out the individual differences and render more apparent the behavior of each element.

(This simple mean could be taken at a glance, but since this mean can have no actual signification, it is not inserted in Table I.) *Fe*, then, enjoys the distinction, not only of being represented by the greatest number of lines, but also of having the power to produce lines in the solar spectrum, both Fraunhofer and chromospheric, with greater facility than that possessed by any other element considered in Table I. The difference between *Fe* and the rare earths as a whole is very marked. By reference to Exner and Haschek's tables, it is seen that the rare earths have very many lines in their spectra. It needs a line of approximately intensity 5 in either arc or spark before a corresponding line is found in the Fraunhofer or chromospheric spectrum, while for *Fe*, if there is a line of intensity 1 in the arc, there is found a line in the solar spectrum corresponding to it with an intensity 1 in Rowland, while a line of intensity 1 in the spark spectrum will have a line in the chromospheric spectrum corresponding to it which will likewise be of intensity 1.

That the systematic variations in intensities noted are not merely the results of chance will be more evident if the quantities are arranged according to the atomic weights of the elements.

In Table IV (*op. cit.*, p. 485) was given the periodic table of atomic weights taken from the *Encyclopaedia Britannica*, 11th edition, Vol. 9, p. 258, under "Element," to which was added to each element the total number of lines of the chromospheric spectrum. Part of the Periodic Table only is given in Table II, where in addition to symbol and atomic weights there are given the total number of lines, and the two ratios of the intensities of chromosphere to Rowland and arc to spark, respectively. Elements in parentheses are not found in the chromosphere.

If we enter the periodic table at *Ca*, we find an element which is represented by the very strongest lines in the sun, in the chromosphere, in the arc, and in the spark. The lines in the sun due to *Ca* are stronger than those of the very light elements *H* and *He*. No satisfactory explanation for this has ever been given. That *Ca* is not dissociated, thus giving rise to a very light element, as has been sometimes thought, is proved by Table II. The elements present in the sun are in such a condition that the atoms differ from the atom of *Ca* by successive gradations. This will be seen by run-

ning in a horizontal row to the right through Table II. and it will be seen that the ratios of chromosphere to Rowland and spark to arc both show a marked tendency to decrease. This same decrease is seen in each of the horizontal rows as one goes to the right. Moreover, from *Ca* in a vertical column downward the two ratios

TABLE II

PERIODIC TABLE OF ATOMIC WEIGHTS

Under each element are given: the atomic weight; the total number of lines in the 1905 chromospheric spectrum; the ratio of the intensity of chromospheric line to Rowland; and ratio of spark to arc.

He 4 15	(*Li*) 7	(*Be*) 9	(*B*) 11 ..	*C* 12						
(*Ne*) 20	*Na* 23 3	*Mg* 24 10	*Al* 27 3	*Si* 28 7						
(*A*) 40	(*K*) 39	*Ca* 40 42	*Sc* 44 60 1.64 7.33	*Ti* 48 331 1.60 1.06	*V* 51 186 0.92 1.21	*Cr* 52 198 1.00 1.03	*Mn* 55 89 0.93 0.77	*Fe* 56 727 0.54 0.33	*Ni* 59 154 0.40 0.33	*Co* 59 101 0.51 0.72
(*K*) 83 ...	(*Rb*) 85	*Sr* 87 8 4.56	*Y* 89 52 1.72 2.17	*Zr* 91 123 1.29 1.68						
(*Xe*) 131	(*Cs*) 133	*Ba* 137 12 1.04 1.20	*La* 139 48 1.60 1.77	*Ce* 140 87 1.28 0.82	*Pr* 141 1	*Nd* 144 80 1.31 0.90	*Sa* 150 33 1.61 0.56	*Eu* 152 6	*Gd* 157 51 0.89 0.89	
				Dy 162 3	*Yb* ? 1	*Er* 167 24 0.68 0.60				

decrease, also from *Sc* and *Ti* downward. There is thus seen a general decrease in the ratios of the chromosphere to Rowland and of the spark to arc, both to the right and downward, through the periodic table. Above *Ca* in the table is found *Mg*, and on each side of *Mg* are *Na* and *Al*. All three of these elements are represented by very strong lines in all four spectra considered. Above

Ti is found *Si*, and above *Si* is *C*. Though *Si* and *C* are not specially prominent in the sun they are both important elements, *C* from the cyanogen spectrum and carbon bands which appear in sun and chromosphere, and *Si* from the very prominent lines which are found in earlier type stars.

It seems, therefore, that the gradual change from element to element in the periodic table is not one of chance, but one which can find its true explanation only in the constitution of the atom itself. In particular, it seems as if *Ca* was not an exceptional element as has been thought, but is one which occupies its proper place in the periodic table. Likewise it seems as if *V* and *Cr* represent the transition between the group of elements represented by *Ti* and the group represented by *Fe*. The size of the ratios manifested would permit us to put *V* and *Cr* either in Group I with *Fe* or in Group II with *Ti*. For obvious reasons they are put along with *Ti*.

It is with great interest, therefore, that we tabulate the depths of the reversing layers for the various elements, which depths are obtained from the "height of the chromosphere" as given in Table I (*op. cit.*, p. 424). Instead of doing this for each individual element, it was done by groups. Group I contains *Fe*, *Ni*, *Co*, and *Mn*. In Group II are placed not only *Cr*, *V*, *Ti*, *Zr*, *Sr*, and *Ba* of Group II above but also *Ca*, *Mg*, and *Al* of Group IV. (This Group IV is represented by comparatively few lines, and H and K are omitted.) In Group III are the rare earths. In order that the various lines should not be spread among too many classes, the intensities were grouped into four classes as follows: (1) those in which the intensity in Rowland was 2 or less; (2) intensities in Rowland 3 to 5, inclusive; (3) intensities 6 to 10, inclusive; (4) intensities greater than 10. Since the enhanced lines extend to greater heights than those not enhanced, these were treated separately. The results are given in Table III.

In tabulating these heights, it was at once noticed that the heights for all elements were greater at the violet end of the spectrum. Consequently, heights were tabulated separately to the violet side of λ 4900, and to the red side. Though the heights differ, there is shown the same relative behavior throughout the

spectrum, and there are therefore given in Table III the results for the spectrum as a whole.

The table might be explained as follows: 240 of the weakest lines due to the *Fe*-group have an intensity in Rowland of 1.26. In the chromospheric spectrum the average intensity is less and amounts to only 1.07. The average height of these 240 lines is 338 km. Of these 240 lines, only 7 are enhanced. The unenhanced lines extend up to an average of 336 km, while the enhanced lines are relatively much stronger in the chromosphere and extend to greater heights.

Table III shows the following:

1. The average intensity of the lines in the chromosphere is about equal to that from Rowland's tables if the intensity in the latter does not exceed 10.

2. The ratio of the intensities of the chromospheric spectrum to that of Rowland is characteristic for the different groups of elements, this ratio being least for the *Fe*-group and greatest for the rare earths.

3. The difference in heights for the various groups is quite as characteristic. This is best seen by referring to the ordinary or unenhanced lines. The *Fe*-group lines extend to the least heights, the lines of the rare-earth group extend to the greatest heights, in all cases where there are sufficient number of lines to make averages reliable.

4. The enhanced lines are in all cases much stronger in the chromosphere than those not enhanced, and they extend to greater heights. The heights actually depend on the degree of enhancement.

5. The enhanced lines share the same characteristic increase of height from the *Fe*-group to the group of rare earths, as was exhibited by the lines not enhanced.

6. Heights more closely correspond to intensities in the chromospheric spectrum than they do to Rowland's intensities. On the average, a line of intensity 1 in the chromosphere has an approximate height of 350 km, and there is an increase of about 80 km for each unit increase in intensity. Though the determination of heights by measures of the lengths of the chromospheric cusps

TABLE III

AVERAGE HEIGHTS OF THE CHROMOSPHERE ARRANGED ACCORDING TO INTENSITIES AND GROUPS

	Solar Intensity 2 and Less				Solar Intensity 3 to 5				Solar Intensity 6 to 10				Solar Intensity Greater than 10			
	Total No. of Lines	Average Intensity Rowland	Average Intensity Chromosphere	Average Height	Total No. of Lines	Average Intensity Rowland	Average Intensity Chromosphere	Average Height	Total No. of Lines	Average Intensity Rowland	Average Intensity Chromosphere	Average Height	Total No. of Lines	Average Intensity Rowland	Average Intensity Chromosphere	Average Height
Fe-group				km				km				km				km
All lines	240	1.26	1.07	338	243	4.18	1.82	505	110	7.42	3.54	512	23	20.5	0.0	780
Enhanced lines only	7	1.36	3.14	400	7	3.86	0.43	728	2	6.5	11	650				
Unenhanced lines	233	1.26	1.01	336	236	4.10	1.59	395	114	7.43	3.41	510	23	20.5	0.0	780
Ti group																
All lines	275	1.07	1.30	385	132	3.70	4.23	588	20	7.45	12.45	1690	8	19.6	27.5	3712
Enhanced lines only	44	1.30	2.78	484	45	4.87	7.94	639	6	8.17	28.0	4350	3	17.3	38.3	6333
Unenhanced lines	231	1.03	1.02	366	87	3.75	2.31	562	14	7.14	5.8	550	5	21.0	21.0	2140
Rare earths																
All lines	150	0.82	3.10	417	29	3.59	6.15	783	2	6.5	7.0	625				
Enhanced lines only	11	1.41	4.28	610	7	3.59	12.29	1464								
Unenhanced lines	139	0.77	3.01	401	22	3.59	4.21	566	2	6.5	7.0	625				

admits of no great precision, nevertheless it is felt that the large number of lines considered makes the results fairly accurate.

If the sun be considered as a hot body surrounded by cooler vapors extending to various heights but densest nearest the photosphere, then, as was first pointed out by Evershed,[1] the flash spectrum is a progressive exposure. The vapors extending to greatest heights will have relatively longer exposures, and as a consequence flash-spectrum intensities increase with the increase in the heights of the vapors. It would seem, therefore, that the *Ti* lines are stronger in the flash than the *Fe* lines, mainly because the *Ti* vapors extend higher on the average than do the *Fe* vapors. Similarly, it seems that the rare earths, in spite of their higher atomic weights, ascend to still greater heights than do the *Ti* vapors. However, it should not be forgotten that the rare earths are very rich in lines, and in general only the stronger lines are found in either the ordinary solar spectrum or in the flash spectrum.

In his paper on "The Distribution of Velocities in the Solar Vortex," St. John[2] confirms the discovery by Evershed[3] of the displacement of the Fraunhofer lines in the penumbrae of sunspots, and arrives at the following conclusions:

1. The proportionality between displacements and wavelengths shows that the phenomenon is due to the Doppler effect of material of the reversing layer flowing out of spots and of the chromospheric material flowing into the spots.

2. The increase of displacements indicating an outward flow corresponds to a decrease in the intensity of the lines of the reversing layer, and the increase of the displacements indicating an inward flow of the chromospheric gases corresponds to an increase in intensity of the latter lines. A satisfactory explanation seems to be found in a difference in level.

It therefore seems that St. John and the writer both use the differences in heights or differences in level to explain differences in the lines of the solar spectrum.

[1] *Philosophical Transactions*, **197** A, 393, 1901.

[2] *Astrophysical Journal*, **37**, 322, 1913.

[3] *Kodaikanal Observatory Bulletin*, No. 15, 1909.

According to St. John's[1] ideas, however, the rare earths are found in the low-lying regions of the reversing layer. Since they are found comparatively close to the photosphere, they are at a rather high temperature. As a result of this high temperature there is little absorption by the rare-earth vapors, and the Fraunhofer lines are not strong. These high temperatures, however, make more brilliant the lines of the flash spectrum, with the result that the intensities of the rare earths in the flash spectrum are much greater than they are in the Fraunhofer spectrum. Measures of the 1905 eclipse spectrum confirm the increased intensities in the flash spectrum demanded by St. John's theory, *but these measures do not show that the rare earths are found in shallow layers*, but exactly the reverse.

Again, St. John finds that with increasing intensities of the *Fe* lines in the Fraunhofer spectrum there is a decreasing difference of Doppler effect in the spots, which is interpreted as due to a difference of level. This effect seems so uniform that the intensities of the Fraunhofer lines become to St. John a scale whereby he can sound the depth of the reversing layer. In other words, an *Fe* line of intensity 4 in Rowland finds its maximum absorption always at the same level above the photosphere, while lines of intensity 8 have their maximum absorption always in the same level above the photosphere, but this level for lines of intensity 8 is different from the level for lines of intensity 4, and in fact exists above the latter. This idea leads to some important consequences. All lines of *Fe* of intensity 4 in Rowland, and not enhanced lines, must therefore be represented in the flash spectrum by lines of the same height, and this equality in height must entail an equal intensity in the flash spectrum. *Fe* lines of intensity 8 likewise must be represented in the flash spectrum always by lines longer and of greater intensity. Although in general it may be said that the stronger the *Fe* lines are in Rowland, the stronger are those lines when reversed in the flash spectrum and the higher do they extend, yet there are so many exceptions to this, without any apparent cause, that it does not seem possible to make a general rule.

[1] *Astrophysical Journal*, **38**, 174, 1913.

It would seem, therefore, that the question of heights as given in Table III, and that of relative intensities as given in Tables I and II, must be considered together. It would seem that the process of stellar evolution had advanced to such a stage with the sun that calcium (for some reason) extends higher than any other element, even hydrogen. On account of this great height, H and K are the strongest lines of the solar spectrum. It would further seem that the elements differ from *Ca* and from each other by successive gradations. These differences in the elements seem to find their explanation in the ultimate constitution of the atom itself. In addition to the gradual variation in intensities and heights already shown, we should expect, if the cause is to be found in the atom itself, that the elements close to each other in the atomic-weight table would show also additional common properties. We should expect, for example, that the elements close to *Ca* in Table II would show strong lines comparable with H and K. In fact, they do. Above *Ca* in the table, *Mg* has the *b*-group, *Na* the D lines, *Al* the lines at λ 3944 and λ 3961. Even *Si* has strong lines in stellar spectra at λ 4128 and λ 4131. To the right and below, we have *Sc* with its strong line at λ 4247, *Ti* with its strong lines such as λ 3757, λ 3761, λ 4395, λ 4443, λ 4468, λ 4501, λ 4563, and λ 4572. *Sr* has a strong line at λ 4077 and another at λ 4215. Even *Y* has a strong line at λ 4375 and *Ba* one at λ 4554.

It would seem, therefore, that the elements of the *Ti*-group and those of the rare-earth group both extend to greater heights than do the elements of the *Fe*-group mainly because the atoms of the *Ti*-group of elements and the rare earths are more closely related to the *Ca* atom than are the atoms of *Fe*, *Ni*, *Co*, and *Mn*. If information could be had regarding the changes of pressure, of temperature, and of electrical conditions depending on the differences of elevation, we should advance a long way toward solving some of the curious phenomena connected with the solar spectrum.

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January 1914

Reviews

Die Atomionen chemischer Elemente und ihre Kanalstrahlen-Spektra. By J. Stark. Berlin: Julius Springer, 1913. Pp. 43; 1 pl.

Many valuable researches have been carried out recently by Stark and his co-workers at Aachen regarding the spectra of the canal rays of various elements. The detailed results are reported in some nineteen articles[1] which have appeared during the present year. As the general conclusions have a significant relation to the problem of the electrical structure of the chemical atom, they are of interest to many who have not the time to read the original papers. For such, Stark has written the pamphlet under consideration, in which he reviews briefly the main experimental results and discusses their significance and bearing on fundamental problems.

From deflection experiments it is known that canal rays are partly neutral atoms and partly positive ions, univalent, bivalent, or multivalent according as they have lost one, two, or more electrons. Stark discovered in 1905 that certain lines of a canal ray spectrum show a Doppler effect, and ventured the hypothesis that such lines are emitted by the positively charged rays. Though this appealed to many as probable, there was really no convincing evidence for it before an improved technique, and especially the discovery by Stark of the advantage of diluting the gas to be investigated with helium made the recent careful study of the shifted lines possible. The valency of the rays emitting a given line cannot be determined from the maximum shift of the line, as was at first assumed; but from a study of the variation of the intensity distribution of the line with the cathode fall of potential, Stark not only is able to show that different series of lines have different carriers but can determine the probable electrical charge of the carriers.

[1] Stark, *V. Deutsch. Phys. Gesell.*, **15**, 800–812, 813–820; *Phys. Zeitschr.*, **14**, 102–106, 454–456, 497–498, 768–770, 770–780, 961–965, 965–966; *Annal. d. Phys.*, **42**, 163–180, 231–237, 238–240; Stark and Kirschbaum, *Phys. Zeitschr.*, **14**, 433–439; *Annal. d. Phys.*, **42**, 255–277; Stark and Wendt, *Phys. Zeitschr.*, **14**, 507; Stark, Wendt, and Kirschbaum, *Phys. Zeitschr.*, **14**, 770–779; *Annal. d. Phys.*, **42**, 278–302; Stark, Fischer, and Kirschbaum, *Annal. d. Phys.*, **40**, 499–541; Stark, Kunzer, and Wendt, *Annal. d. Phys.*, **42**, 241–254.

Thus in the case of argon, he found that the red spectrum is emitted by the univalent positive ray, the blue spectrum partly by bivalent and partly by trivalent rays. *Al, B, C, Cl, H, He, Hg, I, K, Mg, N, Na, O, S*, and *Si* canal ray spectra have also been carefully studied. The mercury spectrum is especially interesting as five groups of lines have been distinguished. From their behavior in the canal ray spectrum, Stark concluded that $\lambda\lambda$ 2537 and 1849 are emitted by neutral atoms and then verified the hypothesis by finding that mercury vapor at low pressures shows a sharp strong absorption line at λ 2537.

Another interesting result to which our author calls attention is that the valency of canal rays is apparently unrelated to their chemical valency. Helium rays may be doubly charged, argon rays trebly charged, and mercury rays quadruply charged. The maximum valency observed under ordinary circumstances seems to depend rather on the atomic weight.

Stark's suggestions as to the arrangement of positive and negative electricity in atoms of various valencies are interesting though, of course, they are meant merely to help make the discussion more concrete and are not to be taken too seriously. He distinguishes two kinds of electrons in every atom, detachable electrons, including the valence electrons, and undetachable electrons. According to Stark's hypothesis, the band spectra are due to the vibrations of the more loosely attached valence electrons, whereas the series lines are emitted by the second group of electrons. Thus he would explain the difference of the Zeeman effect for the two types of spectra and other phenomena he has observed. But though he speaks of two kinds of electrons, he does not mean, of course, to imply any essential difference between them, other than a difference in situation with reference to the atom—a dynamic difference. He concludes with some spectrum evidence to prove the non-elasticity of the collisions of fast canal rays with gas molecules, and with a discussion of the difference between ionization by shock and ionization in electrolytes.

Stark is to be congratulated for having demonstrated the truth of his hypotheses regarding the carriers of series and band spectra, which, when they were advanced some years ago, seemed mere guesses. He not only has been extraordinarily active in accumulating careful experimental evidence, but has been unusually successful in interpreting imperfect data and predicting relations which later research has verified.

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November 26, 1913

Researches in Magneto Optics. By P. ZEEMAN. New York: Macmillan, 1913. 8vo, pp. 219, figs. 74. $1.60.

This book is one of the Science Monographs which Macmillan & Co. are publishing. In these monographs the authors are supposed to deal mostly with their own researches. Consequently much of this book has appeared as contributed articles in the journals. For the English-reading public, the *Astrophysical Journal* has been the principal publisher.

CONTENTS

The first chapter is very brief and ends with a few remarks upon electromagnets. A student who is unfamiliar with the subject would need a more extended treatment.

In chap. ii the author gives the account of his original discovery. Although the author considers that the work possesses only a retrospective interest, I think it the most entertaining part of the book. For it reveals that clear insight and critical analysis without which he would have arrived at no results whatsoever. Chap. iv is closely related to chap. ii. For so brief a statement of spectral series, the treatment could not be surpassed. Lastly appears a discussion of Runge's Rule upon the periodic distances between components of complex types of separation. The same topic is renewed in chap. x.

Chaps. iii, v, and ix discuss the "Inverse Effect" which follows from Kirchhoff's Law of Emission and Absorption. Upon this particular

topic the author has made extensive observations. From the nature of the case, therefore, the "Inverse Effect" is thoroughly discussed.

In chap. vi we find that a grating may show selective reflection of polarized light in some sections of a given order of the spectrum and not in another. This conclusion has also recently been reached by Ellerman (*Astrophysical Journal*, **38**, 72, 1913). Some years since the reviewer noticed considerable irregularity in the intensity of the ratios of the *p*- and *n*-components of the magnetic separations, while using a glass condensing lens. These irregularities did not appear when a quartz condensing lens was substituted for the glass lens. The reason appeared self-evident. The rays which pass through at different angles and quite different thickness of quartz had suffered very different amounts and kinds of orientation. Most observers, like the reviewer, have used quartz condensing lenses, and a selective polarization of the grating would effect the *p*- and *n*-components in the same way. The experiments of Stock (*Physikalische Zeitschrift*, **10**, 694–697, 1909) remove all doubt upon that point. Dissymmetries and shifts need a larger treatment. The subject needs also a larger investigation with greater resolving power. When this is done, I am prepared to believe that the dissymmetries which I have observed will either disappear or show the spacings to be unequal multiples of small values (intervals).

There is an excellent description and explanation in chap. viii of the broadening of the solar spot lines, observed by W. M. Mitchell; and an equally good brief application of overlapping components, to the magnetic field of sun-spots, discovered by Hale.

This book should be in the possession of everyone interested in magneto-optics. First, the beginning student will find the work clearly outlined, and in it a preparation for the theoretical work of such contributors as Lorentz, Voigt, and Ritz. Secondly, the student who wishes to get a conception of the work in magneto-optics, but has not the time, or perhaps the ability, to take up the theoretical discussions, will find this book to meet his needs, although the non-mathematical student will omit portions of the present volume. Thirdly, the thorough treatment of the "Inverse Effect" will, just now, particularly appeal to students and workers in astrophysics, where a new domain of work has opened up. Fourthly, of course, it will appeal to the physicist. Lastly, there is a complete bibliography, which will be an enormous time-saver to workers in magneto-optics and spectroscopy.

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The Atmosphere. By A. J. BERRY. The Cambridge Manuals of Science and Literature. Cambridge University Press. New York: Putnam, 1913. Pp. 146; figs. 5. 40 cents.

This little book belonging to the long list of Cambridge Manuals of Science and Literature maintains the same standard as the many excellent manuals which have preceded it. Following the style of the other members of the series, the aim has been to present in readable and attractive form some of the salient facts concerning the earth's atmosphere. The development of the science of the atmosphere, from Galileo through phlogistic chemistry to present-day conceptions, has been interestingly traced. As the chief facts of the composition of the atmosphere are a matter of pretty general knowledge, the greatest interest will perhaps be found in the newer and more speculative phases of the subject, especially the character of the upper atmosphere, the primitive atmosphere of the earth, the planetary atmospheres, and atmospheric radioactivity. Here there will be considerable difference of opinion. But such a summary and yet simple outline giving the gist of the subject will be welcomed by many readers.

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THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXXIX | APRIL 1914 | NUMBER 3

THE COMPLEX STRUCTURE OF SPECTRUM LINES

By CH. WALI-MOHAMMAD

The following investigation was carried out under the kind supervision of Professor W. Voigt in the physical laboratory of the Göttingen University. It represents the checking and the extending of the results published by Dr. L. Janicki.[1] The object was the comparison of the results obtained by Dr. Janicki by means of crossed parallel interference-plates with those yielded by an echelon grating of very high resolving power.

A. SOURCE OF LIGHT

Introduction.—In 1892 A. A. Michelson showed that some of the spectral lines are not simple but possess a complex structure. Since 1892 several spectroscopic instruments of very high resolving power (e.g., interferometer, echelon grating, parallel interference-plates, etalon, etc.) have been constructed and the structure of several lines investigated. But so far no satisfactory source of light, which could be used with the above instruments, has been found. (1) Flames are obviously useless for this purpose. (2) Spark spectra have yielded practically no results worth mentioning. (3) Geissler tubes, which are so useful in the cases of gases, have a very limited application and cannot be successfully

[1] *Annalen der Physik*, **29**, 833, 1909.

used in the case of metals. (4) Arc spectra are the only ones that have yielded any results, and these must be excited in a vacuum. Here may be mentioned the amalgam-lamps of different makes, but the number of substances that can be used with them is extremely limited.

A. Wehnelt[1] discovered the properties of his oxy-cathode in 1904 and Wehnelt and Wiedemann[2] showed later on how the oxy-cathode could be used in conjunction with an anode of a given metal to melt and vaporize the anode and thus produce an arc in vacuum.

Janicki (*loc. cit.*) took up the subject and constructed a discharge tube which gave very satisfactory results. The writer modified the form and size of Janicki's tube in such a way as to enable it to be placed in a magnetic field and thus to observe the Zeeman effect not only on the principal lines but on the satellites which accompany some of the principal lines, as well.

The oxy-cathode possesses the following advantages: (1) The source of light is an arc in vacuum—an absolute condition for producing very sharp lines. (2) The spectrum is pure in the sense that neither the carbon bands nor the air lines are present. (3) The lines are extremely *sharp* and most suitable for use with instruments of very high resolving powers. (4) The lines are intensely bright and consequently require very short exposure. (5) Nearly all metals can be used with the necessary alterations. (6) The rate of vaporization of the metal is under control and can be regulated as desired. (7) The temperature arrived at is very high and the spectrum extends farther into the violet than is the case with other sources of light.[3] (8) The source of light is capable of being placed in the magnetic field, and thus the Zeeman effect on the satellites can be easily observed.

Discharge tube (Fig. 1). The discharge tube AB is of Jena glass 30 cm long and 30 mm in cross-section. The upper end is closed by means of a glass plate P and through this the light passes out to the spectroscope. The lower end is attached by means of

[1] *Op. cit.*, **14**, 425, 1904.

[2] *Physikalische Zeitschrift*, **6**, 690, 1905.

[3] Wehnelt and Wiedemann, *loc. cit.*

sealing wax to a small tube *BC*, through the ground end *C* of which passes the tube *T* carrying the cathode and the anode. *A′A′B′B′* is the cooling mantle through which a constant flow of water takes place.

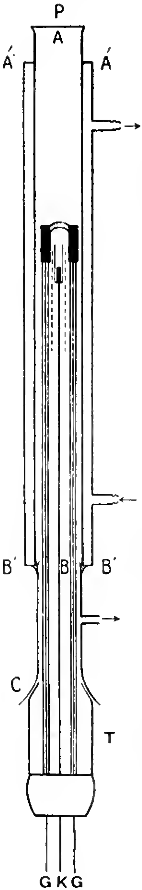

FIG. 1

Cathode (Fig. 2).—The cathode consists of platinum foil 0.015 mm thick and 30 mm long held in a semicircular form between two thick brass pieces *DE* and *DE*. The breadth of the cathode varies between 4 and 6 mm according to the current flowing between the cathode and the anode. The platinum foil is dipped in a solution of calcium nitrate and barium nitrate and heated in order to reduce the salts to their oxides. The cathode was then heated by means of a current from an accumulator battery (40 volts); the heating current was usually about 15 or 16 amperes.

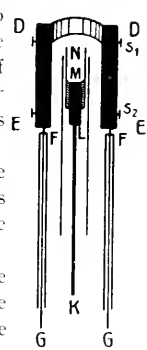

FIG. 2

It may be remarked here that the cathode has a very short life—it gets burned through after it has been in use for some time.

Anode.—The anode consists of the given metal placed in a porcelain tube *NM* and is in electrical contact with the brass piece *ML* and the wire *LK*.

Pressure.—The necessary vacuum was produced by means of Dr. Gaede's rotary pump. The pressure was generally less than 0.01 mm *Hg*.

Current.—A potential difference of 220 volts was set up between the anode and the red-hot cathode. When the vacuum had arrived at the proper value an arc was set up between the cathode and the anode and a current passed between them. By means of suitable water-cooled resistances this current could be regulated easily and controlled. Different metals require different current-strengths in order to set up the arc and to vaporize them properly. According

to Wehnelt the passage of 3 amperes produces the heat-equivalent of 60 to 90 watts at the anode and suffices to melt and to vaporize most of the metals. Evidently the amount of current necessary depends on the specific heat and the melting-point of the given metal. The current varied from 0.02 to 0.6 ampere (for zinc, cadmium, etc.) to 3 to 5 amperes (for cobalt, chromium, etc.).

B. INSTRUMENTS OF OBSERVATION

Spectroscope.—An echelon grating constructed by A. Hilger of London was used in conjunction with a monochromator. The echelon consists of 35 plates. Each plate is 9.945 mm thick and the breadth of the step is 1 mm. Its resolving power lies between 285,000 and 665,000 for the Fraunhofer lines A and H respectively and the limit of the wave-length difference ($d\lambda$) capable of being resolved is 0.027 and 0.006 Å respectively.

Method of observation.—All the lines under discussion were photographed and measured by means of a Zeiss microscope. For photographing the lines, plates of various makes were used. I employed Lumière's plates for the blue and violet, Viridin plates of Dr. Schleussner (Frankfurt) for the green and yellow, and Pinacyanol bathed and Panchromatic plates of Wratten & Wainwright (London) for the red end of the spectrum.

As already mentioned, the source of light is extremely bright and the time for photographing a line comparatively short. The following table shows the advantage of an oxy-cathode over other sources of light:

BISMUTH LINE $\lambda = 4722$

Observer	Source of Light	Observing Instrument	Exp.
(1) Lunelund	Quartz amalgam-lamp	Echelon	$1\frac{1}{2}$ to $2\frac{1}{2}$ hrs.
(2) Gehrcke and von Baeyer	Amalgam-lamp	Parallel plates	$4\frac{1}{4}$ hrs.
(3) Author	Oxy-cathode	Echelon	$\frac{1}{2}$ min.

C. RESULTS

1. ALUMINIUM

Current, 1 to 2 amperes — Melting-point, 66° C.

The only lines in the visible spectrum are λ 3901.7 and λ 3944.2. Both these lines are bright, sharp, and simple. With

a current of 2 amperes they show reversal. In addition to these two lines there are three strong bands in the blue which all shade down toward the red end of the spectrum.

2. ANTIMONY

Current, 0.5 to 4.5 amperes Melting-point, 625° C.

It is remarkable that antimony in spite of great evaporation gives no line spectrum. Only a band spectrum rich in lines is to be seen. An increase in the current fails to bring out any lines that are strong enough for observation.

An alloy of antimony with lead and zinc was also employed but without any better results.

3. BISMUTH

Current, 0.5 to 4 amperes Melting-point, 268° C.

The intensely blue line λ 4722 is the first to appear with a current of about 0.5 ampere. With a greater current very few more lines appear. With 4 amperes the lines λ 5552.4, λ 4733.9, and λ 4561.3 are seen and all of them are simple. A band in the green also becomes visible and shades down toward the red end of the spectrum.

The structure of a few bismuth lines has been investigated by von Baeyer[1] alone and by von Baeyer and Gehrcke.[2] They used a quartz bismuth amalgam-lamp of their own construction. Lunelund[3] also investigated the bismuth lines by means of Arons'[3] amalgam-lamp.

λ = 4722 (Blue)

λ = 4722 $d\lambda_{max}$ = 0.345 A Exposure, ½ minute

Crossed Interference-Plates			Echelon Grating			
Gehrcke and von Baeyer		von Baeyer	Lunelund		Author	
			0.166?	1		
			−0.144?	1	−0.144?	1
+0.242(= −0.103)*	2	+0.242(= −0.103)	−0.105?	4	−0.103	3
+0.289(= −0.056)	1	+0.283(= −0.062)	−0.062	3	−0.061	5
+0.316(= −0.029)	1	+0.318(= −0.027)	−0.031	3	−0.029	7
0.000	.	0.000	0.000	10	0.000	10
+0.057	1	+0.058	+0.059	5	+0.057	8
+0.104	3	+0.100	+0.103	2	+0.102	6

* The numbers within the parentheses have been introduced by the author.

[1] *Verh. deutsch. phys. Gesells.*, 1907. [2] *Annalen der Physik*, **20**, 285, 1906.

[3] H. Lunelund, Inaug. Diss., Helsingfors, 1910; *Annalen der Physik*, **34**, 505, 1911.

Since $d\lambda_{max} = 0.345$ A, the values given by Gehrcke and von Baeyer represent the values given within the parentheses (obtained by subtracting from 0.345 the values given by them). In this way the agreement between the results of different observers becomes apparent. Nevertheless the position of the first three satellites is not established beyond doubt.

It may be mentioned here that the formulae for the dispersion of an echelon grating show that there may exist an ambiguity in the results deduced from them. It is sometimes very difficult to say to which principal line a particular satellite belongs. Recently P. Gmelin[1] has suggested a method for removing such ambiguity, but I had no time for availing myself of his method.

Gehrcke and von Baeyer, too, are not sure of their having correctly arranged the satellites of this line. The evidence of Michelson's plane grating, which has already helped in giving the correct order of the satellites of mercury lines, may also throw some light here.

$\lambda = 4122$ (Violet)

According to Gehrcke and von Baeyer, this line is composed of three components of nearly the same intensity and they arbitrarily consider the middle component to be the principal line.

Lunelund had to expose his plate for fully five hours in order to photograph it, and he found that it consists of four components. The figures are:

Gehrcke and von Baeyer	......	−0.21	0.00	+0.15
Lunelund	−0.11	−0.05	0.00	+0.05

The wave-lengths of the line given by Gehrcke and von Baeyer and Lunelund are 412 $\mu\mu$ and 4122 A respectively.

I find that the line is not complex at all. The measurements of the wave-lengths of the line under observation show that there are two distinct lines separated from one another by a distance equal to about $\frac{1}{3}$ or $\frac{1}{4}$ A. The lines given are, according to

Kayser and Runge[2] (Arc)	4122.01 4121.69	Diff. = 0.32 A
Exner and Haschek[3] (Arc)	4122.10 4121.86	Diff. = 0.24
Exner and Haschek[3] (Spark)	4122.08 4121.75	Diff. = 0.33

[1] *Annalen der Physik*, **33**. 17, 1910.

[2] *Abhandlungen Berliner Akad.*, 1893.

[3] *Tabellen der Spektra*, Wien, 1902.

It appears that the above mentioned observers (Gehrcke and von Baeyer and Lunelund) have not distinguished the foregoing two lines from each other. Perhaps Gehrcke and von Baeyer observed a "ghost," since they could not use the crossed interference-plates owing to the faintness of the lines, and had to photograph them by means of a single interference-plate. Gehrcke and von Baeyer required an exposure of 8 hours while Lunelund required one of 5 hours. I exposed my plate for only 5 minutes and got a photograph showing two equally bright and sharp lines which appeared to be two distinct principal lines and not satellites. I believe that λ 4122 is not one line but two simple lines situated near each other.

4. CADMIUM

Current, 0.25 to 0.6 ampere — Melting-point, 325° C.

Cadmium is an easy metal to employ and the lamp burned very smoothly and for a long time.

The following three lines possess satellites:

λ	Half-Silvered Air Plates	Crossed Interference-Plates			Echelon Grating						
	Fabry* and Perot	Gehrcke† and v. Baeyer	Janicki‡		Janicki ‖		Lunelund ¶		Author		Exp
5086 I (ss)	+0.076	+0.081	+0 77	2	+0.076	1/5	+0.078	2	+0.076	2	3 min.
	0.000	0.000	0.000	1	0.000	1	0.000	10	0.000	10	
	−0.024	....	.....	..	−0.026?	1/8	....	..	−0.026	1	
4800 I (ss)	+0.082	+0.063	+0.058	2	+0.059	1/4	+0.060	2	+0.058	6	3 min.
	0.000	0.000	0.000	1	0.000	1	0.000	10	0.000	10	
	0.000	−0.038	−0.034	3	−0.034	1/6	−0.034	3	−0.034	3	
	−0.082	−0.083	−0.081	4	−0.080	1/5	−0.080	2	−0.081	6	
4678.4 (s)	....	+0.035	$+0.030_3$	2	+0.030	1/3	+0.032	3	+0.031	3	1½ min.
		0.000	0.000	1	0.000	1	0.000	10	0.000	10	
		−0.055	-0.055_8	3	−0.056	1/6	−0.056	3	−0.056	6	

(s) = bright (ss) = very bright.

* Ch. Fabry, *Comptes rendus*, **138**, 854, 1904.

† *Annalen der Physik*, **20**, 269, 1906.

‡ *Ibid.*, **29**, 833, 1909.

‖ *Ibid.*, **19**, 30, 1906.

¶ *Loc. cit.*

λ 5086.—It is remarkable that Janicki with his echelon grating finds a satellite at −0.026 but with his parallel-plates fails to observe it. Hamy and Fabry, too, have observed this satellite. Lunelund using an Arons' amalgam-lamp and an echelon and giving an exposure of about one hour could not find this satellite. The

probable reason why Janicki failed to observe it with the parallel-plates is the following:

The principal line is very broad and bright, while the satellite is very fine and faint. The latter is visible with a current of 0.2 or 0.3 ampere; with a greater current than this, it merges into the principal line. On account of the irradiation it is not possible to photograph the satellite as a separate line when an ordinary photographic plate is used. Hence the need of a "non-halation" plate for such cases. In this way I measured the distance of the satellite from the principal line and found it in close agreement with the measurements of Hamy, Fabry, and Janicki.

The following lines are sharp and simple:

6430.3 (ss)	5154.0	4662.7

5. CHROMIUM

Current, 1 to 4 amperes — Melting-point, 1515° C.

The vaporization was very irregular and the emitted light sometimes deepened in color suddenly and many new lines became visible. The spectrum is very rich in lines which lie so near each other that their identification becomes somewhat difficult.

The following lines, in agreement with Janicki, were found to be simple:

5410.0 (s)	4871.0	4565.7
5348.5	4862.0	4546.1 (ss)
5346.0 (s)	4829.5	4544.8
5329.3	4789.5	4540.9
5328.5 (s)	4756.3	4540.7
5298.4	4718.6	4535.9
5296.9	4708.2	4530.9
5276.2	4652.3 (s)	4526.6
5275.9	4651.4 (s)	4497.0 (s)
5275.3	4646.3 (s)	4385.1 (s)
5265.9	4626.3 (s)	4371.4 (s)
5264.3	4616.3 (s)	4359.8
5247.4	4613.5 (s)	4351.9 (ss)
5208.6 (ss)	4600.9	4351.2
5206.2 (ss)	4591.6	4344.7 (s)
5204.7 (ss)	4580.2	4339.8
4922.4	4569.8	4339.6

The three bright lines λ 4289.9, λ 4274.9, and λ 4245.5 show a peculiar behavior. Each of these lines appears to be accompanied

by a so-called "variable satellite," i.e., a satellite of which the distance from the principal line depends on the current-strength. My observations agree with those of Janicki and can be summed up as follows: (1) With a very small evaporation from the anode (i.e., with about 0.7 ampere), the lines are simple. (2) With an increase in the current, the lines become double. (3) The intensities of the components are different, being for the three lines above in the ratio of 1:6, 1:5, and 1:6 respectively. (4) With an increase in the current, the distance between the two components increases. (5) This increase in the distance is, as a rule, only possible when the current-strength is increased from low to high values. When the current is decreased from high to low values, the change in the distance does not occur with the same rapidity.

It appears from the foregoing, that here we have a case of unsymmetrical reversal. W. Hartmann[1] has observed these same lines in a magnetic field and he finds that their intensity increases considerably in the magnetic field. He also found that they show strong reversal and that the distance of the components varies, each component giving rise to a triplet.

In the case of magnesium,[2] where there is not the slightest doubt that the lines show reversal, we find a similar behavior—the only difference being that in this case the reversal is quite symmetrical.

For the line λ 4289.9, the distances between the two components were:

0.7 ampere	0.000 (simple)
2.0	0.017 A
2.5	0.020 A
3.0	0.024 A

The other two lines behave in exactly the same manner.

6. COBALT

Current, 3 to 5.5 amperes Melting-point, 1500° C.

The spectrum is very rich in lines and requires a very high current to produce them. Janicki finds that the four lines in the blue possess satellites. I find that only one of them has a real satellite while the other three are most probably cases of reversal

[1] Dissertation, Halle, 1907. [2] See p. 107.

which arise from the high currents used. It may be mentioned in passing that with such intense currents the porcelain tube containing the anode melts easily.

The following lines, in agreement with Janicki, were found to be simple:

5483.6	5342.9 (s)	4780.0
5444.8	5280.8	4749.9
5369.8	5266.7	4663.6
5369.1	4860.0 (s)	4531.1 (s)
5353.7	4840.4 (s)	4121.5 (ss)
5352.2 (s)	4813.7 (s)	4118.9 (s)
5343.6	4793.0	4110.7
		4092.6 (s)

The following lines were not simple:

λ	Janicki	Author
4629.5	+0.044 (2) 0.000 (1)	+0.045 (3) 0.000 (10)
4581.8	+0.065 (1) 0.000 (1)	Simple (reversal)
4565.7	+0.058 (1) 0.000 (1)	Simple (reversal)
4549.8	+0.048 (1) 0.000 (1)	Simple (reversal)

Janicki remarks that each of the last three lines consists of two equally bright components with a great resemblance to reversal. He found that by increasing the strength, the increase in the distance between the two components could not be noticed.

On the other hand, I found that with a current of about 4 amperes the lines were simple, and with an increase in the current, they became double, both the components being equally bright. It appears that Janicki had chosen a current of 5 or 6 amperes to start with and had consequently found the lines to be double.

7. COPPER

Current, 2 to 3.5 amperes Melting-point, 1080° C.

Copper is not an easy metal to use, as in the molten metal a gas bubble is formed which acts as an insulator between the

metal above and the metal below. To avoid this difficulty an alloy of silver and copper (80 per cent silver) was employed which vaporized quite satisfactorily. The color of the light was rich blu'sh-green.

The following lines, in agreement with Janicki, were found to be simple:

5220.2	I N I 4	4507.8 (s)	
5218.4 (ss)	I N I 4	4480.5 (s)	II N II 4
5153.4	I N II 4	4063.5	I N I 5
5105.8 (s)		4062.9 (ss)	I N I 5
4651.3 (s)		4022.9 (ss)	I N II 5
4531.0 (s)	II N I 4		

The following lines possess satellites:

λ	Janicki		Author		Exp.
5782.3 (ss)	0.000 −0.058 −0.096	1 2 2	0.000 −0.057 −0.095	10 5 5	6 min.
5700.4 (s)	0.000 −0.054 −0.086	1 2 2	0.000 −0.054 −0.090	10 5 5	6 min.
4704.8	+0.072 +0.033 0.000	2 2 1	+0.073 +0.034 0.000	6 6 10	4 min.
4275.3	+0.048 0.000	2 1	+0.048 0.000	4 10	5 min.

The two yellow lines λ 5782 and λ 5700 possess similar structure. From my measurements the components of the latter can be found from those of the first by multiplying it by $\frac{18}{19}$.

The separation of these satellites is rather small and they are probably due to the reversal of only one satellite. Nevertheless, an increase in the current does not seem to affect their distance apart, hence I believe that they are real satellites.

The evidence against this view is that Hartmann (*loc. cit.*) observed that the lines λ 5782 and λ 5700 are double lines and that their distance apart is 0.080 and 0.082 A respectively. Further, the Zeeman effect of these lines as observed by Michelson and by Hartmann shows some peculiarities. Under the influence of the

magnetic field, the components approach each other and give rise to a single narrow line without any broadening. Then they separate from one another, leaving behind a middle line.

8. LEAD

Current, 0.05 to 2.0 amperes | Melting-point, 327° C.

Lead alone and an alloy of lead and tin were used. It was found that the alloy burned much more smoothly than the lead alone. Janicki found that on the surface of lead used at the anode a layer of oxide was formed very quickly and consequently he had to take special precautions to avoid this difficulty. The alloy presents no such difficulties.

The following lines, in agreement with Janicki, were found to be simple:

6041.2	5005.6 (s)
6002.1	4387.3 (s)
5875.0	4168.2 (s)
5547.2 (s)	4062.3
5045.0	4019.7 (s)

The following lines possess satellites:

λ	Janicki		Author		Exp.
6057.3 (s) F	0.000	1	0.000	10	6 min.
	−0.136	2	−0.130	3	
5608.2 (s) F	0.000	1	0.000	10	5 min.
	−0.085	2	−0.085	3	
5373.6 F	+0.166$_3$	3	+0.166$_2$	4	6 min.
	+0.079$_4$	2	+0.077$_4$	4	
	0.000	1	0.000	10	
	−0.111$_9$	4	−0.111$_6$	3	
5201.6	simple		+0.003	2	6 min.
			0.000	10	
4545.2 F	+0.077$_5$	3	+0.077$_2$	4	6 min.
	+0.037$_6$	2	+0.036$_9$	4	
	0.000	1	0.000	10	
	−0.051$_7$	4	−0.052$_0$	3	
4058.0 (ss)	+0.033	2	+0.032	5	5 min.
	0.000	1	0.000	10	
	−0.041	3	−0.041	5	

s = bright   ss = very bright

Janicki has pointed out the similar structure of the lines λ 5373 and λ 4245 and shows that the satellites of the latter can be deduced from those of the former by multiplying them by 0.465.

9. MAGNESIUM

Current, 0.2 to 1.0 amperes — Melting-point, 630° C.

Magnesium powder was used and the lamp burned quite smoothly. The color of the light is intensely green.

The following lines, in agreement with Janicki, were found to be simple:

5711.6		4571.3 (s)	
5528.7 (s)		4352.2 (s)	
5183.8 (ss)	reversal	4167.8	
5172.9 (ss)	reversal	3838.4 (ss)	reversal
5167.6 (ss)	reversal	3832.5 (ss)	reversal
4703.3 (s)		3829.5 (ss)	reversal

The lines forming the triplet in the green and the triplet in the ultra-violet show an easy reversal and it is consequently difficult to get them as simple lines. In this case was applied for the first time the sure test of the difference between a line showing reversal and a line possessing a satellite. Apparently the two cases are very similar. With an extremely small vaporization of the metal, i.e., with a current of 0.2 ampere, the lines were found to be simple. As the vapor-density was increased by increasing the current, the line divided into two equally bright components whose distance from each other increased with the increase in the current.

The measurements made on the line λ 5167.8 gave the following results:

Current	Distance between the Components
0.2 ampere	0.000 Å (simple)
0.4	0.028
0.6	0.049
0.8	0.055

The components are equally bright and are symmetrically situated with respect to the original line. The lines λ 5183 and λ 5172 show similar behavior.

P. G. Nutting[1] has investigated the structure of the magnesium lines. Using an arc for his source of light, he found all the lines to

[1] *Astrophysical Journal*, **23**, 64, 1906; **24**, 111, 1906.

be simple. With greater currents he found each of the three green lines to be double. Evidently this was a case of reversal regarded by Nutting as a change in the structure of the lines.

10. MANGANESE

Current, 0.7 to 2.0 amperes — Melting-point, 1245° C.

The lamp burned smoothly for a very long time with but little consumption of the metal.

The following lines, in agreement with Janicki, were found to be simple:

5255.5	4844.5	4709.0	4257.8
5196.7	4823.7 (ss)	4705.6	4239.9
5151.1	4783.6 (ss)	4502.4	4235.5 (s)
5118.1	4766.6	4499.1	4235.3 (s)
5074.8	4766.0	4491.8	4083.8
5030.8	4762.6 (s)	4490.3	4083.1
5005.1	4761.7 (s)	4436.5	4079.6
4862.3	4754.2 (ss)	4415.1	4070.4
4858.7	4730.3	4281.3	4063.4
4858.0	4727.6	4266.1	

The following lines possess satellites:

λ	Janicki		Author		Exp.
6021.8 (s)	Simple		Simple		5 min.
6016.6	Simple		0.000 −0.052	10 5	5 min.
6013.6	Simple		0.000 −0.035	10 5	5 min.
5538.1	2-3 weak satellites		0.000 −0.046 −0.104	10 4 3	5 min.
5517.1	0.000 −0.073 −0.118?	1 2 3	0.000 −0.070 −0.120	10 5 4	5 min.
5506.1	0.000 −0.047 −0.089	1 2 3	0.000 −0.047 −0.088	10 5 3	5 min.
5481.7	0.000 −0.065 −0.122	1 2 3	0.000 −0.062 −0.115	10 5 3	5 min.

λ	Janicki		Author		Exp.
5470.9	0 000 −0 056 −0 105	1 2 3	0 000 −0.055 −0.104	10 5 3	5 min.
5407.6 (s)	0 000 −0 057 −0.105 −1.144	1 2 3 4	0 000 −0 054 −0 104 −0 147	10 7 5 3	3 min.
5399.7	0 000 −0 055 −0 102 −0 143	1 2 3 4	0 000 −0 054 −0.102 −0 142	10 7 6 4	3 min.
5394 9	0 000 −0 065 One more satellite	1 2	0 000 −0 068 No satellite	10 5	5 min.
5388.7	. . . . 2 satellites	. . . .	0 000 −0.036 −0 066 −0 093?	10 4 3 3	6 min.
5377.8	0 000 −0 035	1 2	0 000 −0 037	10 4	6 min.
5341.2 (ss)	0 000 −0 057 −0 108 −0 140 −0 183	1 2 3 4 5	0 000 −0 058 −0 107 −0 149 −0 190	10 7 7 6 6	3 min.
4061.9	0 000 −0 034 −0.061 −0 082	1 2 3 4	0 000 −0 033 −0 060 −0 082	10 7 7 5	6 min.

It will be seen that all the lines show similar structure. The satellites of most lines become uniformly weaker—as they recede from the principal line and their distance from the principal line also decreases regularly—in other words, they constitute a series. Janicki found that, knowing the first satellite, the others can be calculated from the following empirical formula:

$$d\lambda_n = d\lambda_1 + \frac{d\lambda_n - 1}{1.17}.$$

Janicki found that when the current-strength was increased, the three lines λ 6021.8, λ 6016.6, and λ 6013.6 broadened and lost in sharpness *without* showing a reversal.

On the other hand, I found that the line λ 6021.8 is simple, while each of the other two possesses a satellite. The following observations show that I was not dealing with a case of reversal: (1) The three lines according to Janicki have similar structure but according to my observation they behave differently. (2) The satellite is half as intense as the principal line. (3) The distance of the satellite from the principal line does not increase with an increase in the current.

λ 5538.1.—I have measured the two satellites which Janicki could not.

λ 5394.9.—Janicki thinks that probably another satellite exists but I found no trace of any other satellites.

λ 5388.7.—Janicki found that the satellites could not be easily measured. I have measured them.

Janicki has succeeded in investigating a few more lines in the ultra-violet between λ 4040 and λ 4030. These lines lie very near each other, their wave-lengths differing from one another by a few angstroms only. Here I had three difficulties to face: (1) The prism of the accessory spectroscope employed in conjunction with the echelon grating was of dense, yellow glass and absorbed much of the violet light. (2) The dispersion of the prism was rather small and the identification of the lines so closely situated extremely difficult. (3) Lastly, the great drawback of the echelon grating was noticed here. The distance between the two neighboring orders of the spectrum became very small and the lines almost covered one another. Consequently I was not able to investigate the structure of these lines.

It may be mentioned here that Janicki, too, found difficulties with these lines. He had three different interference-plates at his disposal but he could resolve these lines with one of them only. Moreover, he had the advantage of photographing them all simultaneously.

11. SILVER

Current, 1.5 to 2.5 amperes Melting-point, 960° C.

Pure silver and an alloy of silver and copper were used. The lamp burned for a very long time with a very minute consumption of the metal. The color of the light was intense green.

The following lines, in agreement with Janicki, were found to be sharp and simple:

5471.7 (s)	I X I 4	4212.1 (s)	I X I 5
5465.7 (ss)	I X I 4	4055.5 (s)	I X II 5
5209.2 (ss)	I X II 4	3981.9 (s)	II X I 5
4648.7 (ss)	II X I 4	3841.3 (s)	II X II 5
4476.3 (ss)	II X II 4	3810.6 (s)	I X I 6

It is worth noticing here that all the lines above belong to the first or the second subordinate series.

12. SODIUM

D Lines

There has always been more or less difficulty in getting a proper source of light for observing the D lines. A. A. Michelson[1] says that by using metallic sodium in a heated vacuum tube, the results are so variable and the character of the lines varies so much with a variation in temperature and pressure that a complete investigation is not possible. I found that in my lamp, the D lines were often visible. They arose from the porcelain tube containing the metal used as the anode. I also tried an amalgam containing a minute quantity of sodium.

With a very small current, the two D lines are simple and extremely sharp. With an increase in the current each of the two lines divides into two and the distance between the components so produced increases. The black space between these components remains perfectly sharp but the outer edges of the components become hazy. These changes are evidently due to reversal.

Like Fabry and Perot[2] and Janicki,[3] I too find that each of the D lines is simple and not composed of two components each having a very weak satellite.

13. TELLURIUM

Current, 0.02 to 0.05 ampere Melting-point, 200° C.

The lamp burned for a very long time with an extremely small consumption of the metal, one gram of the metal lasting for several hours. The light is intensely rich green and is due wholly to the green line λ 5350.6.

[1] *Phil. Mag.*, **34**, 280, 1892.

[2] *Comptes rendus*, **130**, 653, 1900.

[3] *Annalen der Physik*, **19**, 36, 1906.

Janicki found that the line λ 5350.6 has one satellite +0.1137 with an intensity equal to one-fourth of the principal line. I find the distance to be +0.114 and the intensity seven-tenths.

The principal line is much broader than any other line examined by me (cf. cadmium λ 5086) and with very small currents (less than 0.02 ampere) appears to be double. When the current is increased, the two components approach each other and give rise to a single line. Since the line appears to be double only with the smallest current available (i.e., the smallest vapor-density) it cannot be due to reversal. Although the ocular observation distinctly showed the line to be double, I did not succeed in photographing it as such.

Following are the observations made on this line:

Michelson*		Fabry and Perot†		Barnes‡		Janicki		Author	
0.00	1	0.000	1	0.00	1	0.000	1	0.000	10
+0.02	1/5	+0.020	1/2	0.04	1/4	...		Satellite	
+0.12	1/2	+0.114	1/2	0.10	3/4	0.113_7	1/4	0.114	7
+0.13	1/10					....			

* *Loc. cit.*
† *Comptes rendu* 126, 407, 1898.
‡ *Astrophysical Journal*, 19, 190, 1904.
Loc. cit.

Michelson and Perot and Fabry used tellurium chloride in Geissler tubes.

It is interesting to note that while observing the Zeeman effect[1] on the above line, the presence and position of another satellite was indirectly determined. This satellite corresponds with +0.020.

14. TIN

Current, 2 to 3 amperes Melting-point, 230° C.

Janicki found that when tin is used at the anode, it yields large quantities of hydrogen and that gas bubbles are formed in the molten metal which, acting as insulators, break the current and thereby extinguish the arc. He therefore took special precautions in order to overcome this difficulty. I avoided this difficulty by using an alloy of tin and lead which burned well and gave no trouble.

Tin gives only one line, λ 4524, in the visible spectrum and this line does not possess any satellite.

[1] Wali-Mohammad, *Annalen der Physik*, 39, 247, 1912.

15. ZINC

Current, 0.3 to 0.5 ampere — Melting-point, 415° C.

Zinc was one of the easiest substances to use. The lamp burned very smoothly, and the color of the light was intense bright blue.

The following lines are sharp and simple:

6364.0 (ss)	4722.3 (ss)
5182.2	4680.4 (ss)
4810.7 (ss)	4630.1

Michelson[1] found a component of λ 4810 while Houstoun[2] found λ 4810, λ 4722, and λ 4680 to be double. There is not the least doubt that what Houstoun observed was the reversal.

It has been settled by the researches of Hamy,[3] Janicki,[4] Gehrcke and von Baeyer,[5] and Lunelund[6] that all the above zinc lines are simple and do not possess a complex structure.

CONCLUSION

It will be seen from the foregoing that of the 15 different metals investigated, comparatively very few possess complex lines. Most of the lines are simple and none of the metals shows the same abundance and complexity of satellites as mercury. Many complex lines (cf. copper, lead, manganese) possess similar structure and form a sort of minor series.

Further, some of the lines show symmetrical, while others show unsymmetrical, self-reversal. Such reversals have to be carefully differentiated from lines possessing real satellites. Complex lines should not be used as standard lines or as lines of reference.

Lastly, the results yielded by the echelon grating agree, on the whole, with those given by crossed interference-plates. The oxy-cathode proved a very useful source of light for the investigation.

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[1] *Phil. Mag.*, **34**, 280, 1892.

[2] *Ibid.*, **7**, 456, 1904.

[3] M. Hamy, Sur le spectra du zinc, *Comptes rendus*, **138**, 959, 1904.

[4] L. Janicki, *Annalen der Physik*, **19**, 36, 1906; **29**, 845, 1909.

[5] Gehrcke und O. von Baeyer, *Ibid.*, **20**, 269, 1906.

[6] H. Lunelund, Dissertation. Helsingfors, 1910.

A POLARIZATION SPECTROPHOTOMETER USING THE BRACE PRISM

By HARVEY BRACE LEMON

The two factors which chiefly control the sensibility in any form of photometer or spectrophotometer are, in the order of their importance, (1) the elimination of the dividing line between the two fields under comparison, and (2) the greatest possible conservation of light so that the illumination of the two fields shall be perhaps as large as 500 meter candles.[1] For the accomplishment of the first of these requirements there are no devices more satisfactory than the Lummer-Brodhun cube or the Brace prism,[2] both of which furnish between the two fields lines of separation of the order of a few wave-lengths only in width. For the purpose of spectrophotometry, however, the Lummer-Brodhun cube must be used with an auxiliary dispersing prism, whereas the Brace prism itself furnishes the dispersion as well as the two fields which are to be matched, and consequently excels the Lummer-Brodhun cube in fulfilling the second requirement noted above.

In working with the Brace prism for a number of years certain very serious objections to the manner of operation as proposed by Brace have been found. In the original form of this instrument changes of intensity in one of the fields are produced by changing the slit-width, and therefore the spectral purity, if a continuous spectrum is under observation. For very small changes in intensity this does not produce a noticeable difference in hue between the two fields, but for considerable variations it does, and the accuracy of the settings is thereby greatly impaired. Moreover, the intensity is proportional to the slit-width only where the spectral luminosity-curve is parallel to the axis of wave-length, unless bilateral slits are employed. When these are used the intensity is proportional to slit-width wherever the luminosity-curve is a straight line. Even this latter condition is fulfilled only

[1] Nutting, *Outlines of Applied Optics*, p. 172, 1912.

[2] D. B. Brace, "On a New System for Spectral Photometric Work," *Astrophysical Journal*, 11, 6, 1900.

at two very limited regions of the spectrum, and consequently for general work a calibration of slit-width readings for true intensity values must be made either with a rotating-sector disk or some similar device which will give known changes of intensity in front of one source. If the value of the slit-width in front of the other source necessary to produce a match is measured and plotted against the known intensity furnished by the sector, then will the curve drawn through all such points constitute one of the curves of calibration.[1] These curves in general are not straight lines;

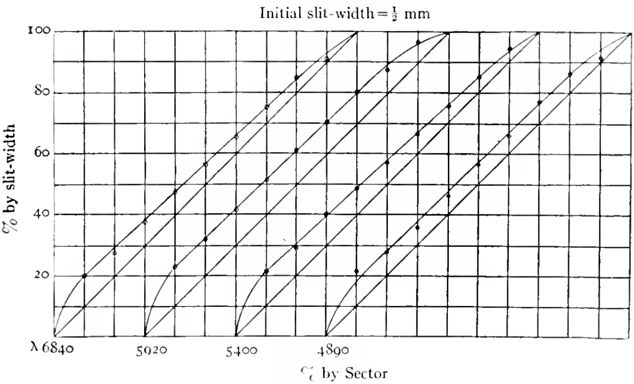


FIG. 1

they differ for every different wave-length and for every different initial value of the slit-widths. An example of them is given in Fig. 1, which shows the curves for a single initial value of slit-widths, 0.5 mm, and for four different wave-lengths. The percentage of intensity as indicated by slit-width readings is plotted against the true percentage of intensity given by the sector. If the two agree, the curve is the 45° straight line. Now a *complete* calibration consists of a double infinite set of such curves and is out of the question, of course. A calibration, to be usable for quanti-

[1] Cf. E. V. Capps, "Calibration of the Slit in Spectral Photometric Measurements," *Astrophysical Journal*, 11, 25, 1900.

tative work, should contain observations at perhaps 10 wave-lengths for each of 5 different slit-widths, making 50 curves. The points on a single curve require about 100 individual settings. The labor involved in such a calibration is very great for the result obtained. Add to this the fact that the accidental closing of a slit too tight will change the origin of co-ordinates by an unknown amount and that any readjustment of the instrument, especially any shift of the prism, necessitates an entirely new calibration, and the great disadvantages in the use of the Brace instrument as put forth by him become apparent.

These disadvantages have been entirely overcome by the introduction into one of the collimators of the simplest sort of a polarizing arrangement. This involves no sacrifice of intensity in the other beam, which is the objection made by Brace[1] to the use of polarizing arrangements in his instrument.

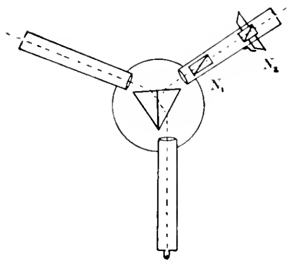


FIG. 2

The modification, Fig. 2, consists in the introduction into one of the collimator tubes of two nicol prisms, one, N_1, fixed, and the other, N_2, capable of rotation about the axis of the collimator, the amount of which is measured by a divided circle. In the mounting of these nicols three precautions must be observed:

1. The prisms must be of the form of rectangular parallelopipeds whose end faces are parallel and can be made perpendicular to the collimator axis, and an adjustment for this, especially in the case of the rotating nicol, N_2, must be provided. If the axis of rotation of N_2 is not coincident with the collimator axis, a shift of the slit-images will occur at the eye-end of the telescope which is equivalent to a shift of wave-length sufficient to make the field under observation variable in hue. With proper adjustment made by observing some brilliant spectral line such as D_3 of helium upon an ocular

[1] *Op. cit.*, p. 17.

slit or cross-hair, and adjusting the set screws provided on N_2 (not shown in the figure), the line under observation may be made to remain absolutely fixed in position while the nicol is rotated.

2. Another equally important condition is that the fixed nicol, N_1, be the nearer of the two to the Brace prism. This effects a stationary azimuth between the plane of polarization and the prism surfaces, and hence no variation in intensity can be produced by the partial polarization suffered by the light in transit through the Brace prism. Unless this precaution is taken, in fact with practically every other arrangement of the nicols in either of the collimator tubes or the telescope tube, the partial polarization of both beams produced by reflection at the glass surface of the prism and from the silver strip within is great enough to make errors in intensity amounting to 100 per cent at low intensities unless it be taken into account. The magnitude of this correction can of course be calculated from the optical constants of the instrument. It is more readily determined by a calibration similar to the one indicated above for slit-width readings. Calibration-curves of this type may consist of either a singly or doubly infinite family, depending on the location of the nicols, and consequently any arrangement of nicols other than one satisfying the condition of constancy of azimuth of the polarized beam with reference to the Brace prism can offer no improvement whatever on Brace's original form. As an example of a polarizing arrangement not satisfying the above condition of constancy of azimuth, note that given by Wallace[1] and used later by him,[2] by him and the author,[3] and by the author.[4] A calibration-curve typical of this arrangement is given in Fig. 3, where the abscissae are intensities in percentage given by the sector and the ordinates the same intensities as measured by the polarizer, i.e., given by $\sin^2 \theta$ where θ is the complement of the angle between the transmitting planes of the two nicols. The calibration-curves are seen to be functions of the wave-length. For this arrangement they are functions of the slit-widths also.

[1] "Studies in Sensitometry, I," *Astrophysical Journal*, **25**, 124, 1907.

[2] "Studies in Sensitometry, II," *ibid.*, **26**, 299, 1907.

[3] "Studies in Sensitometry, III," *ibid.*, **29**, 140, 1909.

[4] "Spectroscopic Studies on Hydrogen," *ibid.*, **35**, 115, 1912.

For these reasons no advantage can be claimed for this arrangement over the original Brace instrument.

3. As a third condition for success the Brace prism should be so placed with reference to the collimator beams that the polarized beam is received on the silver strip as shown in Fig. 2, while the beam from the other collimator passes above and below the silver strip. All absorption due to the silver strip and the nicols is therefore put into the one beam which comes from the standard source, and losses in light can therefore be compensated for by increasing

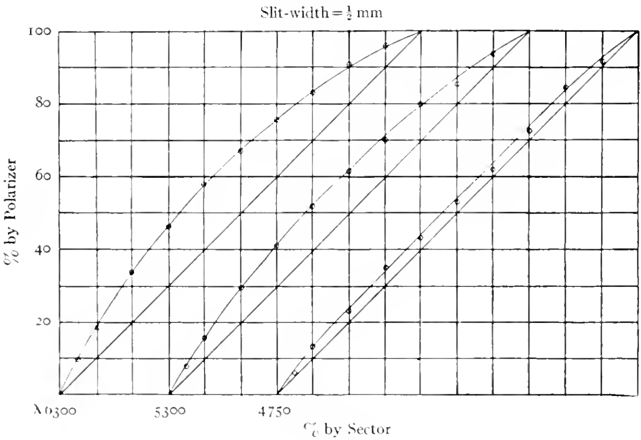


Fig. 3

the intensity of that source in which ample light is available. The other source, which is under comparison with the standard, suffers no more loss in intensity than that necessitated by the two lenses and by its dispersion through the prism. In other words, the economy of light justly claimed by Brace as being greater in his instrument than in many other forms has here in no way been sacrificed except at the standard source, where such economy is entirely unnecessary.

To determine how closely the readings from this instrument as given by $\sin^2 \theta$ conform to the actual intensities, many observations

using the rotating sector have been made. The insertion of these here in numerical detail is unnecessary since the results can be shown with great ease graphically, a single graph representing hundreds of settings. The graphs represent intensities in percentage given by $\sin^2 \theta$ plotted as ordinates against percentages given

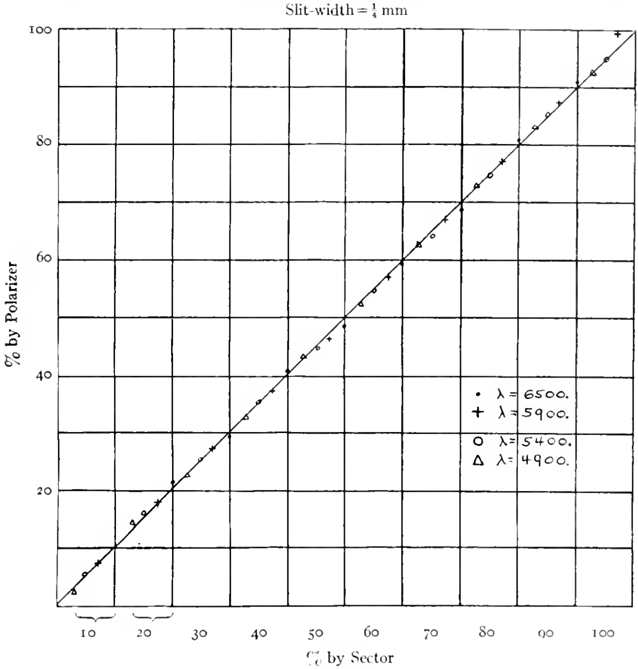


FIG. 4

by the rotating sector as abscissae. If the instrument is to be free from all necessity of having calibration-curves determined, these graphs should be coincident with the 45° straight line for every wave-length, slit-width, and condition of preliminary adjustment. Fig. 4 represents observations made at four different wave-lengths

with slit-widths of 0.25 mm. In order that points for different wave-lengths might not fall too close together, different origins have been selected, one for each color, as shown by the indicated scales of ordinates and abscissae. Thus any set of points corre-

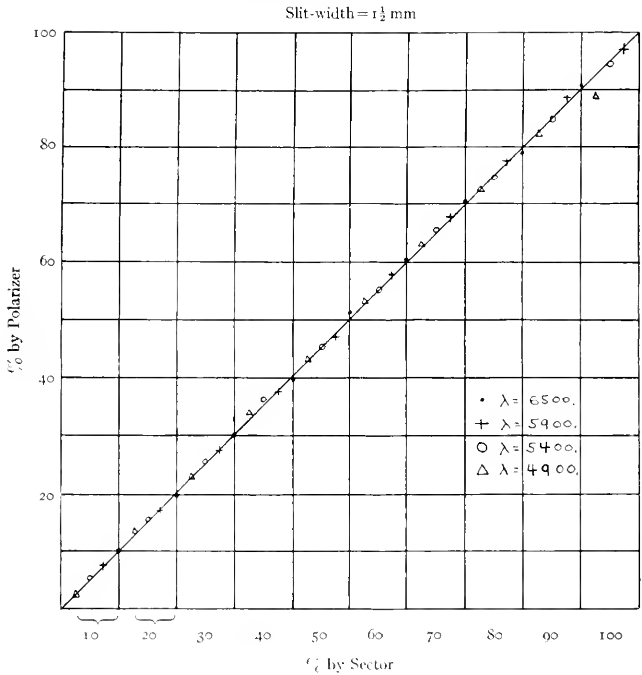


Fig. 5

sponding to a given color and represented by a given symbol (dot, cross, circle, triangle, etc.) is simply shifted with reference to all the other sets along the 45° line so that points are uniformly distributed over its entire length. All points are seen to fit very closely the ideal line and no systematic variation with color is dis-

cernible.[1] Fig. 5 is a similar series of observations taken with slits 1.5 mm in width, and the same remarks apply. As a final test one-half of such a series was taken, the instrument then was entirely dismounted and sent into the shops for refinishing of its mechanical and some of the optical parts. After reassembling, a different

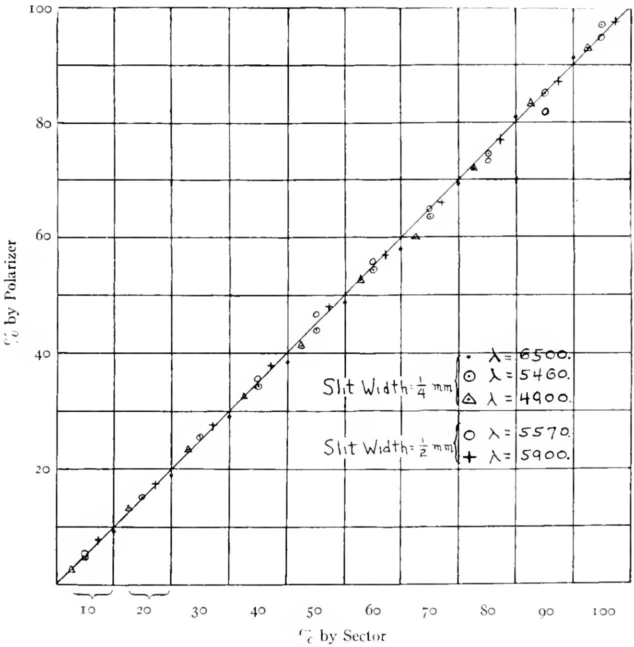


FIG. 6

Brace prism was put in, the slit-widths were made different, and the remaining half of the observations taken at different wave-

[1] Quantitative measurements of the true sector openings have not been made. The sector was carefully laid out and filed to the ruled lines. The errors are less than those of ordinary observation in intensity matches, but they begin to make appearance in measurements compiled from very extensive series of settings, such as these.

lengths from those used before. The same line fits one half of the points just as well as the other half. The departures are accidental in character and not at all systematic. Since the most unfavorable conditions possible were here imposed and speed was sought as well, each point being the mean of only five settings instead of the ten usually taken, it is not surprising that the accidental departures from the true curve are a little more than in the preceding two figures. Figs. 4, 5, and 6 show conclusively that this form of instrument needs no calibration whatever for wave-lengths, slit-width, prism, or condition of preliminary adjustments, that intensities as measured by $\sin^2 \theta$ taken directly from the average of group settings represent actual intensities. The sensibility of the instrument, provided the optical surfaces, especially those of the nicols, are carefully worked and are entirely free from minute scratches, approaches the sensibility of the human eye to differences in intensity. To sum up, then, we have in this form of instrument one in which the great economy of light furnished by the Brace prism is entirely preserved, together with the excellence of elimination of the dividing line, whereas there is *complete freedom from the cumbersome calibrations* hitherto characteristic of that instrument.

This form of photometer is put on the market by the firm of William Gaertner & Co., Chicago, to whom acknowledgments are due for the loan of an instrument for a number of years and for many mechanical changes very obligingly made as this new form was developed. Miss K. T. Aschenbrenner has been of assistance also in taking long sets of observations and aiding in their reduction.

For the benefit of those utilizing nicol prisms for controlling intensities, a table of $\sin^2 \theta$ by degrees and tenths to four places is appended. The pages are unnumbered and perforated along the edge so that they may be removed and mounted on a card for ready reference if desired.

Ryerson Physical Laboratory
University of Chicago
December 1913

TABLE I

Sin² θ for Photometer

°	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.0000	.0000	.0000	.0000	.0000	0000	.0001	.0001	.0002	.0002
1	.0003	0004	.0004	.0005	0006	0007	0008	.0009	.0010	.0011
2	.0012	.0013	.0015	.0016	.0018	0019	.0021	.0022	.0024	.0026
3	.0027	.0029	.0031	.0033	.0035	.0037	.0039	.0042	.0044	.0046
4	.0049	.0051	.0054	.0056	.0059	0062	.0064	.0067	.0070	.0073
5	.0076	.0079	.0082	.0085	.0089	.0092	0095	.0099	.0102	.0106
6	.0109	0113	.0117	.0120	.0124	.0128	.0132	.0136	.0140	.0144
7	.0149	.0153	.0157	.0161	.0166	.0170	.0175	.0180	.0184	.0189
8	.0194	.0199	.0203	.0208	.0213	0218	.0224	.0229	.0234	0239
9	.0245	.0250	.0256	.0261	.0267	0272	0278	.0284	.0290	0296
10	.0302	.0307	.0314	0320	.0326	.0332	.0339	.0345	.0351	.0358
11	.0364	.0371	.0377	.0384	.0391	.0398	.0404	.0411	.0418	.0425
12	.0432	.0439	.0447	.0454	.0461	.0469	.0476	.0483	.0491	.0499
13	.0506	.0514	.0522	0529	.0537	.0545	.0553	.0561	.0569	.0577
14	.0585	0594	.0602	.0610	.0619	.0627	0635	.0644	.0653	.0662
15	.0670	.0679	.0687	.0696	.0705	.0714	.0723	.0732	.0741	.0751
16	.0760	.0769	.0778	.0788	.0797	.0807	.0816	.0826	.0835	.0845
17	.0855	.0864	.0875	.0884	.0894	.0904	.0914	0924	.0935	.0945
18	.0955	.0965	.0975	.0986	.0997	.1007	1017	.1028	.1039	.1049
19	.1060	.1071	.1081	.1092	.1103	.1115	.1125	.1137	.1148	1159
20	.1170	.1181	.1192	.1203	.1215	.1226	.1238	.1250	.1261	.1272
21	.1284	.1296	.1308	.1320	.1331	.1344	.1355	.1367	.1379	.1391
22	.1404	.1415	.1428	.1440	.1452	.1464	.1477	.1490	.1502	.1514
23	.1527	.1540	.1552	.1565	.1578	.1590	.1603	.1616	.1629	.1641
24	.1654	.1667	.1680	.1694	.1707	1720	.1733	1746	.1760	.1773
25	.1786	.1800	.1813	.1826	.1840	1854	1867	.1880	1894	.1908
26	.1921	.1936	.1949	.1963	.1977	.1991	2005	.2020	.2033	.2047
27	.2061	.2075	.2089	.2104	.2117	2132	.2147	.2161	.2175	2190
28	.2204	.2218	.2233	.2248	.2263	.2277	.2292	.2306	.2321	.2336
29	.2351	.2365	.2380	.2394	.2410	2424	.2440	.2455	.2469	.2485
30	.2500	.2515	.2530	.2546	.2561	.2576	.2592	.2606	.2622	.2638
31	.2652	.2668	.2684	.2699	.2714	.2730	.2745	.2760	.2777	.2793
32	.2809	.2824	.2839	.2855	.2871	2887	.2903	.2919	.2935	.2950
33	.2966	.2983	.2998	3015	.3030	3046	.3062	.3079	.3095	.3110
34	.3128	.3143	.3159	.3175	.3192	.3208	3224	3240	3257	.3272
35	.3290	.3307	.3322	.3339	.3356	3373	3389	.3406	.3421	.3439
36	.3455	.3472	.3488	.3504	.3522	.3538	3552	.3571	.3588	.3606
37	.3622	.3639	.3656	.3673	.3690	.3705	3722	3739	3757	.3774
38	.3790	3807	.3825	.3841	.3858	.3874	3892	.3908	.3926	.3943
39	.3961	.3978	.3994	.4012	.4029	.4046	4063	.4079	4098	.4115
40	.4133	.4150	.4167	4184	.4202	4217	.4235	.4252	.4270	.4288
41	.4303	.4321	.4339	.4355	.4373	4391	4408	4426	4442	.4461
42	.4477	.4496	.4512	.4529	.4548	4565	.4582	.4598	.4617	4634
43	.4651	.4669	.4686	.4703	.4721	.4738	4756	.4773	4791	.4808
44	.4826	4844	.4860	.4878	4896	4914	.4930	.4948	4966	.4982

TABLE I—*Continued*

•	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
45	.5000	.5017	.5035	.5051	.5070	.5086	.5105	.5122	.5141	.5157
46	.5174	.5193	.5210	.5226	.5243	.5263	.5280	.5297	.5314	.5331
47	.5351	.5365	.5383	.5400	.5418	.5435	.5453	.5470	.5488	.5506
48	.5523	.5541	.5557	.5575	.5593	.5611	.5626	.5644	.5662	.5678
49	.5696	.5712	.5731	.5746	.5765	.5781	.5800	.5816	.5834	.5851
50	.5869	.5886	.5902	.5921	.5937	.5954	.5970	.5990	.6006	.6023
51	.6046	.6056	.6073	.6090	.6107	.6124	.6140	.6157	6174	.6192
52	.6209	.6226	.6243	.6260	.6280	.6295	.6310	.6327	.6345	.6362
53	.6377	.6394	.6412	.6430	.6445	.6462	.6477	.6495	.6513	.6528
54	.6546	.6561	.6580	.6595	.6610	.6628	6644	.6662	.6677	.6693
55	.6710	.6727	.6742	.6759	.6775	.6792	.6808	.6823	.6841	.6856
56	.6872	.6890	.6906	.6922	.6937	.6953	.6970	.6986	.7002	.7018
57	.7034	.7050	.7065	.7081	.7097	.7114	.7129	.7145	.7160	.7176
58	.7191	.7208	.7223	.7238	.7254	.7270	.7284	.7301	.7316	.7332
59	.7347	.7362	.7377	.7393	.7408	.7423	.7439	.7454	.7470	.7485
60	.7501	.7514	.7530	.7544	.7560	.7575	.7589	.7605	.7621	.7635
61	.7649	.7665	.7679	.7693	.7707	.7723	.7737	.7752	.7768	.7782
62	.7796	.7811	.7825	.7840	.7854	.7869	.7881	.7896	.7910	.7925
63	.7940	.7952	.7967	.7982	.7995	.8009	.8022	.8037	.8050	.8065
64	.8078	.8093	.8106	.8119	.8134	.8147	.8160	8173	.8187	.8200
65	.8215	.8228	.8241	.8255	.8268	.8279	.8293	.8306	.8320	.8333
66	.8346	.8360	.8372	.8385	.8397	.8410	.8424	.8435	.8449	.8461
67	.8474	.8486	.8498	.8511	.8523	.8535	.8549	.8561	.8572	.8584
68	.8596	.8608	.8622	.8634	.8646	.8658	.8670	.8680	.8692	.8704
69	.8716	.8728	.8740	.8750	8762	.8774	.8784	.8796	.8808	.8819
70	.8831	.8841	.8853	.8863	.8876	.8886	.8896	.8908	.8919	.8929
71	.8939	.8952	.8962	.8972	.8983	.8993	.9003	.9014	.9024	.9034
72	.9045	.9055	.9066	.9076	.9087	.9095	.9105	.9116	.9126	.9135
73	.9145	.9156	.9164	.9175	.9183	.9194	.9203	.9213	.9221	.9230
74	.9241	.9249	.9258	.9268	.9277	.9285	.9294	.9305	.9313	.9322
75	.9330	.9339	.9348	.9356	.9365	.9374	.9382	.9391	.9397	.9406
76	.9415	.9423	.9432	.9438	.9447	.9456	.9463	.9471	.9478	.9486
77	.9493	.9502	.9508	.9517	.9524	.9532	.9539	.9546	.9554	.9561
78	.9568	.9574	.9581	.9590	.9596	9603	9609	.9616	.9623	.9629
79	.9636	.9643	.9650	.9656	.9660	9667	.9674	9681	.9687	.9692
80	.9698	.9704	.9710	.9716	.9722	.9728	.9733	.9739	.9744	.9750
81	.9755	.9761	.9766	.9771	.9777	.9782	.9787	9792	.9797	.9801
82	9806	.9811	.9816	.9820	.9825	.9830	.9834	.9838	.9843	.9847
83	.9852	.9856	.9860	.9864	.9868	.9872	9876	9880	.9883	.9887
84	.9891	.9894	.9898	.9901	.9905	.9908	9912	9915	9918	.9921
85	.9924	.9927	.9930	.9933	.9936	.9938	.9941	.9944	.9946	.9949
86	.9951	.9954	.9956	.9958	.9960	.9963	.9965	9967	.9969	.9971
87	.9973	.9974	.9976	.9978	.9979	.9981	9983	.9984	.9985	9987
88	9988	.9989	.9990	.9991	.9992	.9993	.9994	9995	.9996	9996
89	9997	9998	.9998	.9999	9999	9999	1 0000	1 0000	1 0000	1 0000

AN APPLICATION OF THE REGISTERING MICRO-PHOTOMETER TO THE STUDY OF CERTAIN TYPES OF LABORATORY SPECTRA[1]

BY ARTHUR S. KING AND PETER PAUL KOCH

The constantly widening use of photography as a means of recording observations, and the fact that a large proportion of such observations involve the measurement of the density of the photographic image and frequently the gradual variations of this density, give rise to exacting requirements in the application of photometry to the study of photographic plates. Instruments in use—the best known of which is probably the Hartmann micro-photometer[2]—measure with high precision the density of a restricted area of the photographic image, and can, with care and patience, be operated to show the variation of this density from point to point. There is, however, an obvious need for an apparatus which will record photographically the varying intensities of a series of objects, such as the lines in a spectrum or a set of interference rings, and show also their distance apart. This can be done if the photographic objects to be measured are made to move slowly in front of an opening through which a beam of light from a constant source is passed, and the resulting changes in the intensity of this light are recorded on a moving photographic plate. The effects obtained for such a variation in the light-energy transmitted will then correspond closely to those yielded by the recording bolometer for variations in heat-energy.

This principle is applied in the registering micro-photometer which was designed by one of the writers and constructed under his direction in the physical laboratory of the University of Munich. A detailed account[3] of its construction and of the preliminary tests has been published, together with succeeding papers[4] on the use of the instrument in various photometric studies.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 77.

[2] *Zeitschrift fur Instrumentenkunde*, **19**, 97, 1899.

[3] Koch, *Annalen der Physik*, **39**, 705, 1912.

[4] *Ibid.*, **40**, 797; **41**, 115; **42**, 1, 1913.

In the summer of 1913, arrangements were made, with the ready consent of Professor Röntgen, by which this apparatus was brought by the designer to the Mount Wilson Solar Observatory and mounted in the Pasadena laboratory, where it was operated steadily for several weeks in registering intensity-curves for a variety of subjects represented in the photographs made in the different lines of investigation carried on at the Observatory. The object of the present paper is to give an account of the action of the photometer when applied to the study of several types of spectrum lines on plates made in the physical laboratory. The photographs thus examined have already been described in papers[1] on the various phenomena of the electric furnace and the tube-arc. The results of this photometric study serve to supplement the examination previously made, and also to demonstrate the usefulness of the micro-photometer in a branch of spectroscopy where quantitative measures of line-intensity and structure are highly desirable.

The observations to be described were necessarily preliminary in character and of very limited scope. The heavy demands on the instrument during the short time it was available did not permit of the repetition of slightly defective records, which would have added to the finish of the results.

APPARATUS AND METHODS

The construction of the instrument need be described only in outline here, as a detailed account[2] has already been published. A beam of light from a Nernst lamp passes through a slit, across which the photographic plate to be examined is moved by clockwork connected with the platform on which it rests. The light then passes to a photo-electric cell, which responds, by varying activity of the discharge from its potassium-coated electrode, to the change in density of the photographic plate as the latter passes across the beam of light. The other electrode of the photo-electric cell is connected to the vertical filament of a string electrometer, the plates of which are maintained at a potential difference of about

[1] King, *Contributions from the Mount Wilson Solar Observatory*, Nos. 60, 73, 76; *Astrophysical Journal*, **35**, 183, 1912; **38**, 315, 1913; **39**, 130, 1914.

[2] Koch, *loc. cit.*

200 volts. The varying charge on the filament, resulting from the changing activity of the cell, causes a horizontal movement of the filament. This movement is recorded by an image of the filament being projected on a photographic plate which moves vertically downward in a box having a horizontal slot on the side toward the electrometer. The rate of fall of this registering plate is controlled by the same clockwork that moves the plate to be registered across the illuminated slit, and the motion results in a curve being imprinted on the registering plate, the highest point of which (as the plate is usually oriented when examined) represents the maximum density of the photographic image which is being recorded. The relative rates of movement of the negative and of the registering plate are arranged to give a convenient scale for the intensity-curve. The methods of adjusting the sensitiveness according to the subject, the proper slit-widths and the reduction of inertia in the recording system have been described by the designer.

In the tests of which an account is to be given, the plates recorded were negatives made by double-copying on small plates, in order to avoid cutting the large original negatives. The plate was clamped on the moving platform of the apparatus so that the spectrum line to be recorded was parallel to the slit, which for this work was 0.038 mm wide and 1.5 mm long. The movement across the slit by means of the clock-drive then gave the variation in the light transmitted to the photo-electric cell, enabling the intensity-curve to be traced by the recording apparatus. The adjustments permitted of a ratio of either 7.65 or 46.4 between the movement of the negative and of the registering plate. The latter ratio was used throughout the present work, so that a registering plate 12 cm long gave the density-curves for a range of about 2.5 mm on the plate under examination. Except for the inconvenience of shifting the negative on its platform and piecing together the intensity-curves on the short registering plates, the apparatus as originally constructed proved well adapted to the study of the several types of spectra to be described.

The relation between electrometer deflection and density of the photographic image has been investigated,[1] and found to be almost

[1] Koch, *Annalen der Physik*, **39**, 734, 1912.

linear up to a certain degree of blackening, above which the sensitiveness of the instrument decreases rapidly as complete blackness is approached. The machine can be made to register the deflection corresponding to maximum photographic density whenever necessary if the Nernst lamp which illuminates the negative be covered for a few seconds at the beginning or end of a registration. A line showing the ordinate of total blackness will then be traced on the registering plate. The ordinates of all intensity-curves on the plate should then stop short of this height by an amount depending on the sensitiveness for which the instrument is adjusted. It can be made sensitive close to the point of maximum blackness, but in this case the illumination must be decreased for the lower part of the curve by the introduction of glass plates whose absorption is known. This somewhat difficult operation can usually be avoided by the selection of negatives of such exposure that all objects to be registered are within the range of density for which the deflection is proportional to the degree of blackening.

To place the classification of spectrum lines according to their intensity on an accurate quantitative basis, the directly measured density distributions of the photographic image must be reduced to intensity distributions, using properly chosen standards of relative intensity.[1] This presents a problem which can probably best be solved by relating the intensity distribution of the spectrum lines directly to the distribution of the black body.[2] Further, it will be important to see how far the influence of the spectrograph employed modifies the intensity distribution of the type of spectrum line under examination, and if there is need, to apply a correction for this. The latter problem is under investigation by one of the writers, the first results having been published.[3] The spectrum lines whose density curves are now to be discussed were not photographed with the data necessary to reduce the densities to intensities in a simple manner, so that the results are quantitative to a limited degree only.

[1] Koch, *Annalen der Physik*, **30**, 841, 1909; **34**, 377, 1911.

[2] *Ibid.*, **39**, 740, 1912.

[3] *Ibid.*, **42**, 1, 1913.

RESULTS

1. *Curves for sharp and diffuse lines.*—Values for the intensities of spectrum lines are usually given with tables of wave-length measures, very rough estimates in general being made to serve. The requirements are more exacting when the variations of line-intensities caused by changes in the physical condition of the source are considered, and any gain in accuracy in work of this kind is to be welcomed. The more distinctive differences between arc and spark spectra and between furnace spectra for different temperatures are usually well marked, and a practiced observer may attain a fair degree of accuracy in eye-estimates of intensities for lines which are similar in their distribution of intensity over the width. Serious difficulties arise, however, in the comparison of "sharp" lines with those variously described by the words "diffuse," "nebulous," "hazy," "*unscharf*," and similar terms. The latter are much wider in proportion to their density than are the sharp lines, and the eye does not readily combine the qualities of density and width so as to compare accurately lines in which these elements vary in different proportions. The purpose of the registering micro-photometer is to show, by the height and width of the density-curve, how black and how wide the spectrum line is in the negative, and also to measure by its area how much blackness is produced by the light from the line in question. Evidently this last property, if photographic differences are minimized by taking lines from the same plate, will give much closer values for the relative amounts of luminous energy emitted by the spectrum lines than can be attained by eye-estimates.

A set of lines to test the micro-photometer in this regard was selected from the arc spectrum of titanium near λ 4000 and the curves for eight typical lines, four sharp and four diffuse, are given in Fig. 1. The diffuse lines are not so extreme in this quality as can be found in other spectra, such as that of the copper arc, and the fast plate used gives a less abrupt rise at the base for the curves of the sharp lines than could be attained with greater contrast in the negative; but the typical differences, given by the gradual slope of the diffuse-line curves, resulting in a larger area in proportion to the height, are very distinct. The areas of the

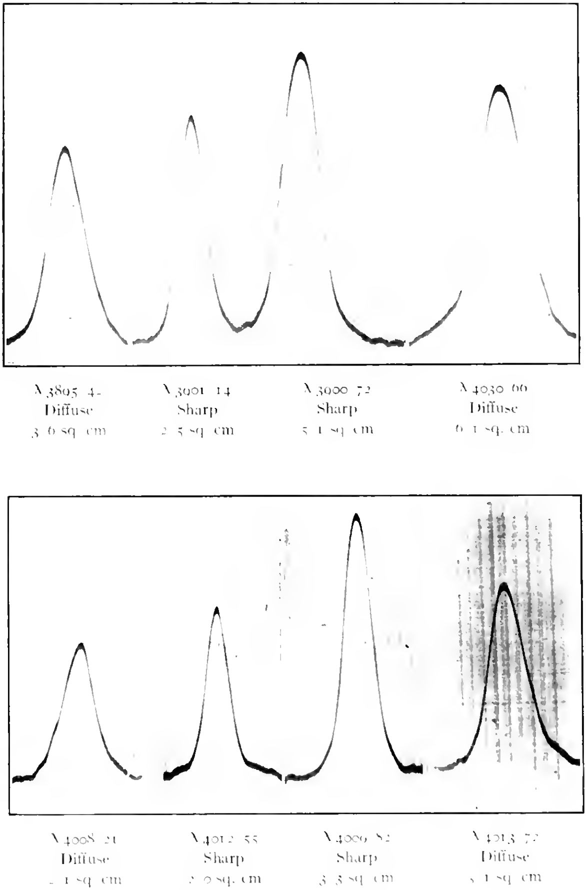


FIG. 1.—Curves for sharp and diffuse lines of the titanium arc, with areas inclosed by each curve.

curves (before being reduced for reproduction) were measured by means of a transparent scale ruled in millimeter squares. These areas, in sq. cm, are given for each line.

In Fig. 1 the curves for diffuse and sharp lines are placed adjacent to each other. They were traced from lines on the same negative, with the sensitiveness of the photometer unchanged throughout. None of the lines was of maximum photographic density in any part. It is seen that for each pair the diffuse line has its maximum distinctly lower than the sharp line, but the diffuse member of the pair has the greater area, except for λλ 4009.82 and 4013.72, where the difference in height is large and the areas nearly equal. The inaccuracy of considering the density of the maximum as representing the amount of photographic action produced by a given spectrum line is quite evident, as is also the difficulty of estimating by eye to what degree the width of a diffuse line makes up for the lack in density of its center.

For lines having a similar intensity distribution over the width, the blackness of the centers will be closely proportional to the total intensities of the several lines. Thus in the lower four curves of Fig. 1 the ratio of the heights for the two diffuse lines and for the two sharp lines is in each case about 2:3, which is approximately the ratio of their respective areas. Such a proportionality does not hold for the upper four lines of this figure and there is in general enough difference in the degrees of sharpness and diffuseness of individual lines to render unsafe an estimate of the relative intensities based on the densities of the maxima, even for lines apparently of the same type. The area of the intensity-curve is in general required as the measure of the strength of a line, due regard being paid to the decrease in sensitiveness of the instrument at higher densities mentioned on p. 216.

2. *Electric furnace lines for different temperatures.*—The curves given in Fig. 2 are for strong titanium lines at temperatures of about 2400° C. and 2100° C. respectively. The areas of the curves for 2100° represent the intensities more correctly than those for the higher temperature, as the ordinates of the latter reach the height for which the sensitiveness of the instrument decreases. The unequal intensities of the members of the close

doublet λλ 4536.12 and 4536.25 at low temperature are well shown by the curves. The one-sided notching of the curve representing this doublet at 2400° shows that the difference persists in some degree at the higher temperature. The general relation shown in

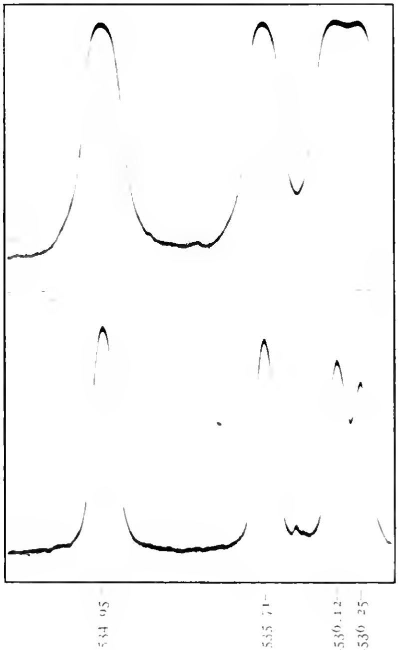


FIG. 2.—Curves for electric furnace lines of titanium, produced at 2400° C. (above) and 2100° C.

this figure is one which has been noted in the electric furnace studies of one of the writers, i.e., that low-temperature lines, when given high density by long exposure, still remain narrow; while at higher temperatures, there is a decided change in the direction of greater

width, if the exposures are timed to give the same general density at both temperatures. When the quantity of vapor is sufficient, the lines which are given at low temperature undergo a gradual softening of their centers and often pass into reversal as the temperature rises.

Time permitted the registration of only a few curves for the electric furnace spectra, and while these have shown the main problems involved in adapting the method to this work, it was not possible to carry out the experiments necessary to solve these difficulties. In addition to timing the exposures so as to obtain the proper range of density for the lines at each temperature, the varying contrasts in the negatives must be dealt with. Slight differences in contrast are given by different developments for plates of the same kind, and still larger variations are encountered in the different plates adapted to certain regions of the spectrum. To eliminate the effects of such differences will probably require the exposure of a part of each spectrum plate to lights whose relative intensities are known. Such a scale may be based on an absolute unit of light-intensity. The registration by the micro-photometer of these different degrees of blackening will then reduce the curves for spectrum lines on different plates to a common scale.

3. *Curves for lines displaced by pressure.*—The intensity distribution in spectrum lines when displaced by pressure around the source is of interest in connection with the question as to how far, if at all, the pressure-effect is to be considered as an unsymmetrical widening. It is well known that some lines become very unsymmetrical under pressure, but these belong in general to the class of diffuse lines and are probably never quite symmetrical. They show dissymmetry in the arc very readily at atmospheric pressure when the vapor density is increased or when the discharge conditions are altered. Sharp lines should be used to test the general effect of pressure, and for this purpose three sharp lines of iron, $\lambda\lambda$ 5371.734, 5397.344, and 5405.989 (the first and third being international standards of the second order), photographed with the electric furnace[1] at pressures of 8, 16, and 24 atmospheres, were

[1] King, *Contributions from the Mount Wilson Solar Observatory*, No. 60; *Astrophysical Journal*, **35**, 183, 1912.

registered by the micro-photometer. The curves for the first two lines are reproduced in Fig. 3. The total width of a line, which in this case is greatest for 16 atmospheres, bears no direct relation to the pressure, unless the conditions of the source, especially as to temperature and quantity of vapor, are kept strictly constant for different pressures. Other photographs were made at each pressure for which the lines were both wider and narrower than those reproduced here. The main point of interest is the close symmetry maintained by the reversed line at the three pressures. The degree of symmetry can be tested by means of the reference line traced by the machine during the descent of the plate, the distance of the curve from this line measuring the intensity at a given point. The line is shown for each of the curves of λ 5371.734 and for the 16-atmosphere curve of λ 5397.344, but is omitted from the two others on account of defective plates. The relative areas of the two sides of the reversed line were measured by bisecting the reversal and counting the number of millimeter squares included between the curve and the reference line for equal distances right and left until the curve became low. The areas thus measured for λ 5371.734 at the three pressures are as follows:

	Area in Sq. Cm		
	8 Atm.	16 Atm.	24 Atm.
Violet side of reversal	16.7	20.5	20.5
Red side of reversal	17.5	21.0	20.9
Difference	0.8	0.5	0.4

The small differences that appear are probably real and give slightly larger areas for the red side of the line. They are so nearly of the same magnitude that evidently there is no direct connection with the pressure. The extra area on the red side is chiefly on the outer part of the curve and may fairly be ascribed to the action of the grating, which gave a slightly hazy edge on the red side of strong lines unless the area of the ruled surface was greatly reduced.

As a result of the close approximation to symmetry for lines at these varying pressures, it is clear that the narrow line emitted by the cooler and less dense vapor near the end of the furnace tube is

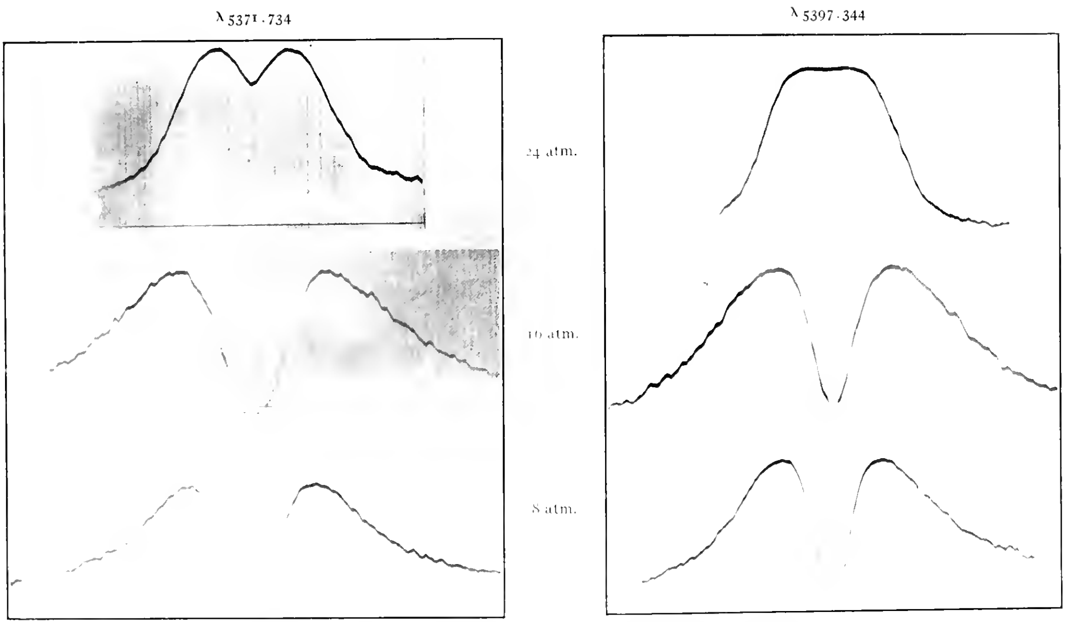


FIG. 3.—Curves for iron lines emitted by the electric furnace under pressure

displaced by pressure to the same degree as is the wider line given by the vapor near the middle of the tube's length, since the absorption of the latter radiation due to the cooler vapors gives the reversed line observed. Unsymmetrical broadening is in general much more pronounced for wide lines, and should increase with the pressure. The close symmetry which these measurements show for the reversed line at 24 atmospheres indicates a definite effect by pressure regardless of the width of lines or of the temperature or density of the vapor which produces them. This is in harmony with the conclusions previously arrived at by direct experiments[1] on variation of temperature and vapor density for given pressures, and adds to the evidence that the pressure effect is a real displacement of the maximum of the line, at least for those lines which are classed as "sharp" in spectra given with the source in vacuum or at atmospheric pressure.

4. *Intensities of reversed lines.*—The estimation by the eye of the relative intensities of reversed lines and the comparison with sharp lines involve much uncertainty. In the case of clearly reversed lines, one is inclined to lay stress on the relative widths of the reversals. This is probably safer with furnace lines than with those of other sources, by reason of the slow gradient in the condition of the vapor from center to end of the tube, which remains nearly constant during an exposure. Unless there are lines in various stages of partial reversal on the same plate, there is no simple means of connecting the intensity estimates for reversed and unreversed lines.

The registering micro-photometer promises a distinct advance in this direction. A line may be very broadly reversed, but if enough structure remains in the two sides to give its total width and the slope of the curve from the violet and red sides respectively, the central portion may be filled out by graphical methods so as to inclose an area which will represent with considerable closeness the intensity which the emission line would have if unaffected by the absorption which brings about the reversal. A reversal much wider than those shown in Fig. 3 by λ 5371.734 at 8 and 16 atmospheres would still give the intensity of the line if measured in this

[1] King, *op. cit.*, pp. 100, 202.

way, since the slope is nearly constant above a point less than half of the maximum ordinate of the curve.

5. *Intensity curves for tube-arc lines.*—From the foregoing study, it is evident that this form of micro-photometer is well adapted to show differences in structure and intensity of spectrum lines. Both of these elements vary for different parts of the same line in the case of the spectrum of the tube-arc, described by one of the writers.[1] This arc takes place when the furnace tube burns apart, causing a heavy arc current to pass at low voltage. The enhanced lines are given strongly in this source and large differences appear in lines produced at the center and near the wall of the ruptured tube.

Curves were traced with the micro-photometer for a number of typical lines given by the tube-arc when the spectrum was photographed according to the method described in the second paper, in which the slit coincided with the vertical diameter of the tube's image. The line on the negative was then about 20 mm long. By parallel diamond scratches running the length of the plate, each line was divided into four equal sections. The successive registration of these sections gave the curves shown for several lines in Fig. 4. The slit of the photometer, being 1.5 mm long, integrated the density of this length of the spectrum line, and as the tube-arc lines sometimes change rapidly both in strength and in structure over a short portion of the length, the form of the curve does not quite represent the true condition of the line at any point. The four sections are indicated by *a*, *b*, *c*, *d*, to correspond with successive parts of the line given from bottom to top of the tube's cross-section. The portion given by the axis of the tube would come between sections *b* and *c*.

The several curves of Fig. 4 may be considered in order. No. 1 represents the enhanced line of magnesium, λ 4481, which the tube-arc photographs have shown to be double, with the violet component the stronger. The photographs which gave the best definition of the components of the doublet were too strong to be used in the micro-photometer, the plate selected for this purpose

[1] King, *Contributions from the Mount Wilson Solar Observatory*, Nos. 65 and 73; *Astrophysical Journal*, **37**, 119; **38**, 315, 1913.

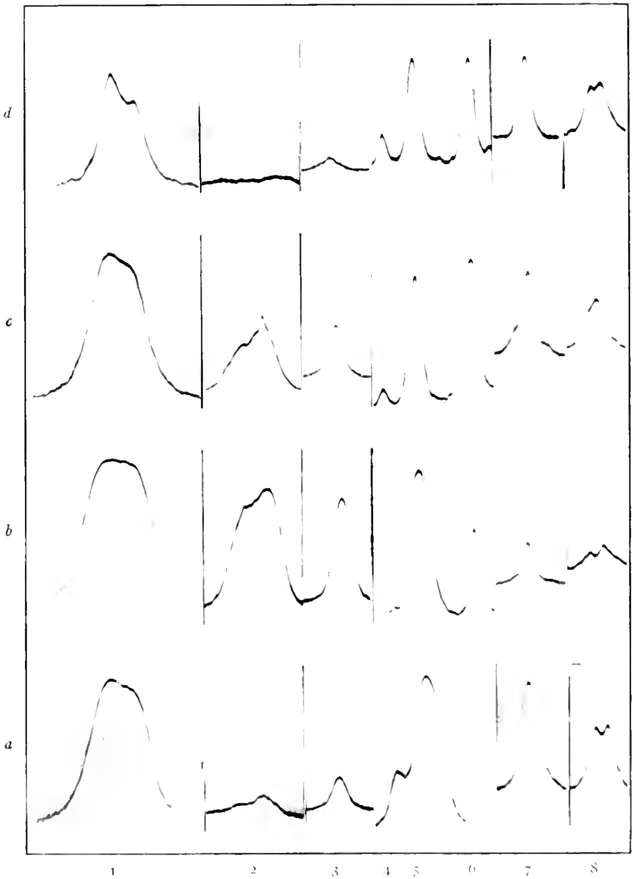


FIG. 4. Curves for lines emitted by different parts of the tube arc:

1 *Mg* (enhanced, double)
2 *Ca* (enhanced, double)
3 *Pb* (enhanced)
4 *V* (arc)
5 *Ti* (enhanced)
6 *Fe* (arc)
7 *Ti* (arc)
8 *V* (arc, reversed)

showing the components somewhat diffuse. However, the resolution is shown in *d*, while the dissymmetry toward the violet is distinct in all parts and the relative intensities of the doublet as a whole at the different heights are typical of the usual behavior of the line in the tube-arc. The greatest strength is seen to be at *b*, slightly below the center of the tube, but the line retains considerable intensity toward each end.

A much more rapid gradation from the maximum is shown by λ 4267 of carbon, represented by the curves of No. 2. This line, usually appearing only in the spark, was also shown by the tube-arc to be double, with its stronger component to the red. The dissymmetry is shown by the curves. The maximum is very distinctly at *b*, retaining some strength at *c*, and being relatively very weak at *a* and *d*, near the wall of the tube.

The enhanced line of lead, λ 4247, a symmetrical, single line, shows in No. 3 an intensity-gradation very similar to that of No. 2. These three sets of curves serve to show the variation of intensity for spark lines; the maximum enhanced-line radiation being given at the height *b*. The hydrogen lines in the tube-arc would give very broad curves, similar in relative areas to Nos. 2 and 3.

The remaining curves of Fig. 4 are for lines strong in the arc as well as in the spark. None of them shows the greatest strength at *b*, though No. 5, an intermediate type represented by the enhanced lines of titanium and vanadium, is strongest in the lower half of the tube, weakening at *c* and *d*. The iron arc line, No. 6, shows a gradation typical of the arc lines of several elements, with its maximum at *c*.

The titanium and vanadium arc lines, Nos. 4, 7, and 8, are strongest near the wall of the tube at *a* and *d*, showing a distinct minimum at *b*, where the enhanced lines are strongest. Throughout this figure, the entire curve for a given portion of a line is shown, and no attempt is made to represent the different heights from the same reference line.

The set of curves, No. 8, is of special interest, showing the variable dissymmetry in the reversal of a vanadium arc line. It is a general effect for reversed lines that the dissymmetry, when present, is most decided near the center of the tube.

These curves serve to supplement the material presented in the second paper[1] on the tube-arc phenomena, where the probable relation of the effects to those observed for the arc and spark under different conditions was discussed. It is evident that if a set of curves were traced for successive portions of the tube-arc line from end to end, with the slit of the micro-photometer as short as possible, a plot could be made with the areas of the curves as ordinates and distances along the line as abscissae which would represent with considerable accuracy the intensity variation given by different parts of the tube-arc for the line in question. Such a plot was made, from eye-estimates of the relative intensities, in the paper referred to (p. 321).

SUMMARY

The material offered in this paper is the result of a preliminary application of the micro-photometer in registering the distribution of photographic density over the width of the spectrum line. The types of lines examined have shown that the instrument may be used effectively to bring out several features which are important in the comparison of spectra.

1. A quantitative measure is obtained of the relative strengths of lines having very different appearance, especially the sharp and diffuse lines often occurring in the same spectrum, when the maximum density of the line cannot be taken as the measure of its intensity. An extension to the measurement of the intensity of reversed lines appears to be possible.

2. The characteristics of lines given at different temperatures of the electric furnace have been examined, together with the requirements necessary that spectra of this sort may be comparable for measurements of relative intensity.

3. Intensity-curves for electric furnace lines when displaced by pressure show that a line, if originally sharp, may maintain a structure very nearly symmetrical through a wide range of pressures. Thus the pressure-effect does not appear to be the result of unsymmetrical widening, and the symmetry of reversals is

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 73; *Astrophysical Journal*, 38, 315, 1913.

further evidence of a lack of dependence on temperature or vapor density.

4. The use of the instrument in the study of tube-arc lines of different classes shows that it registers very satisfactorily the various peculiarities of intensity and structure in lines from this source.

Mount Wilson Solar Observatory
November 1913

THE FUNDAMENTAL LAW OF THE GRATING

By JANET TUCKER HOWELL

THE COINCIDENCE METHOD

The fundamental law of the grating is that $N\lambda = w\ (sin\ \theta + sin\ \phi)$ where N is the order of the spectrum, λ is the wave-length, w is the distance apart of the lines of the grating from center to center, θ is the angle between the incident light and the normal to the grating, and ϕ is the angle between the diffracted light and the normal. With the concave grating mounting used by Rowland $\phi = 0$ and the law of the grating becomes $N\lambda = w\ sin\ \theta$. It follows therefore that, for any one angle θ, $N\lambda$ is constant, and it is possible, knowing standard wave-lengths in one order, to measure the wave-lengths of lines in the superimposed spectra of other orders. This procedure, known as the method of coincidences, was used by Rowland in determining his Table of Standards; and in parts of the spectrum in which standards have not been established by independent interferometer measurements, it is still the only method for the determination of wave-lengths.

SUMMARY OF PREVIOUS WORK ON GRATING MEASUREMENTS

Kayser in his first paper on "Standards of Wave-Length"[1] took up the question of the errors in Rowland's Table of Standards. These errors are due largely to the fact that Rowland shifted different parts of the spectrum to make his solar and arc spectra agree, but Kayser thought they were due partly to a second cause, namely, the unreliability of the coincidence method, and the object of his work was to test this method. Using the method of coincidences Kayser determined the wave-lengths of a group of third-order lines from second-order standards with two different gratings and found a systematic difference in his values amounting to 0.033 A. This could be accounted for by supposing one of the gratings to have suffered a linear displacement of the rulings of one part

[1] *Astrophysical Journal*, 19, 157, 1904.

with respect to the rest,[1] were it not for the fact that in calculating the Fabry and Perot standards by the coincidence method he found an error amounting to 0.019 A even for his better grating. As there was no reason for doubting the values of Fabry and Perot. Kayser came to the conclusion that neither of his gratings was available for coincidence work and thought it highly improbable that gratings existed perfect enough for accurate measurements by this method. He therefore confined his work on wave-length measurements to direct interpolation between Fabry and Perot standards of the same order.[2]

Since Kayser's last paper, work has been done on this subject in several directions. Papenfus in 1911[3] published two papers on the availability of the coincidence method for wave-length measurements. Working with a 13-ft. concave grating having 10,000 lines to the inch he determined the values of the tertiary standards from λ 6065 to λ 6678 by direct interpolation and from λ 4076 to λ 4376 by the coincidence method. He compared his values in both cases with Kayser's values determined by direct interpolation and found the same order of difference, which showed that no appreciable error had been introduced by the use of the coincidence method in the case of his grating.

Goos has done some recent wave-length work with a plane grating having 7000 lines to the inch.[4] He investigated the effect of varying the light-source, using first a short arc of 3 or 4 mm and then a long arc of 8 or 9 mm. With the short arc the lines were broader and when the broadening was unsymmetrical the values found with the two types of arc differed sometimes by as much as 0.02 A. So he arrives at the conclusion that a variation in the light-source used will account for large variations in the values found for tertiary standards and that before these tertiary standards can be established with any degree of accuracy a more exact definition of the standard arc is necessary.

The most recent work on tertiary standards has been done at

[1] *Ibid.*, **18**, 278, 1903. [2] *Ibid.*, **32**, 217, 1910.

[3] *Zeitschrift fur wissenschaftliche Photographie*, **9**, 332, 340, 1911.

[4] *Astrophysical Journal*, **35**, 221, 1912; **37**, 48, 1913; **38**, 141, 1913.

Mount Wilson by St. John and Ware.[1] They worked both at Pasadena and at Mount Wilson and found that owing to the difference in pressure amounting to about one-fifth of an atmosphere, due to the height of Mount Wilson, some of the lines were appreciably displaced and unsymmetrically widened. The mean difference for lines suffering a great displacement amounted to about 0.05 A per atmosphere. The secondary standards were displaced only 0.006 A per atmosphere, and in the region investigated they found 25 lines which were symmetrically widened and only slightly displaced under pressure which they recommend as tertiary standards.

The work of Goos on the variation of wave-length with the length of arc and that of St. John and Ware on the displacement and unsymmetrical broadening of some lines under pressure shows that there is a chance for large errors in wave-length measurements if the experimental conditions vary to any extent. It leads one to suspect that, after all, the inadaptability of one or both of Kayser's gratings to coincidence work may not have been altogether responsible for the errors found. In calculating the Fabry and Perot standards by the coincidence method it is possible that, although the standards themselves might not be subject to any appreciable variation in wave-length, the intermediate tertiary standards used might belong to the type that is subject to displacement and unsymmetrical broadening under varying experimental conditions. As these intermediate tertiary standards lie between λ 3550 and λ 3630, a region which has not been investigated in this respect, there is as yet no conclusive evidence on this point.

There is one source of error in coincidence work for which correction should be made in any absolute determinations of wave-length. This error is due to the fact that the dispersion of air varies with the temperature. The following example will show the magnitude of the effect.

$n=\frac{\lambda_e}{\lambda}$ where n is the refractive index, λ_e is the wave-length in ether and λ is the wave-length in air. Suppose that the wave-lengths λ 5640, first order, and λ 2820, second order, coincide exactly at 0° C. and 760 mm pressure.

[1] *Op. cit.*, **36**, 14, 1912.

Then

$$\lambda_{e1} = n \times 5640 = 1.0002924 \times 5640$$

and

$$\lambda_{e2} = n \times 2820 = 1.0003091 \times 2820.$$

At 10° and 760 mm the values for n are different.

$$\lambda_1 = \frac{\lambda_{e1}}{n} = \frac{1.0002924 \times 5640}{1.0002761} = 5640.09187$$

$$\lambda_2 = \frac{\lambda_{e2}}{n} = \frac{1.0003091 \times 2820}{1.0002919} = 2820.04847$$

$$\therefore\ 2\lambda_2 - \lambda_1 = 5640.0969 - 5640.0919 = 0.005\ \text{A}.$$

So that two lines which coincide exactly at 0° C. will differ by 0.005 A at 10° C. This shows the importance of keeping the temperature constant and may account to a large extent for the variations found in measurements by the coincidence method. In exact determinations of wave-length by this method all values should be reduced to standard conditions.

The work of Papenfus showed that his grating is adapted to coincidence work and it is possible that one of Kayser's gratings may be also, but the coincidence method is far from being reinstated in the position of importance it enjoyed at the time of Rowland's work. It was with the purpose of further testing the method that the piece of work here reported was undertaken.

RESULTS OF THIS INVESTIGATION

The work consists of two parts. In the first part, using the international standards from λ 5266 to λ 5434 in the second order, I determined from them by the method of coincidences the superimposed third-order lines in this region with two gratings, one having 15,000, the other 20,000 lines per inch. This is the same region that Kayser used in comparing his 20,000- and 16,000-line gratings. My results agreed remarkably well and show that the two gratings examined are perfect enough for wave-length determinations by the method of coincidences. But this part of the work did not in itself constitute a thorough proof of the law of gratings. In the second part of my paper I have shown that it is possible, starting with one standard line and assuming the law of

gratings, to determine the value of secondary standard lines to such a degree of accuracy that it would be feasible—if the secondary standards were not already determined by independent methods—to start from one known line and build up a whole system of wave-lengths from it, provided of course that the gratings used in the work were as good as the ones with which I have been working. This second part of my work shows that a system of wave-lengths can be built up from one line by means of the coincidence method which will attain the degree of accuracy originally claimed for this method by Rowland and therefore verifies the fundamental law of gratings.

APPARATUS

After the death of Rowland his two ruling machines were not used until 1910, when they were again put into working order by Dr. J. A. Anderson. Several of the gratings ruled last year by Dr. Anderson have been at my disposal. I have had therefore exceptional opportunities for carrying on grating work. I only regret that I have not had the time to test a larger number of them in the same and in different directions. In the first part of my work, comparing tertiary standards obtained by the coincidence method with two gratings, I used one of Rowland's six-inch concave gratings with a 21-foot radius and 20,000 lines to the inch, and one of Anderson's six-inch concave gratings with approximately the same radius and 15,000 lines to the inch. In the second part of my work it was necessary to work with superimposed spectra of higher orders, so I used one of Anderson's concave gratings having a radius of 21 feet and 7,500 lines per inch.

The grating mounting was that used by Rowland and Jewell in their work. The experimental conditions were kept as constant as possible, so that any line-displacement due to a variation in the light-source would occur to the same extent throughout the work. In the event therefore of a standardization of the light-source to be employed in wave-length measurements, my values could not be used for tertiary standards. They are, however, correct in relation to each other and this is all that is necessary in comparing the values obtained from different gratings in a proof of the coincidence method. The iron arc used was about 4 mm long and was operated at 110 volts with a direct current of between 5.5 and 6.5 amperes.

Owing to the fact that the Johns Hopkins University is situated in the middle of a city, it was found impossible to use plates taken in the daytime, as the vibrations due to the traffic seriously impaired the sharpness of the lines. All the plates used for measurement were therefore taken between 12 P.M. and 6 A.M. The method of measuring the plates was as follows: The secondary standards λ 5266.569 (group *d*) and λ 5434.527 (group *a* of Gale and Adams) were used to determine the scale of each plate in the second order. Using the scale thus determined, the values of the intermediate standard lines λ 5371.495 (group *a*) and λ 5405.78 (group *a*) were calculated. From the differences between the calculated and standard values a calibration-curve of each plate was drawn from

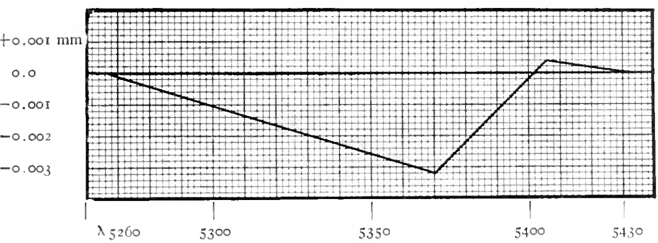


FIG. 1.—Typical calibration-curve for a plate between λ 5266 and λ 5434

which the readings for all other lines were corrected. A typical calibration-curve is shown above in Fig. 1.

The measuring engine was specially constructed for use in this piece of work. The screw was ground by the Rowland method until all periodic errors were sufficiently eliminated for the purpose and it was estimated that the errors outside of the error of the screw did not amount to more than one-fifth of a division on the head of the screw. One division on the screw-head = 0.001 mm and, as it was found impossible to count on successive readings agreeing to within less than 0.005 mm, the error of the instrument is negligible in comparison with the error of setting on a line. The readings were made to thousandths of a millimeter, and from twelve to fifteen readings were made on each line. The plates

were reversed after the whole region had been measured in one direction three times.

The scale of my plates was such that for the 20,000-line grating 1 mm = 0.9702 A and for the 15,000-line grating 1 mm = 1.30729 A, in the second order. After correcting the readings for each line from the calibration-curve of the plate the wave-lengths of the lines were found in terms of this scale in the second order and then reduced to third-order values by multiplying by the factor $\frac{2}{3}$.

FIRST PART—RESULTS

The region between λ 5266 and λ 5434 was measured as described above. In this region there is only one second-order line which is on the list recommended by St. John and Ware as tertiary standards. It was therefore the only second-order line I measured, and my values are given below in Table I. together with those obtained by other observers. This line belongs to group *a* in the classification of Gale and Adams.

TABLE I

20,000 Grating	15,000 Grating	Kayser	Goos	St. John
5429.7005	5429.7001	5429.701	5429.700	5429.702

As there was no way of telling which of the third-order lines were the best for this work and the least affected by varying experimental conditions, I measured all those which were sharp on both sets of plates. The reading for any one line on one plate did not differ from the mean value of the four plates used by more than 0.003 mm. The greatest difference found between the values of any third-order line calculated by the two gratings amounted to 0.005 A, a difference which occurs only once in the 47 lines measured. The sum of the differences amounts to +0.0620 and -0.0237 A, the average difference being only $+\frac{0.0383}{47}=+0.0008$ A and the average variation $=\pm\frac{0.0857}{47}=\pm 0.0018$ A. So the difference between two lines measured on different gratings by the coincidence method is of the same order of magnitude as the

difference between values of the same line measured on different plates. This proves that the two gratings employed were perfect enough to be used in wave-length measurement by the coincidence method. The tabulated results are given in Table II.

TABLE II

20,000 Grating	15,000 Grating	Difference	Kayser	Rowland
3513 8220	.8226	+0 0006	821	97
3521.2669	2674	+ 0005	26	41
3524 0803	0809	+ 0006	07	22
3524.2463	.2469	+ .0006	24	39
3526 0470	0453	− .0017	03	18
3526.1705	.1721	+ 0016	.23	38
3526.3843	3839	− .0004	38	.53
3526 6785	6806	+ .0021	66	81
3527.7999	.8033	+ 0034	.78	93
3529 8235	.8265	+ .0030	.80	95
3533 0115	.0103	− .0012	00	15
3533.2028	.2057	+ .0029	19	.34
3536.5597	5632	+ .0035	55	70
3540 1314	1349	+ .0035	.12	27
3541.0925	0939	+ 0014	.09	24
3542 0819	.0787	− .0032	08	.23
3545.6437	6487	+ 0050	62	77
3553 7459	.7432	− 0027	72	.87
3554 1250	1238	− 0012	11	26
3554 9299	.9265	− 0034	3554 92	3555 07
3556.8822	.8831	+ 0009	3556.880	3557 03
3558.5213	5211	− 0002	52	67
3565 3862	.3880	+ .0018	38	53
3570 1049	.1045	− .0004	.12	27
3571 9985	.9968	− 0017	3572.00	3572 15
3575.3745	3750	+ 0005	35	50
3581 2017	.2024	+ 0007	.20	35
3582 2069	.2061	− .0008	.19	34
3584 6657	.6672	+ .0015	.65	80
3584 9628	9636	+ 0008	3584 96	3585 11
3585 3255	.3274	+ 0019	32	47
3585 7118	7142	+ 0024	71	86
3586 1172	.1165	− 0007	.11	26
3586 9921	9918	− 0003	3586 97	3587 12
3589 1115	1103	− 0012	10	25
3594 6419	.6397	− 0022	62	77
3603.2095	2106	+ 0011	20	35
3605 4595	.4615	+ 0020	47	62
3606 6846	6865	+ .0019	682	83
3608.8645	8621	− 0024	3608 85	3609 01
3610 1584	1597	+ 0013	16	31
3612 0805	0851	+ .0046	08	23
3617.7899	.7939	+ 0040	.78	93
3618 3899	3923	+ 0024	38	53
3618.7723	7730	+ 0007	77	92
3621 4634	4655	+ .0021	.46	.61
3622 0061	0088	+ .0027	00	15

SECOND PART—THEORY

In this part of my work I have tried to furnish a real proof of the accuracy with which the fundamental law of the grating holds. The grating used in this part was one of Anderson's with 7,500 lines to the inch and a radius of 21 feet. For my fundamental line I chose λ 5232.957, one of the international secondary standards (group *d*) and proceeded as follows to determine the scale of my plate, knowing

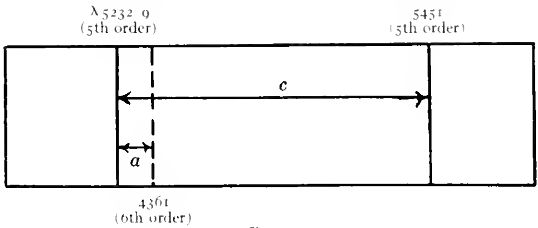


FIG. 2

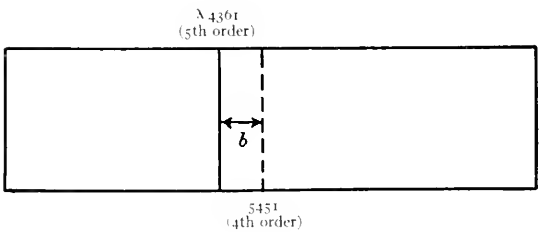


FIG. 3

only this one wave-length and the law of gratings. I first photographed this line in the 5th order. A plate of this type is illustrated in Fig. 2. On Fig. 2 we know the wave-length of line λ 5232.957 in the 5th order. Near it is a line in the 6th order which we shall call 4361, although its wave-length is not known. The distance between λ 5232.957 (5th) and 4361 (6th) is *a* mm. Then, if x mm = 1 A in the 5th order,

$$\text{the distance } a = \frac{a}{x}\ \text{A}.$$

The wave-length of the line 4361 will then be $\lambda_1 = (5232.957 + \frac{a}{x})$ A in terms of the 5th order, which gives

$$\lambda_1 = \left[5232.957 + \frac{a}{x}\right]\frac{5}{6} \text{ A}$$

in the 6th order.

The grating is now shifted so as to bring the line 4361 in the 5th order on the middle of the plate. A plate of this type is illustrated in Fig. 3. At a distance b mm from 4361 in the 5th order there will be a line in the 4th order which we shall call 5451. This line also occurs on Fig. 2 in the 5th order at a distance c mm from λ 5232.957, the standard line:

$$b = \frac{b}{x} \text{ A}$$

in the 5th order

$$\therefore 5451 = \lambda_2 = \left[5232.957 + \frac{a}{x}\right]\frac{5}{6} + \frac{b}{x} \text{ A}$$

in the 5th order

$$\lambda_2 = \left(\left[5232.957 + \frac{a}{x}\right]\frac{5}{6} + \frac{b}{x}\right)\frac{5}{4} \text{ A}$$

in the 4th order.

Also from Fig. 2,

$$\lambda_2 = 5451 = \left(5232.957 + \frac{c}{x}\right) \text{A}$$

in the 5th order

$$\therefore \left(5232.957 + \frac{c}{x}\right) = \left(\left[5232.957 + \frac{a}{x}\right]\frac{5}{6} + \frac{b}{x}\right)\frac{5}{4}$$

$$\therefore 5232.957x = 24c - 25a - 30b$$

$\therefore x =$ no. of mm per A in the 5th order

$$x = \frac{24c - 25a - 30b}{5232.957}.$$

In practice I used four lines in the neighborhood of λ 5232.9 instead of the one referred to above as 4361. Then for the line given as 5451 I used one of the international standards, λ 5455.614 (group a), so that my scale, obtained entirely by the coincidence method, might more easily be compared with the scale obtained directly from the two standards λ 5232.957 and λ 5455.614. In order to identify the lines it was of course necessary to take plates

with color-filters to cut out all except certain orders. I then took four plates of Fig. 2 and four of Fig. 3. The values for b for each of the four lines used in place of 4361 were then averaged for the four plates of Fig. 3, each line being read nine times. These average values for b were then used with the different values for a and c obtained from each of the four plates of Fig. 2. The values for a and c and the average values of b are given in Table III. The four lines used for 4361 are denoted by the symbols I, II, III, IV. On two plates there were only three lines good enough for accurate measurement.

The values for x, the number of mm per angstrom in the 5th order are given in Table IV. The subscripts I, II, III, IV indicate which of the four lines was used in obtaining x and the mean value of x obtained by the method of coincidences is then compared with x (standard), the value found by assuming the two international secondary standards λ 5232.957 and λ 5455.614 in the usual way.

The greatest error for the scale of the plates determined by this method amounts to one part in a hundred thousand and the mean error to three parts in a million. It is easier, however, to judge the method by comparing the values for the standard line λ 5455.614 as found by the coincidence method with the accepted value as found by independent interferometer measurements. This comparison is made in Table V.

We therefore see that the maximum error introduced by calculating one standard line from another by the method of coincidences is 0.0029 A and the mean error amounts to only 0.0007 A.

In calculating the line λ 5455.6 from the original line λ 5232.9 there has of course been no calibration-correction made. But even in starting with a set of the international standards it is necessary to use two of them without calibration-correction of any kind in order to get the scale of the plate. In order to carry out the scheme of building up a whole system of wave-lengths from one line it would be necessary, after establishing a second line in the manner described above, to determine the intermediate standard lines by coincidences in the same way, starting from each of the two fundamental standards as base line and also from each new

standard as it is determined. In this way, by averaging the results, each line will be established with sufficient accuracy to be used as a standard line and from these standards a calibration-curve may

TABLE III

	I	II	III	IV	
Plate I(*a*)					
a	−9 2226	+7 7708	+17 3393	+26 0642	
b	+11 2161	−2.9545	−10 0230	−18 1077	
c					212 5540
Plate I(*b*)					
a	−9 2228	+7.769	+17 342	+26 0616	
b	+11.2161	−2 9545	−10 0230	−18 1077	
c					212 5507
Plate I(*c*)					
a		+7 7761	+17 3419	+26 067	
b		−2.9545	−10 0230	−18 1077	
c					212 550
Plate I(*d*)					
a	−9 2215		+17 3412	+26 066	
b	+11 2161		−10 0230	−18 1077	
c					212 5534

TABLE IV

Plate	$X_{(I)}$	$X_{(II)}$	$X_{(III)}$	$X_{(IV)}$	X_{mean}	$X_{standard}$	Difference
I(*a*)	0 054607	0 954656	0 954631	0 954649	0 954636	0 954628	−0 000008
I(*b*)	95459	954614	95460	954614	954604	954610	+ .000006
I(*c*)		.954598	95461	954614	954607	954610	+ 000003
I(*d*)	0 954595		0 954617	0 954618	0 95461	0 95462	+0 00001

TABLE V

Plate	$C_{(mm)}$	$\frac{c}{x_{mean}} = C_A$	5232 957+C_A St Line Calc.	Standard I A	Difference from St 5455 614
I*a*—	212.5546	222 6553	5455 6123	5455 614	−0 0017
I*b*—	212 5507	222 6585	5455 6155	5455 614	+ 0015
I*c*—	212.550	222 65707	5455 61407	5455 614	+ .00007
I*d*—	212.5534	222 6509	5455 6169	5455 614	+0 0029

be drawn for the plate. Then the other lines of all the superimposed spectra in the region can be determined. These lines will give starting-points for similar work in other parts of the spectrum. Of course as the steps involved in the work carry one farther and

farther from the two original lines, errors might accumulate to a very appreciable extent, but at the same time every new line that is established will give another reference line from which the next lines may be determined. So that by tying up each new line by coincidences with many other lines, some of greater, some of less, wave-length, the errors introduced will offset each other.

My original intention in taking up this work was to build up, in some part at least, a system such as this. To build up an entire system, determining each line with a sufficient number of coincidences to insure the desired accuracy, would be a stupendous task. It would also be an unnecessary one, because the values obtained for the standard lines could not obtain an accuracy greater than that of the international system. It is therefore simpler to use these standards to begin with. I have shown, however, that, having these standards throughout part of the spectrum, it is possible to investigate the region into which they do not extend by means of the coincidence method with an accuracy as great as has been obtained as yet in wave-length determination by direct interpolation. Before a grating can be used for this work it must be tested by the method given in Part II of this paper in order to see how closely it obeys the law of gratings. The results obtained with my three gratings have been so good that it is perhaps not too much to say that a grating which is unfit for coincidence work is the exception and not the rule.

In conclusion I wish to thank Dr. J. S. Ames and Dr. J. A. Anderson, under whose direction this research has been conducted, for the help and advice they have given me and the interest they have taken in my work.

Johns Hopkins University
June 1913

THE INFRA-RED ABSORPTION SPECTRA OF SOME ALKALOIDS

By B. J. SPENCE

During the last two decades an extended study of the absorption of many chemical compounds has been made by various investigators. The field has been not only that covered by the visible spectrum, but the ultra-violet and the infra-red regions have been subjected to close scrutiny. The attempt has been chiefly to relate absorption spectra to molecular or atomic grouping and also to relate a variation in the absorption spectra to a variation in molecular or atomic groupings. The attempts have not been as fruitful and the results as clearly defined as one would perhaps anticipate. The infra-red region possesses an advantage which the ultra-violet and visible regions do not, namely, that of extent, consequently the infra-red is more likely to yield data for any given substance from which inferences may be drawn.

Organic compounds have been the chief source of study owing to their variety and in many instances to their apparently simple molecular groupings. Coblentz (*Carnegie Institution Publications*, No. 35) has made a systematic study of a large group of substances and his results taken in conjunction with those of other investigators have led to many interesting conclusions, but the question as to the relation of molecular groupings and absorption spectra has been by no means clearly established. The field of investigation covered has not been complete, in fact only a small portion of the whole has been worked over. Perhaps with additional data, more concise statements may be made as to the relationship of absorption and molecular constitution.

The ultra-violet absorption spectra of many of the so-called alkaloids were studied by Hartley (*British Association Report*, 1901). His work in the ultra-violet region was both comprehensive and searching, and the constitution of many compounds was established by this method where chemical methods have failed. He not only located the position of the absorption bands but

measured the absorption limits at decreasing concentration until complete transmission was obtained. The infra-red absorption cannot be studied in this manner owing to the lack of diathermanous solvents for the alkaloids, yet it possesses the advantage of extent, namely, from $0.5\,\mu$ to $15\,\mu$, the working limits of the rock-salt prism.

The study of the alkaloids about to be described was undertaken chiefly because the alkaloids constitute a large group of organic substances showing alkali reaction and basic properties similar to ammonia. They contain carbon, hydrogen, and nitrogen and most of them contain oxygen. Those studies were, generally speaking, derivatives of pyridine, pyrrolidine, quinoline, and piperidine and contain the pyridine ring. Behaving like ammonia, they combine directly with acids to form salts without the elimination of water and hydrogen.

The alkaloids studied were divided into four groups (Richter, *Organic Chemistry*, 2), namely, the pyridine, quinoline, tropine, and isoquinoline groups. They were particularly chosen owing to the fact that like the benzene derivatives they have much in common in their deportment. The number in each group studied was limited largely by the fact that many of them could not be obtained or put into such a form that transmission was not accompanied by scattering or diffusion.

APPARATUS

The investigation was carried on by means of a spectrometer of special design, constructed by the university mechanician. The chief feature of the instrument was that it did not possess the customary divided circle. The divided circle was replaced by a carefully turned disk and rotated by an invar strap attached to one point of its periphery. One end of the strap was fastened to the nut of a carefully ground screw, which when turned drew the nut forward or backward, thus rotating the disk. The screw had fastened to one end a disk graduated in degrees. To the other end of the strap was fastened a weight which served to take up at all times the slack of the invar strap. This combination was constructed so that a rotation of the graduated disk through an angle of one degree rotated the disk replacing the divided circle through an arc of five seconds.

The rock-salt prism and mirror were placed on the disk replacing the divided circle and adjusted according to the method described by Wadsworth (*Philosophical Magazine*, 38, 1904). The rock-salt prism was one with a 60° angle and faces 40×50 mm. The concave mirrors were of 50 cm focal length and of 5 cm aperture.

The recording instruments were a thermopile and galvanometer. The galvanometer was the four-coil Thomson astatic system type similar to one described previously (*Physical Review*, 28, 1909) except that, instead of being shielded by the heavy soft iron cylinders, this one had its coils imbedded in a soft iron core which effectually performed the same function as the cylinders.

The thermopile used was similar to one described by the author (*Physical Review*, 1, 1910) but was somewhat better adapted to precision work. Instead of being bound together by ivory strips, it was held together by pieces of paper shellacked to the sides of the thermopile. This gave the instrument sufficient rigidity and decreased the thermal capacity to a large extent. The thermopile-galvanometer-spectrometer combination was of sufficient sensibility to give a galvanometer deflection of 35 mm in the region of 12 μ with the galvanometer-scale at a distance of a meter from the galvanometer. The source of light was a Nernst glower burning at normal power and the width of the spectrometer slit was 0.7 mm. At this sensibility the galvanometer-thermopile combination assumed a steady deflection almost immediately and returned to its initial zero position almost immediately.

The instrument was calibrated by means of the known dispersion constants of rock salt and the angle of the prism. As a means of verification of the calibration, the absorption spectra of films of selenite, mica, and collodion were obtained with the instrument and these spectra compared with the spectra of the same substances obtained by others. The two calibrations were in as perfect accord as could be determined.

EXPERIMENTAL

In order to proceed with the study of the absorption spectra it was necessary to get the substances in some clear form before the spectrometer slit. This was the chief source of difficulty. When the substances were in liquid form, it was an easy matter to include the liquid between two rock-salt plates separated by tin foil. When

the substances were in the solid state and possessed melting-points such that melting took place without decomposition, they could be melted between two rock-salt plates, and a thin film obtained by this means. Many substances crystallized to such an extent upon solidification that the film scattered the light so greatly that this means of preparing the film was prohibited. Some substances could be dissolved in ether and this solution spread over the rock salt a sufficient number of times to obtain a film of minute crystals in which scattering was not too great.

The rock-salt plates were fastened to a carrier so that the film could be drawn before the slit at will. The absorption of the rock salt was eliminated by drawing before the slit the plate containing the film and then replacing this plate and its film by another plate or plates equivalent in thickness to the salt plate containing the film. The ratio of the two galvanometer deflections obtained by these operations gave the transmission of the film under consideration for a particular wave-length.

Table I states the groups of alkaloids studied, the alkaloids studied under each group, and the position of the absorption bands found for each. With the pyridine group were studied the non-alkaloidal substances pyridine and α-picoline.[1] These were studied to learn whether any relation existed between them and the alkaloids of that group. More particularly pyridine was studied because the alkaloids of this group are derivatives of it. Then α-picoline was studied owing to its relation to pyridine. It is known as a homologous pyridine.

Under the tropine group are included the substances benzoic acid and cocaine hydrochloride. Benzoic acid was studied because cocaine yields, when treated with hydrochloric acid, ecgonine and benzoic acid as two of its decomposition products; the point being that cocaine hydrochloride might show bands characteristic of these. This would undoubtedly be true should cocaine hydrochloride proceed so far in its decomposition that these two products were present. It is also to be mentioned here that belladonna is composed of two closely related alkaloids, hyocine and hyoscyamine, both belonging to this group.

[1] These have also been recorded by Coblentz.

TABLE I

PYRIDINE GROUP

PYRIDINE*...	C_5H_5N......	3.25, 5.22, 6.35, 6.05, 8.32, 8.80, 9.50, 9.77, 10.15.
PIPERIDINE .	$C_5H_{10}NH$......	3.45, 6.10, 6.93, 7.60, 7.07, 8.75, 9.10, 9.65, 10.70, 10.90, 11.60.
Coniine.	$C_8H_{17}N$........	3.45, 6.15, 6.40, 7.00, 7.45, 9.05, 9.65, 10.25, 10.55, 10.85, 11.30.
Nicotine.......	$C_{10}H_{14}N_2$......	3.63, 6.42, 7.05, 7.62, 8.45, 9.18, 9.81, 11.15.
Pilocarpine.. .	$C_{14}H_{16}N_2O_2$....	3.15, 5.75, 6.88, 7.37, 8.20, 8.60, 9.10, 9.80, 10.20, 10.70.
Piperine.......	$C_{17}H_{19}NO_2$......	3.42, 6.25, 6.00, 8.03, 8.98, 9.80, 10.05, 10.55, 10.80, 11.25, 11.65.
α-Picoline*...	C_6H_7N..	3.40, 5.25, 6.32, 6.92, 7.85, 8.80, 9.20, 9.70, 10.10, 10.35.

TROPINE GROUP

Atropine....	$C_{17}H_{23}NO_3$	3.32, 5.82, 6.90, 7.40, 8.35, 8.60, 9.45, 9.72, 10.20, 10.70, 10.85, 11.33.
Cocaine........	$C_{17}H_{21}NO_4$ ..	3.43, 5.85, 6.90, 7.95, 9.10, 9.75, 10.12, 10.72.
Cocaine HCl....	$C_{16}H_{21}NO_4(HCl)_2$	3.25, 5.90, 6.95, 7.95, 9.05, 9.95.
β-Eucaine......	$C_{17}H_{21}NO_2$......	3.80, 5.97, 6.42, 6.95, 7.32, 7.95, 9.12, 9.40, 9.88, 10.18, 10.65, 11.12, 11.20.
Ecgonine HCl .	$C_9H_{15}NO_3HCl$	3.60, 5.95, 7.05, 7.75, 8.35, 9.00, 9.70, 11.20.
Homatropine....	$C_{16}H_{21}NO_3$. ..	3.40, 5.75, 6.85, 8.35, 9.12, 9.70, 10.33, 10.80, 11.50.
Belladonna....		3.10, 6.00, 7.00, 8.40, 9.57, 9.80, 10.30, 10.85, 11.45.
Benzoic Acid...	$C_7H_6O_2$...	3.57, 5.92, 7.03, 7.75, 8.58, 8.94, 9.45, 9.80, 10.72.

QUINOLINE GROUP

Quinoline*.... ..	C_9H_7N. . .	3.25, 5.25, 6.35, 6.80, 7.62, 8.15, 8.90, 9.77, 10.55.
Quinine........	$C_{20}H_{29}N_2O_2$.....	3.25, 6.15, 6.82, 7.48, 8.10, 9.15, 9.72, 10.08, 10.35, 10.75, 11.00, 11.35, 11.70.
Quinine Sulphate	$(C_{20}H_{24}N_2O_2)_2$.. H_2SO_4	3.63, 6.10, 6.88, 7.45, 8.15, 8.88, 9.75, 10.90, 11.65.
Brucine*.......	$C_{23}H_{26}N_2O_4$....	3.40, 4.85, 6.03, 6.97, 7.87, 8.32, 9.05, 9.62, 9.85, 10.85, 11.30, 11.70.
Chinconidine. .	$C_{19}H_{22}N_2O$ . ..	3.50, 6.30, 6.88, 7.50, 8.10, 9.15, 9.95, 11.15.
Quinidine	.. ..	3.32, 6.25, 6.88, 7.57, 8.10, 9.05, 9.80, 10.05, 11.07.

* Coblentz, *Pub. Carn. Institution*, No. 35.

TABLE I—*Continued*

ISOQUINOLINE GROUP

Codeine.........$C_{18}H_{21}NO_3$......3 45, 6.35, 6.05, 7.33, 8.00, 9.10, 9.65, 10 37, 10.65, 10 95, 11.45.

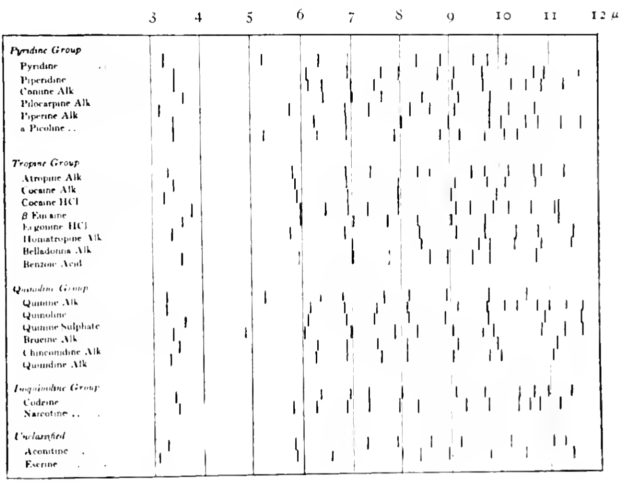
Narcotine.......$C_{22}H_{23}NO_7$......3.50, 5.77, 6.25, 6.88, 7.32, 7.95, 8.32, 9.32, 9.75, 10.35, 10.77, 11.22.

UNCLASSIFIED

Aconite.........$C_{13}H_{45}NO_{12}$.....3.25, 5.80, 7.25, 7.87, 8.55, 9.18, 10.18, 11.05, 11.30.

Eserine.........$C_{15}H_2N_3O_2$......3.10, 5.85, 6.55, 8.00, 8.35, 8.92, 9 80, 90 50, 11.45.

TABLE II



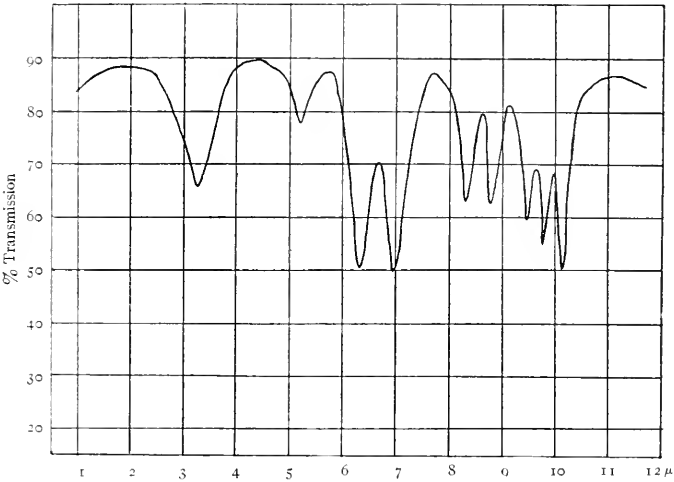


FIG. 1.—Pyridine

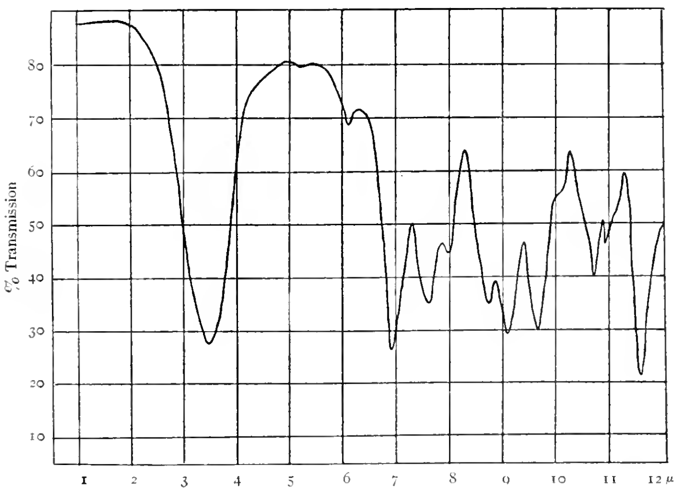


FIG. 2.—Piperdine

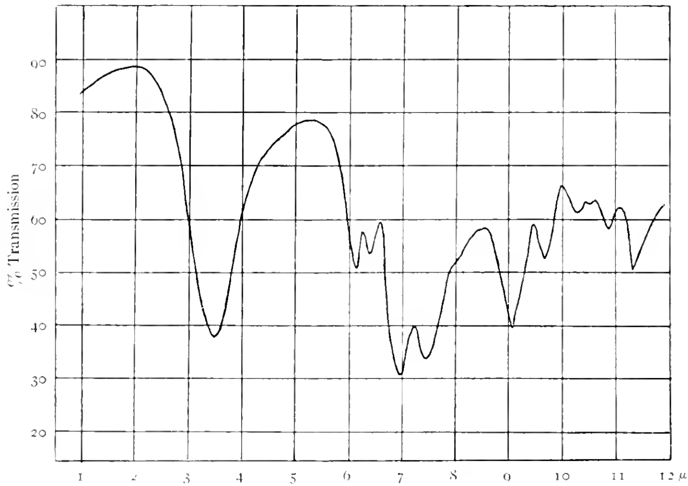


FIG. 3.—Coriine

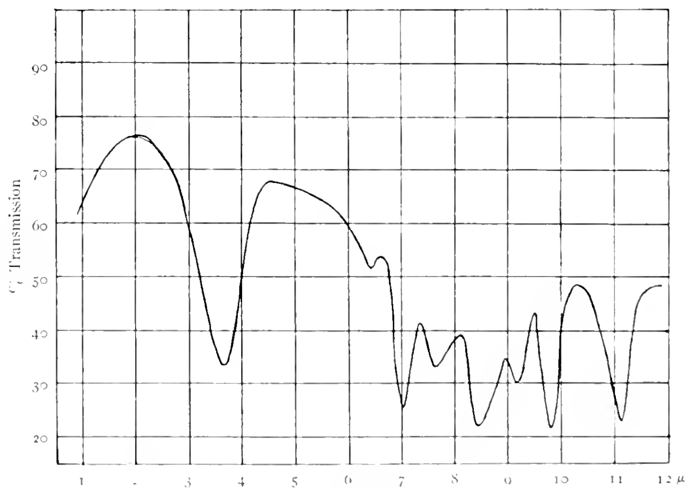


FIG. 4.—Nicotine

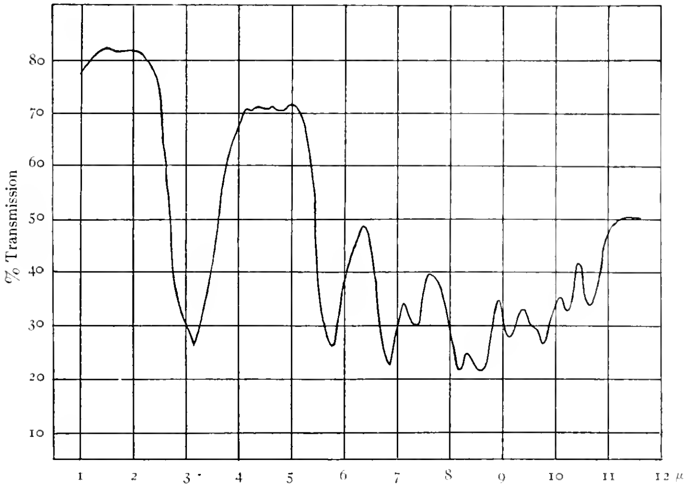


FIG. 5.—Pelocarpine

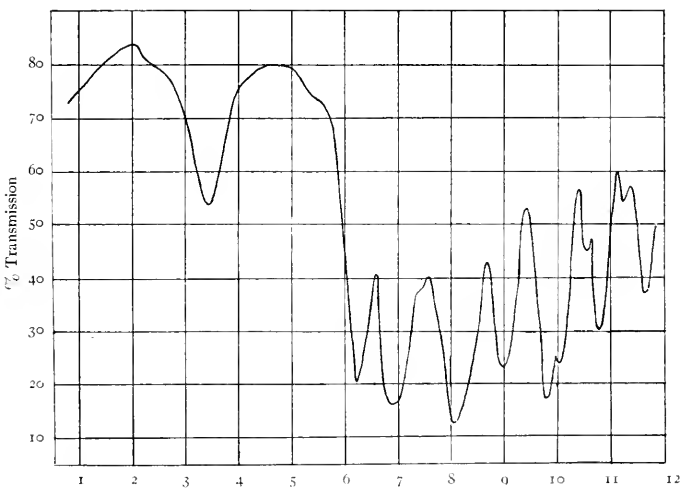


FIG. 6.—Piperine

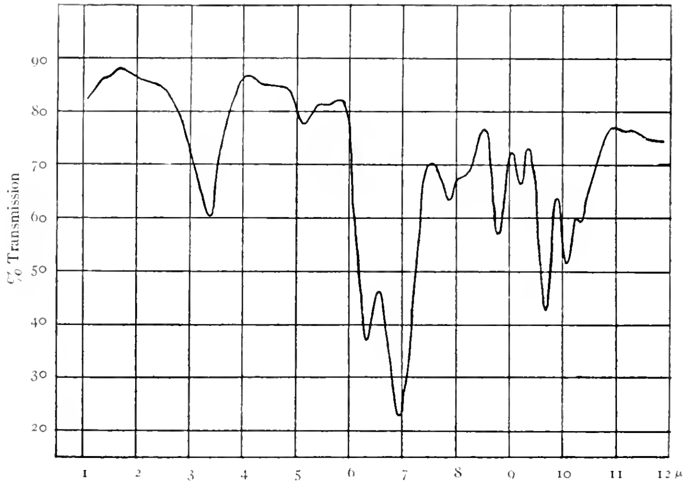


FIG. 7.—α-Picoline

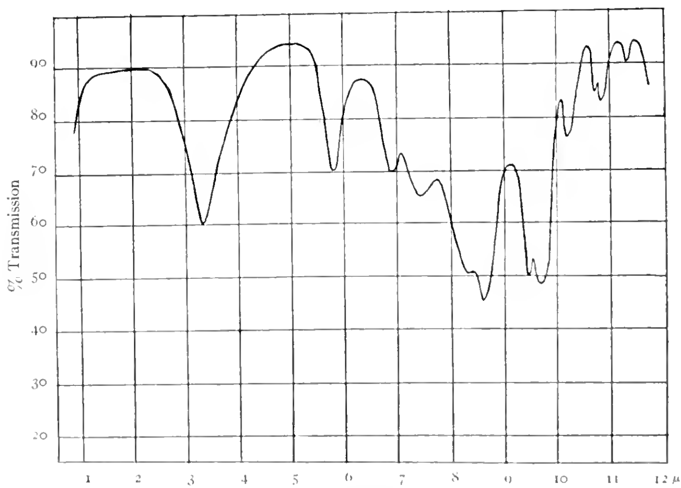


FIG. 8.—Atropine

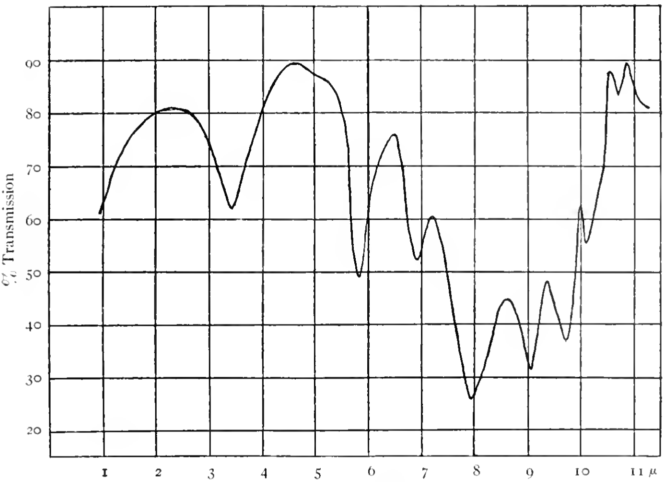


FIG. 9.—Cocaine

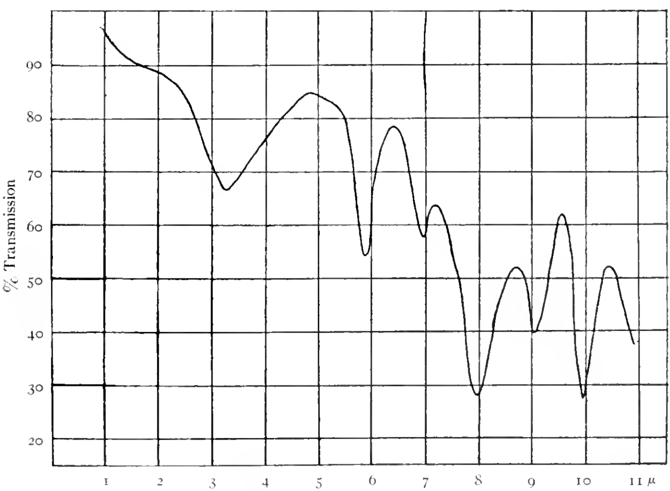


FIG. 10.—Cocaine hydrochloride

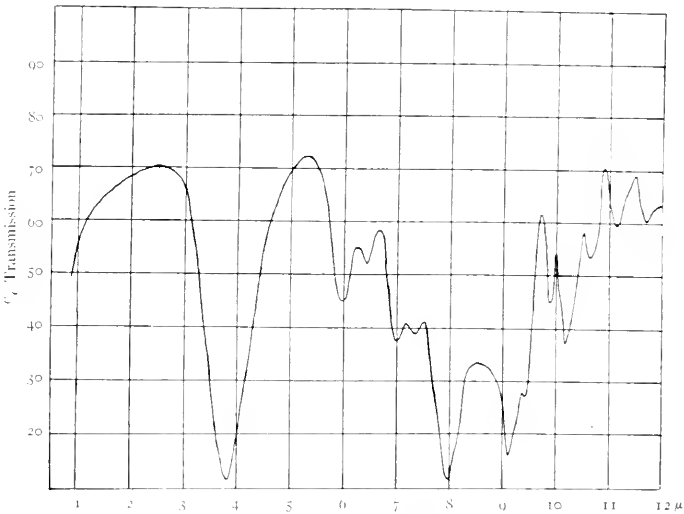


FIG. 11.—Beta eucaine

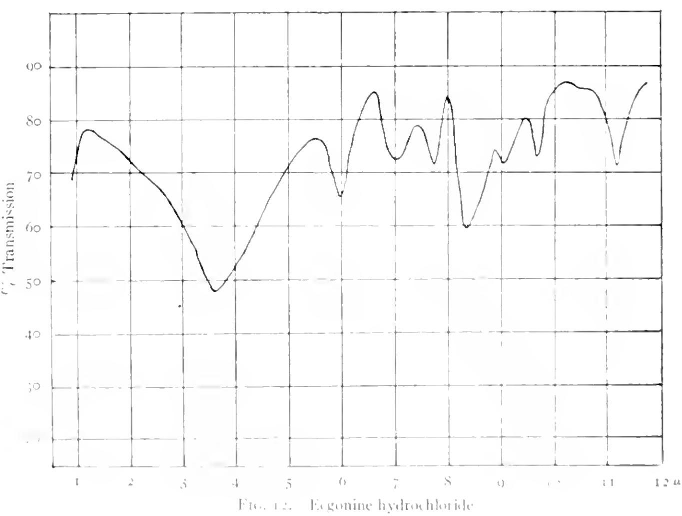


FIG. 12.—Ecgonine hydrochloride

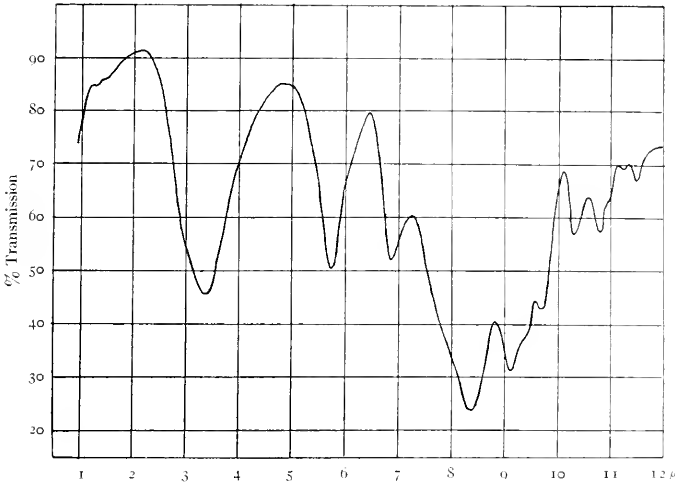


FIG. 13.—Homatropine

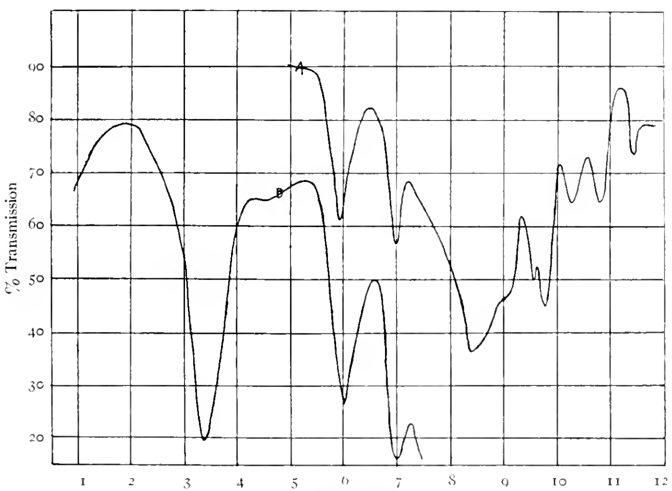


FIG. 14.—Belladonna

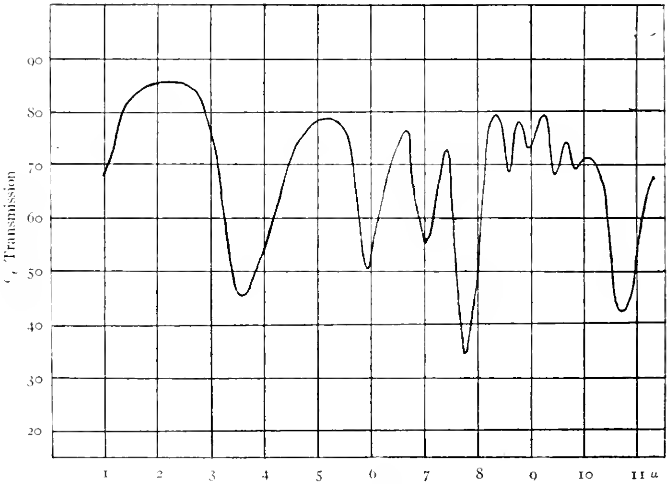


FIG. 15.—Benzoic acid

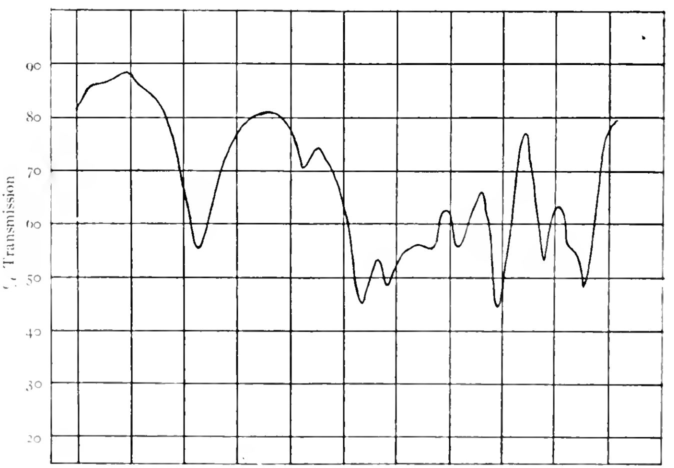


FIG. 16.—Quinoline

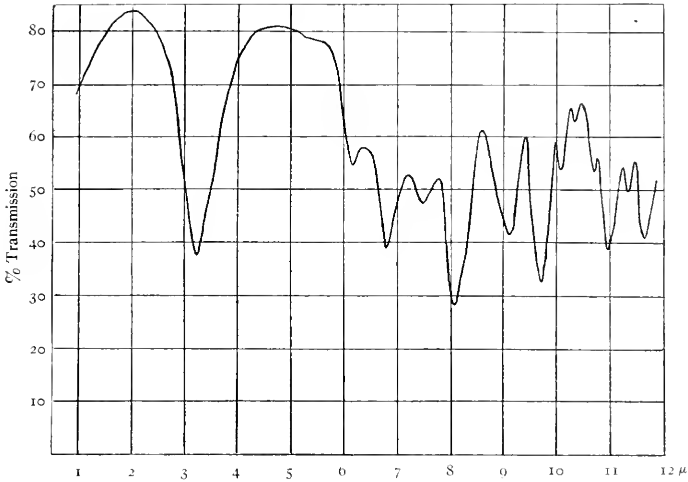


FIG. 17.—Quinine

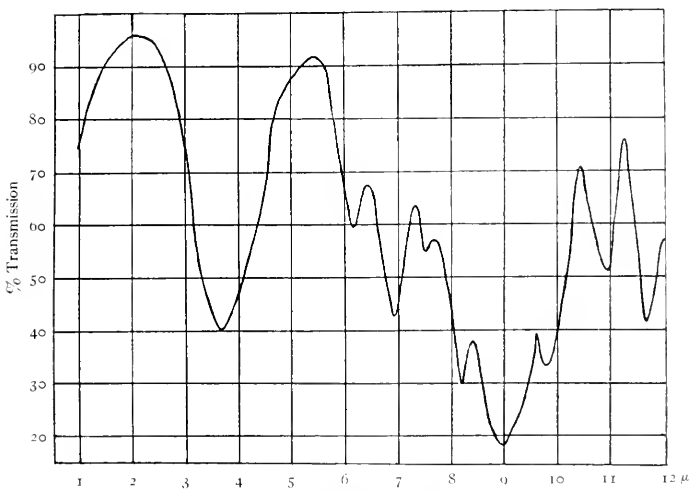


FIG. 18.—Quinine sulphate

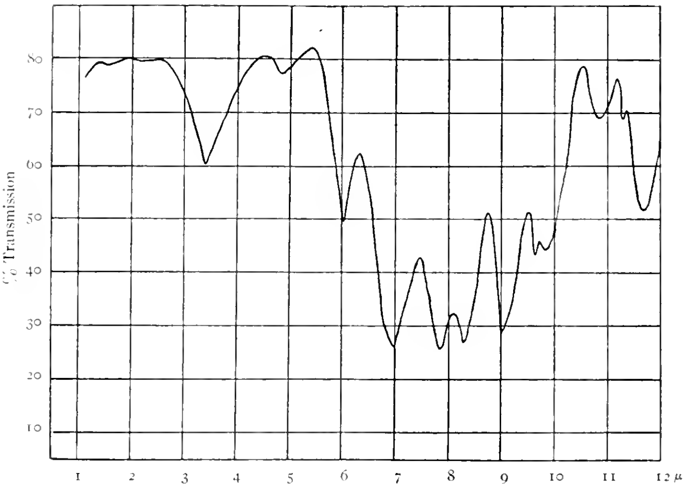


FIG. 19.—Brucine

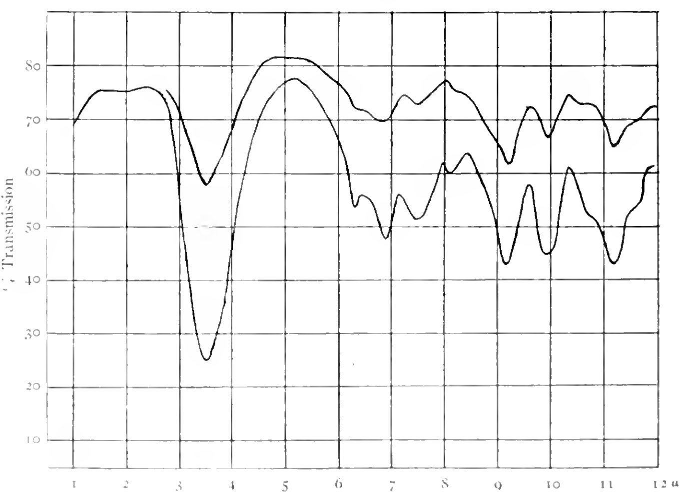


FIG. 20.—Chinconidine

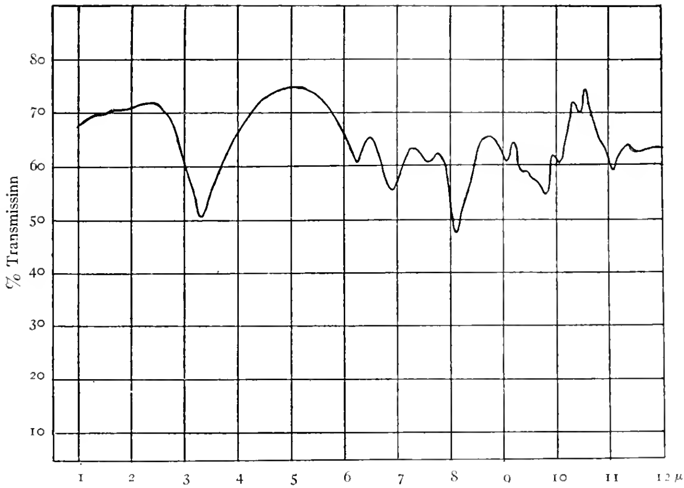


FIG. 21.—Quinidine

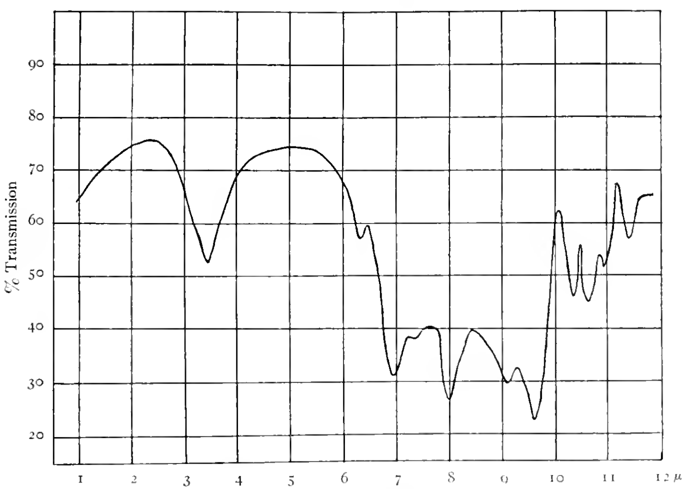


FIG. 22.—Codeine

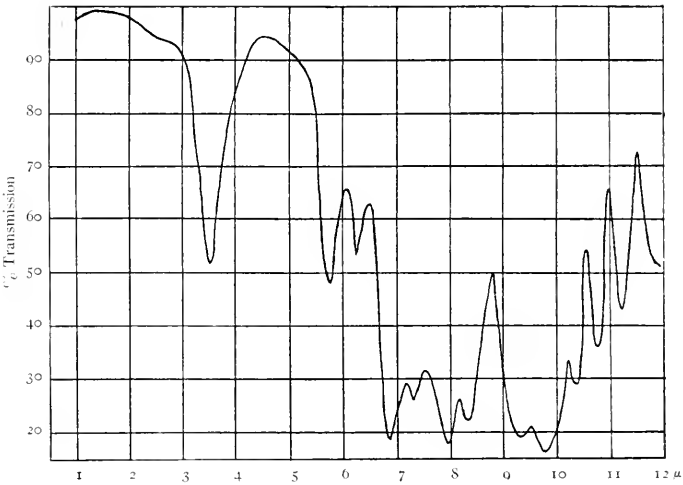


FIG. 23.—Narcotine

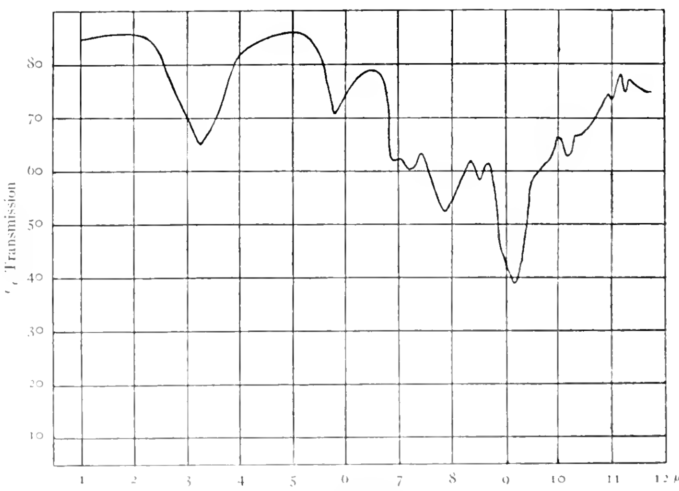


FIG. 24.—Aconitine

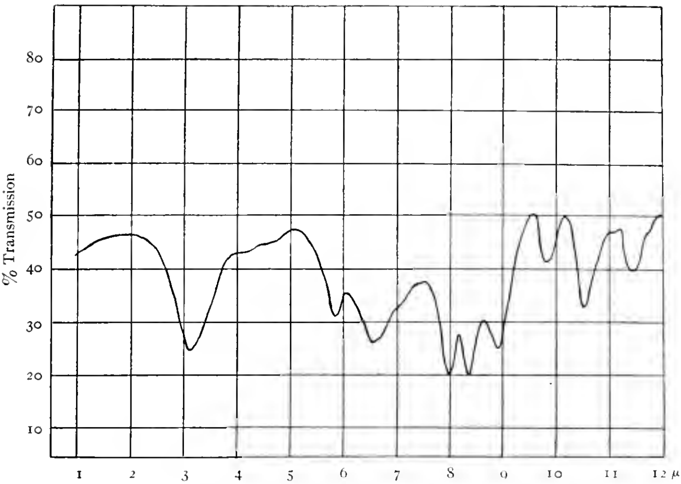


FIG. 25.—Eserine

Under the quinoline group was studied quinine sulphate to learn what relation might exist, if any, between the alkaloid and its salt. Quinine and its salts, which are numerous, would have formed an interesting group to study, but unfortunately most of the salts are of such a nature that it was impossible to put them in any transparent form before the slit of the spectrometer.

Of the isoquinoline group only two alkaloids were studied, namely codeine and narcotine. In general, the melting-points of this group are too high for them to melt without decomposition. A number of well known alkaloids such as caffeine and theine constitute this group.

Two alkaloids, aconitine and eserine of rather hazy grouping were studied. Little is known, comparatively speaking, of the chemistry of either of these alkaloids.

RESULTS

A reference to the accompanying curves will show the various absorption curves for the alkaloids studied. Table II was arranged

for the convenience of comparing the absorption spectra. It states the group, alkaloid, and the location of the absorption bands. A casual glance shows one that there are no identical spectra. Many of them are similar in appearance and a great many have bands in common. In comparing these spectra with spectra of other compounds obtained by other investigators, for example the work of Coblentz (*loc. cit.*), which contains over one hundred and fifty absorption spectra, one finds nothing identical in the way of spectra.

In general the spectra of the alkaloids show a large band between the limits 3.10 μ and 3.60 μ. In the large majority of cases this band lies between the limits 3.25 μ and 3.50 μ. In very few instances a band lies at the wave-length 3.43 μ which Coblentz attributes to the CH_2 or the CH_3 groups which in all of these alkaloids are quite prominent. The band at 3.43 μ attributed to the foregoing groupings might easily have been masked by the presence of groups which influence the absorption in that region.

All of the spectra show a band between the limits 6.80 μ and 7.05 μ. The absorption in this region is in all probability due to the CH_2 or the CH_3 groups as shown by Coblentz, who places the band at 6.86 μ. One finds again a characteristic absorption in the region of 9.75 μ, the limits being 9.65 μ and 9.85 μ. A band in this position is attributed to the C_6H_6 group, but it is difficult to see how this band may be attributed to the C_6H_6 group when this group is not shown chemically to be present.

Studying the groups separately one finds that they possess somewhat more individuality, yet nothing very striking is to be observed. The pyridine group possesses a large band in the region limited by the wave-lengths 3.15 μ and 3.60 μ. Another region lies between the limits 6.10 μ and 6.40 μ with pilocarpine as the exception. Again one finds a region having the wider limits 6.85 μ and 7.05 μ and also a narrower region of limits 9.65 μ and 9.85 μ.

The atropine group possesses a band in the region comprising the wave-lengths 3.10 μ and 3.80 μ, a narrower region comprised between the wave-lengths 5.75 μ and 6.10 μ. Two more regions having the limits 6.85 μ and 7.05 μ and 9.70 μ and 9.90 μ are present.

Homatropine appears to behave anomalously. Benzoic acid studied in connection with this group possesses the bands which are characteristic of this group, namely, the bands at 5.90 μ, 7.05 μ, and 9.80 μ. Benzoic acid is a derivative of benzene but shows only the band at 9.80 μ attributed to benzene.

The quinoline group possesses a number of regions where absorption is common. These regions have the following values: 3.25 μ to 3.50 μ, 6.00 μ to 6.35 μ, 6.80 μ to 6.95 μ, 7.45 μ to 7.65 μ, 8.10 μ to 8.15 μ, 8.90 μ to 9.15 μ, and finally 9.75 μ to 9.95 μ. Brucine forms an exception to the above statement in two instances. Quinine sulphate has only three bands in common with quinine from which it is derived.

Of the isoquinoline group little may be said, for not many members of the group have been studied. Of the two unclassified alkaloids it is unsafe to make predictions as to grouping. They may be members of groups not studied.

In a few instances water of crystallization was present, but in no instance was the band attributed to water of crystallization at 3 μ present. Undoubtedly the presence of the band was masked by the presence of other free periods whose frequencies were not far removed from that at 3 μ.

On the whole, little information can be obtained from the groups studied concerning the relation of infra-red absorption to molecular grouping. Our knowledge of the mechanism of absorption is very meager and it may be that we are working in the dark with reference to the relation of absorption to molecular grouping. However, until the problem is put upon a more firm theoretical basis, our only resource is the statistical method which ultimately will aid the solution of the problem of absorption.

University of North Dakota
October 1913

MINOR CONTRIBUTIONS AND NOTES

SPECTROSCOPIC BINARIES UNDER INVESTIGATION AT DIFFERENT INSTITUTIONS

The following letter was recently sent out to the principal institutions at which spectrographic observations are now being made.

YERKES OBSERVATORY
WILLIAMS BAY, WISCONSIN
September 2, 1913

DEAR SIR:

It seems desirable to collect and print in the *Astrophysical Journal* statements from the different observatories as to spectrographic binaries which are under special observation at the several institutions with a view to a determination of the orbits. This was done in 1908 (Vol. 27, p. 161) and presumably prevented unnecessary duplication of effort both in securing spectrograms and in measuring them. It also serves a purpose in indicating the present state of research in this direction. Will you therefore be kind enough to communicate to me at your earliest convenience such a statement, in form for publication, regarding the work at the ——— observatory?

Very truly yours,
EDWIN B. FROST

The replies that have been received follow, translated when not in English, and in some cases somewhat abridged. Some of the letters contain information not strictly pertaining to spectroscopic binaries, but of interest to those engaged in spectrographic work.

ALLEGHENY OBSERVATORY OF THE
UNIVERSITY OF PITTSBURGH
September 8, 1913

Here is the list of spectroscopic binaries that we are at present observing, with the approximate Right Ascensions:

1	*Persei*	$1^h 40^m$	*50*	*d Serpentis*	$18^h 22^m$
RZ	*Cassiopeiae*	2 40	*43*	φ *Draconis*	18 22
B.D.	$-1^\circ 943$	5 20	*11*	δ^1 *Lyrae*	18 50
19	*Lyncis*	7 16	*1*	*Vulpeculae*	19 12
18	*Ursae Majoris*	9 10	*44*	σ *Aquilae*	19 35
55	*Ursae Majoris*	11 14		*Boss 5070*	19 47
95	*Leonis*	11 51		*Boss 5113*	19 54
12	*Comae*	12 17	*30*	*Cephei*	22 30
7	δ *Scorpii*	15 55	*12*	*Lacertae*	22 38
10	λ *Ophiuchi*	16 20	*10*	*Lacertae*	22 52
108	*Herculis*	18 18			

In addition, we have finished observing the following stars, but the results have not yet been published:

4	*g Persei*	1^h55^m	*14*	*ι Coronae*	15^h57^m
26	*β Persei*	3 01		*U Ophiuchi*	17 12
45	*ε Persei*	3 51	*111*	*Herculis*	18 43
35	*λ Tauri*	3 55	*18*	*Aquilae*	19 02
15	*S Monocerotis*	6 36	*30*	*δ Aquilae*	19 20
	R Canis Majoris	7 15	*26*	*θ Pegasi*	22 05
B.D.	*+3°2867*	14 07	*6*	*Lacertae*	22 26
B.D.	*+6°2875*	14 19	*1*	*o Andromedae*	22 57
25	*Serpentis*	15 41			

We have abandoned *43* θ^2 *Orionis* (5^h30^m) at the request of the Ottawa observers; and we have also abandoned *41* θ^1 *Orionis* (5^h30^m) on account of the character of the lines in its spectrum.

F. SCHLESINGER

DETROIT OBSERVATORY
UNIVERSITY OF MICHIGAN
ANN ARBOR, September 13, 1913

The following list includes those announced binary stars on which the spectroscopic work at the Detroit Observatory has progressed far enough to justify an announcement. The order in which these stars are arranged in the list will give some indication of the degree of advancement of the work. Those named first have received the most attention.

14	*γ Lyrae*	18^h55^m	*80*	*g Ursae Majoris*	13^h21^m
	ζ¹ Ursae Majoris	13 20	*20*	*Tauri*	3 40
	ζ² Ursae Majoris	13 20	*47*	*ρ Leonis*	10 27
8	*β Cephei*	21 27	*50*	*α Cygni*	20 38
10	*β Lyrae*	18 40	*55*	*α Ophiuchi*	17 30

R. H. CURTISS

DOMINION ASTRONOMICAL OBSERVATORY
OTTAWA, CANADA
September 17, 1913

In reply to your circular letter of the 2d inst., I would say that the present condition of the orbital determination of spectroscopic binaries is not very satisfactory.

The process of selection of binaries for investigation was to choose first of all those with a high range of velocity and with good or moderately good spectra. Consequently the binaries now available for investigation consist practically wholly of stars with poor spectra or with a low range of velocity or with both of these drawbacks. I am convinced that all workers in this line of research, especially those like ourselves with small aperture, have felt that the binaries which offer any prospect of yielding satisfactory orbits are very few indeed.

We at Ottawa have made a large number of spectra of several binaries, in one or two cases more than a hundred plates of a star, without being able to obtain a period, and it is very likely that several stars on our present list will prove equally unsatisfactory.

For convenience the binaries under investigation at Ottawa are tabulated into different groups.

SPECTROSCOPIC BINARIES WITH LARGE NUMBER OF PLATES OBTAINED

51	*μ Persei*	4^h08^m	*47*	*ρ Leonis*	10^h27^m
66	*ν Tauri*	4 18	*4*	*γ Corvi*	12 11
86	*ρ Tauri*	4 28	*23*	*Comae*	12 30
20	*τ Orionis*	5 13	*B.A.C.*	*5890*	17 19
136	*Tauri*	5 47	*17*	*ι Andromedae*	23 33
18	*ν Geminorum*	6 23			

SPECTROSCOPIC BINARIES RECENTLY ADDED, ONLY FEW PLATES OBTAINED

15	*κ Cassiopeiae*	0^h27^m	*107*	*μ Virginis*	14^h38^m
34	*ζ Andromedae*	0 42	*13*	*δ Serpentis*	15 30
8	*δ Trianguli*	2 11	*67*	*Ophiuchi*	17 56
113	*α Piscium*	1 57	*68*	*Ophiuchi*	17 57
83	*φ Geminorum*	7 47	*41*	*ι Aquilae*	19 32
65	*α Cancri*	8 53	*12*	*Lacertae*	22 37

SPECTROSCOPIC BINARIES PROPOSED FOR INVESTIGATION

43	*ω Cassiopeiae*	1^h48^m	*50*	*α Cygni*	20^h38^m
25	*χ Aurigae*	5 26	*30*	*Cephei*	22 36
B.D.	*−1°943*	5 28			

SPECTROSCOPIC BINARIES, WORK DISCONTINUED, INABILITY TO OBTAIN PERIOD

77	*σ Leonis*	11^h16^m	*58*	*ν Cygni*	20^h53^m
72	*Ophiuchi*	18 03	*48*	*γ Aquarii*	22 16
17	*ζ Aquilae*	19 01			

The measures of the stars in the last list will be published in due course, enabling an approximation to the velocity of the system to be obtained, and giving opportunity to others to attempt a determination of the orbit.

J. S. Plaskett

Harvard College Observatory
Cambridge, Mass., September 5, 1913

Your letter of September 2, addressed to Professor Pickering, who is still in Europe, is received. I think the statement made in 1907 by Professor

Pickering,[1] as given in the *Astrophysical Journal*, **27**, 162, still holds true, but I will call his attention to your letter on his return, the latter part of this month.

S. I. Bailey

Königliche Sternwarte
Bonn, September 26, 1913

In reply to your inquiry of September 2, I would state that at present no spectroscopic binaries are under special observation with a view to the determination of their orbits. But in the general series of observations of stars of type F to M there are doubtless contained isolated observations of a large number of spectroscopic binaries. I give these stars, with the number of plates obtained. These spectrograms are measured and reduced, and results will be gladly placed at the disposal of those interested.

31	δ *Andromedae*	(3)	0^h34^m		A *Bootis*	(3)	14^h14^m
34	ζ *Andromedae*	(3)	0 42	*3*	β *Coronae*	(5)	15 24
38	η *Andromedae*	(3)	0 52	*13*	θ *Draconis*	(3)	16 00
43	β *Andromedae*	(4)	1 04	*44*	η *Herculis*	(3)	16 39
85	φ *Piscium*	(3)	1 08	*22*	ε *Ursae Minoris*	(8)	16 56
65	ξ¹ *Ceti*	(2)	2 08	*47*	o *Draconis*	(2)	18 50
8	δ *Trianguli*	(3)	2 11	*113*	*Herculis*	(4)	18 51
12	*Persei*	(3)	2 36	*6*	*Hev.* β *Scuti*	(2)	18 42
18	τ *Persei*	(4)	2 47	*60*	τ *Draconis*	(3)	19 17
1	o *Tauri*	(3)	3 19	*12*	φ *Cygni*	(5)	19 35
51	μ *Persei*	(3)	4 08	*7*	δ *Sagittae*	(3)	19 43
47	*Tauri*	(3)	4 08	*55*	η *Aquilae*	(2)	19 47
88	*d Tauri*	(7)	4 30	*63*	ε *Draconis*	(3)	19 49
8	ζ *Aurigae*	(3)	4 55	*32*	*Cygni*	(4)	20 12
16	*Aurigae*	(5)	5 12	*71*	*l Aquilae*	(3)	20 32
58	α *Orionis*	(3)	5 50	*6*	β *Delphini*	(3)	20 33
1	*Geminorum*	(3)	5 58	*62*	ξ *Cygni*	(4)	21 01
7	η *Geminorum*	(4)	6 09	*10*	κ *Pegasi*	(4)	21 40
43	ζ *Geminorum*	(2)	6 58	*21*	ζ *Cephei*	(2)	22 07
4	γ *Canis Minoris*	(3)	7 23	*24*	*Cephei*	(3)	22 08
75	σ *Geminorum*	(4)	7 37		$+38°4711$	(3)	22 10
11	ε *Hydrae*	(4)	8 41	*27*	δ *Cephei*	(3)	22 25
53	ξ *Ursae Majoris*	(5)	11 13	*33*	π *Cephei*	(4)	23 05
5	*Canum Venaticorum*	(3)	12 10	*70*	*q Pegasi*	(3)	23 24
12	*d Boötis*	(3)	14 06				

F. Küstner

[1] "The only spectroscopic binaries likely to be investigated at the Harvard College Observatory are those of Class A, in which both components are bright. They have been photographed here for many years, and the plates obtained will permit a very precise determination of their periods. No investigations of spectroscopic binaries of Class B, in which only one component is bright, are contemplated here at present.—Edward C. Pickering."

Under date February 9, 1914, Professor Pickering writes: "Professor Bailey's letter represents my present views."

KGL. ASTROPHYSIKALISCHES OBSERVATORIUM
POTSDAM, October 21, 1913

At the request of Director Schwarzschild, I advise you as follows regarding the spectroscopic binaries under observation at Potsdam.

Of the observations obtained with Spectrograph IV attached to the 32.5 cm refractor, there are yet unpublished those of *7 ϵ Aurigae* (about 180 plates), *50 α Cygni* (about 180 plates), and *24 γ Geminorum* (about 70 plates). The radial velocity of *7 ϵ Aurigae* cannot be represented by a simple elliptical orbital motion and it seems highly probable that the same is true of *50 α Cygni* and of *24 γ Geminorum*. I shall continue the observations of these three stars.

With Spectrograph III, attached to the 80 cm refractor, certain spectroscopic binaries, among other objects, have been observed by Dr. Münch and myself in the course of the past year in order to test the efficiency of the spectrograph in its new form (with short camera). The stars are *63 Tauri*, *108 Herculis*, and *111 Herculis*. A preliminary orbit of *63 Tauri* has been completed by Dr. Jantsen, and sufficient material is available for a similar orbit of *108 Herculis*, upon which I am now at work. Plates of *111 Herculis* have not yet been measured.

Observations with the 80 cm refractor are at present interrupted, as the figure of the objective is receiving correction by Steinheil. The spectrograph is meanwhile being reconstructed. A new program cannot be laid out until these two operations are completed.

H. LUDENDORFF

LICK OBSERVATORY
MOUNT HAMILTON, CALIFORNIA
September 26, 1913

The following is the list of spectroscopic binaries upon which members of our staff are now working:

	U Ophiuchi	17^h12^m		*h Centauri*	13^h48^m
	X Cygni	20 40	*10*	*κ Pegasi*	21 40
46	*υ Sagittarii*	10 16	*12*	*Persei*	2 30
41	*υ_4 Eridani*	4 14	*10*	*α Canis Minoris*	7 34
	H Velorum	8 53	*1*	*α Ursae Minoris*	1 23

W. W. CAMPBELL

MOUNT WILSON SOLAR OBSERVATORY
September 26, 1913

At present we are planning no work at Mount Wilson on the determination of the orbits of spectroscopic binaries. Our observational program consists almost wholly of stars fainter than the sixth magnitude, and comparatively few spectroscopic binaries as faint as this are known. It is possible that we may make a few observations of some of the fainter Algol variables, such as *RW Tauri*, but apart from this we have no plans in view for work of this character.

W. S. ADAMS

PARIS, September 28, 1913

The sky at Paris is continually disturbed in summer, as well as in winter, and does not permit us to undertake researches requiring continuity in the observations. Evenings without clouds are very few, and still more rare are those in which the images are good. It is for this reason that I have eliminated spectroscopic binaries from our program. I am observing the list of bright stars given in the *Annuaire du Bureau des Longitudes*, with the exception of those which are given as double in Campbell's *Catalogue* which appeared in 1911. Accordingly, the letter which you recently wrote me does not bear on the observations at Paris.

M. HAMY

PULKOWA
October 3, 1913

On account of climatic conditions, and particularly on account of the unfavorable arrangement of our dome, I am not in a position to observe objects according to my own choice. My program is to observe everything which is accessible with the weather we have. If I am able to repeat an observation after a few days, I obtain material for discussion; otherwise, the observations are lost and I have to begin over again. Therefore if I were to refrain from observing an object which is under observation elsewhere, I should practically have to cease observations entirely.

I have selected a number of stars not fainter than 3.5 mag., from Campbell's *Catalogue*, as well as some which have special interest for me.

21	α *Andromedae*	0^h03^m	77	ε *Ursae Majoris*	12^h50^m
33	α *Persei*	3 17	27	γ *Boötis*	14 28
7	ε *Aurigae*	4 55	36	ε *Boötis*	14 41
34	β *Aurigae*	5 52	40	ζ *Herculis*	16 38
43	ζ *Geminorum*	6 58	14	γ *Lyrae*	18 55
66	$α^1$ *Geminorum*	7 28	55	η *Aquilae*	19 47
41	$γ^1$ *Leonis*	10 14	50	α *Cygni*	20 38
41	$γ^2$ *Leonis*	10 14	53	ε *Cygni*	20 42
29	$γ^1$ *Virginis*	12 37	53	β *Pegasi*	22 59
29	$γ^2$ *Virginis*	12 37		*Saturn*	
12	α *Canum Venaticorum*	12 51		*Venus*	

and the standard velocity stars.

Also, for instrumental purposes:

3	α *Lyrae*	18^h34^m

and stars for a scale of spectral type:

4	β *Trianguli*	2^h04^m	58	α *Orionis*	5^h50^m
44	ζ *Persei*	3 48	10	α *Canis Minoris*	7 34
87	α *Tauri*	4 30	78	β *Geminorum*	7 39
24	γ *Orionis*	5 20	16	α *Boötis*	14 11
46	ε *Orionis*	5 31	64	$α^1$ *Herculis*	17 10

A. BELOPOLSKY

ROYAL OBSERVATORY
CAPE OF GOOD HOPE
October 3, 1913

In reply to your inquiry dated September 2, we have no observations specially made here for the purpose of investigating spectroscopic binaries, but I inclose for your information a list of stars included in our current working-list of which one or more spectrograms have been obtained.

S. S. HOUGH

CAPE OF GOOD HOPE

	ε *Phoenicis*	*H.P.* 821	*Eridani*		*I Puppis*
8	ι *Ceti*		α *Caeli*		ε *Puppis*
	ζ *Toucani*	3	π^3 *Orionis*		ζ *Volantis*
	β *Hydri*		γ *Caeli* (du.)		ξ *Puppis* (var.?)
	α *Phoenicis*	2	ε *Leporis*		*a Puppis*
	β^2 *Toucani*	19	β *Orionis*		*j Puppis*
31	δ *Andromedae*	9	β *Leporis*		ρ *Puppis*
16	β *Ceti*		ε *Columbae*		h^2 *Puppis*
63	δ *Piscium*	11	α *Leporis*	17	β *Cancri*
	β *Phoenicis*	40	ϕ^2 *Orionis*		ε *Carinae*
31	η *Ceti*		β *Doradus*		θ *Chamael.*
45	θ *Ceti*	13	γ *Leporis*		β *Volantis*
	γ *Phoenicis*	15	δ *Leporis*		*e Velorum*
99	η *Piscium*		β *Columbae*		β *Pyxidis*
	δ *Phoenicis*		γ *Pictoris*		*d Velorum*
52	τ *Ceti*	58	α *Orionis*	11	ε *Hydrae*
	χ *Eridani*	16	η *Leporis*		γ *Pyxidis*
78	ν *Ceti*	*S.H.P.* 1319	*Doradus*	16	ζ *Hydrae*
13	α *Arietis*		η *Columbae*		ω *Velorum*
68	ο *Ceti*	7	η *Geminorum*		*c Velorum*
	ι *Eridani*		κ *Columbae*		λ *Velorum*
87	μ *Ceti*	13	μ *Geminorum*		*g Carinae*
	β *Fornacis*		δ *Columbae*		*G Carinae*
3	η *Eridani*		α *Carinae*		ι *Carinae*
92	α *Ceti*	24	γ *Geminorum*	30	α *Hydrae*
	α *Fornacis*	27	ε *Geminorum*		ψ *Velorum*
16	τ^4 *Eridani*	9	α *Canis Majoris*		*N Velorum*
	e Eridani		α *Pictoris*		*M Velorum*
1	ο *Tauri*		τ *Puppis* (var.)	35	ι *Hydrae*
18	ε *Eridani*		*A Carinae* (var.)	14	ο *Leonis* (var.)
	y Eridani	16	o^1 *Canis Majoris*	17	ε *Leonis*
27	τ^6 *Eridani*	21	ε *Canis Majoris*		υ *Carinae* (du.)
	β *Reticuli*		σ *Canis Majoris*	24	μ *Leonis*
	g Eridani	43	ζ *Geminorum* (var.)		*m Velorum*
	γ *Hydri*	25	δ *Canis Majoris*	41	λ *Hydrae*
34	γ *Eridani*		γ^1 *Volantis*		*q Carinae*
	δ *Reticuli*		*J Puppis*	41	γ *Leonis* (du.)

CAPE OF GOOD HOPE—*Continued*

	α *Horlogii*
	α *Reticuli*
	γ *Doradus*
	δ *Tauri*
	d *Eridani*
77	θ^1 *Tauri*
	υ^1 *Eridani*
87	α *Tauri*
	υ^2 *Eridani*
H.P. 812	*Eridani*
12	δ *Crateris*
78	ι *Leonis*
	ξ *Hydrae*
	λ *Muscae*
S.H.P. 3966	*Centauri*
5	β *Virginis*
9	o *Virginis*
	η *Crucis*
1	α *Corvi*
	p *Velorum*
2	ε *Corvi*
	ε *Muscae*
	ε *Crucis*
	γ *Crucis*
8	η *Corvi*
9	β *Corvi*
29	γ *Virginis* (du.)
	c *Centauri*
43	δ *Virginis*
	δ *Muscae*
	ε *Virginis*
H.P. 2232	*Com. Ber.*
46	γ *Hydrae*
	m *Centauri*
	d *Centauri*
	i *Centauri*
	g *Centauri*
5	υ *Boötis*
8	η *Boötis* (var.)
	υ^2 *Centauri*
49	π *Hydrae*
5	θ *Centauri*
	δ *Octantis*
16	α *Boötis*
	τ^2 *Lupi*
	α^2 *Centauri*
	π *Puppis*
	δ *Volantis*
60	ι *Geminorum*
	σ *Puppis*
H.P. 1431	*Puppis*
10	α *Canis Minoris*
H.P. 1452	*Monocerotis*
75	σ *Geminorum*
77	κ *Geminorum*
78	β *Geminorum*
3	β *Coronae*
	ε *Trianguli*
38	γ *Librae*
	ν *Librae*
	ω *Lupi*
	ψ^1 *Lupi*
	g *Lupi*
24	α *Serpentis*
35	κ *Serpentis*
41	γ *Serpentis*
	ξ *Scorpii*
	δ *Trianguli Australis*
1	δ *Ophiuchi*
	γ^2 *Normae*
2	ε *Ophiuchi*
	γ *Apodis*
21	α *Scorpii*
20	γ *Herculis*
	H *Scorpii*
	α *Trianguli Australis*
	η *Arae*
26	ε *Scorpii*
	ζ *Arae*
27	*Herculis*
	η *Scorpii*
27	κ *Ophiuchi*
64	α *Herculis*
	β *Arae*
	Q *Scorpii*
55	ξ *Serpentis*
60	β *Ophiuchi*
	ι *Scorpii*
	G *Scorpii*
3	υ *Ophiuchi*
S 3308	*Velorum*
42	μ *Hydrae*
	I *Carinae*
	α *Antliae*
	s *Carinae*
	μ *Velorum*
	υ *Hydrae*
	μ *Carinae*
7	α *Crateris*
	χ *Carinae*
39	o *Sagittarii*
40	τ *Sagittarii*
	δ *Coronae Australis*
	β *Coronae Australis*
41	π *Sagittarii*
	β^2 *Sagittarii*
50	γ *Aquilae*
55	η *Aquilae* (var.)
	ι *Sagittarii*
60	β *Aquilae*
	δ *Pavonis*
	α^1 *Capricorni*
	α^2 *Capricorni* (var.?)
	β *Capricorni* (var.)
	α *Indi*
6	β *Delphini*
	ψ *Capricorni*
12	γ *Delphini*
	β *Indi*
8	α *Equulei*
	γ *Pavonis*
	ζ *Capricorni*
	b *Capricorni*
22	β *Aquarii*
	ν *Octantis*
	γ *Capricorni*
8	ε *Pegasi*
34	α *Aquarii*
	α *Toucani*
	δ^1 *Gruis*
	β *Gruis*
46	ξ *Pegasi*
	τ *Aquarii*
73	λ *Aquarii*
	δ *Piscis Australis*

CAPE OF GOOD HOPE—*Continued*

	α^1 *Centauri*		γ^2 *Sagittarii*		α *Piscis Australis*
	α *Circini* (du.)	70	*Ophiuchi* (Bi)		ζ *Gruis*
	α *Apodis*		η *Sagittarii* (du.)		c^2 *Aquarii*
	c^1 *Centauri*	10	δ *Sagittarii*	6	γ *Piscium*
36	ϵ *Bootis*	58	η *Serpentis* (var.?)		γ *Sculptoris*
	γ *Scorpii*		ζ *Telescopii*		b^1 *Aquarii*
20	σ *Librae*	22	λ *Sagittarii*	28	ω *Piscium*
	ζ *Lupi*		ζ *Pavonis*		ι *Gruis* (var.)
	ϕ^1 *Lupi*	37	ξ^2 *Sagittarii*		

UNIVERSITY OBSERVATORY
VIENNA, October 3, 1913

At the conclusion of the year 1912 a single-prism spectrograph was mounted in connection with the Rothschild coudé telescope of the Vienna Observatory. The spectrograph hangs on a track freely in the observing room and is not connected with the telescope, even during observations. On account of its constant position the instrument may be designated as perfectly free from flexure. The focal length of the collimator is 1007 mm, and the two cameras have focal lengths of respectively 300 mm and 580 mm. The instrument has been in use since February 1913, when the adjustments were completed. The observing program for it includes the stars, about 200 in number, to the 6th magnitude, which are contained in the zone from the north pole to +60° Dec., and of which the radial velocities have not been published in the Bulletins of the Lick Observatory, Nos. 195, 211, 212, 214, and 229. I expect also to undertake the determination of the orbits of spectroscopic binaries which may be discovered in this zone.

ADOLF HNATEK

YERKES OBSERVATORY
WILLIAMS BAY, WIS.
February 4, 1914

The following spectroscopic binaries, all of which were originally or independently detected here, are under observation or measurement with a view to the determination of the orbit, when and if the materials shall be adequate for the purpose. The accumulation of spectrograms of a particular star is slow, as on the average not more than about 60 to 70 full nights are obtained in the year with the spectrograph, owing to the pressure of other work with the 40-inch equatorial and to weather conditions. The spectrograms of several of the stars have been measured and will be discussed by men recently members of our staff but not now connected with this observatory, especially

by Professor S. A. Mitchell, now director of the McCormick Observatory of the University of Virginia.

4	β *Trianguli*	2^h04^m	*78*	o *Virginis*	13^h29^m
82	δ *Ceti*	2 34	*17*	κ *Boötis*	14 10
48	ν *Eridani*	4 31		*55°1703 Draconis*	15 55
37	ϕ^1 *Orionis*	5 29	*96*	*Herculis*	17 58
2	*Monocerotis*	5 54	*14*	γ *Lyrae*	18 55
61	μ *Orionis*	5 57	*1*	*Vulpeculae*	19 12
40	*Aurigae*	6 00	*46*	*Sagittarii*	19 16
21	*Lyncis*	7 19	*22*	*Cygni*	19 52
95	*Leonis*	11 51	*65*	τ *Cygni*	21 11
	78°412 Draconis	12 08	*8*	β *Cephei*	21 27
51	θ *Virginis*	13 05	*7*	*Andromedae*	23 08
79	ζ seq. *Ursae Majoris*	13 20	*17*	ι *Andromedae*	23 33

EDWIN B. FROST

REVIEWS

Die Spektren der Elemente bei normalen Druck. Von FRANZ EXNER und EDUARD HASCHEK. Leipzig: Franz Deuticke, Bd. I, 1911, "Hauptlinien der Elemente und Codex der Starken Linien im Bogen und Funken." pp. 216, M. 18; Bd. II, 1911, "Die Bogenspektren," pp. 347, M. 28; Bd. III, 1912, "Die Funkenspektren," pp. 332, M. 28.

Spectroscopists who used the earlier edition of Exner and Haschek's tables, now ten years old, frequently realized how very unsatisfactory it was to have these tables stop at λ 4600. Such investigators will be glad to welcome the present edition, appearing in three volumes, which carries measures well into the red, to λ 7500, which is made possible by using plates made sensitive to the red by bathing with pinacyanol. In order to have their results quite accordant, the authors repeated their measures for the earlier edition from about λ 4300. In the last ten years there have been new subdivisions of the elements especially in the rare earths. Ytterbium has disappeared as an element and instead we find neoytterbium and luteeium. Holmium has likewise disappeared as an element, but terbium, dysprosium, and neoholmium have been added.

Measures were made on the spectra of both arc and spark at atmospheric pressure from the ultra-violet to the deep red, and in their tables 61,580 lines in the arc spectrum are enumerated, and in the spark 60,252 lines. Photographs were taken by means of a Rowland 4-inch grating of 15 feet radius and 20,000 lines per inch. Exposures were made always in the first order on plates 4×30 cm, and each spectrum was photographed in ten sections. The camera was so arranged that on each plate three exposures were made, the arc spectrum, the spark spectrum, and the comparison spectrum of iron. The latter was placed between the other two and this middle spectrum was so arranged that it slightly overlapped the spectra above and below.

Instead of measuring these plates by the ordinary method, by means of a micrometer microscope, the photographs were projected on a screen. By very ingenious devices, the authors were able, by suitable additions to the lantern and a screen and scale of certain construction,

to project the photographs in such a way that one angstrom unit was equal to one centimeter on the screen. The scale attached to the screen was graduated to millimeters and the positions of the lines were estimated to tenths, so that it was possible to read their wave-lengths directly from the screen to 0.01 angstroms, 250 A. appearing on the screen at once; but of these 100 A. only were measured at the center of the screen.

As a result of their many years of experience with this method of measurement, the authors regard it to be quite equal in accuracy, but decidedly quicker than the actual measurement by means of micrometer. It was possible for them to measure a spectrum of 64 lines in 1^h30^m, a spectrum of 963 lines in 5^h30^m, and a spectrum of 2600 lines in 8^h22^m. As a matter of fact, it would be almost impossible to measure the 120,000 lines enumerated in their table by the well known and generally used method of measuring by micrometer.

Investigators in astrophysics will naturally question the accuracy of this method of measurement. The authors were able to make a great number of comparisons by measuring the same spectrum twice, by investigating the wave-lengths of the impurities which appear in a great number of different spectra, etc., and they find that their average error for 6995 lines is 0.016 A. They made a similar comparison for different authors who have published similar measures and they find the results to be about equal in the two cases.

Those who have occasion to investigate wave-lengths will be interested in the following table, which gives the number of lines measured by them in the 77 elements investigated. These elements are arranged in order of their atomic weights.

One will notice at once that these measures are in some respects sadly lacking: hydrogen, for instance, has but one spark line, and carbon only one line in the arc and 28 in the spark.

These tables of Exner and Haschek cover somewhat the same field as in Kayser's *Handbuch der Spectroscopie*, Vols. 5 and 6. The difference is that Exner and Haschek give all the lines of all the elements; Kayser tabulates only the stronger lines, from the measures by various authors, but with a critical discussion.

While investigating the spectrum of the chromosphere from the 1905 eclipse, the reviewer had need to find the origin of the lines in the chromosphere and in the sun. Comparisons with Rowland's tables left much to be desired in the way of identifications. Fortunately the present edition of Exner and Haschek appeared before this work was completed. The

Symbol	Atomic Weight	No Lines		Symbol	Atomic Weight	No Lines	
		Arc	Spark			Arc	Spark
H......	1	0	1	*Rh*...	103	1002	948
Li......	7	13	12	*Pd*	106	268	532
Be......	9	9	10	*Ag*	108	27	380
Bo......	11	2	3	*Cd*.	112	38	129
C......	12	1	28	*In*..	114	28	30
N...	14	0	142	*Sn*.	119	44	103
O......	14	0	113	*Sb*.	120	38	200
Fl......	19	0	69	*Te*.	127	4	111
Na.....	23	25	13	*I*.	127	0	172
Mg.....	24	52	58	*Cs*.	133	14	66
Al......	27	28	115	*Ba*	137	207	148
Si......	28	40	49	*La*	138	512	356
P......	31	15	85	*Ce*..	140	2894	1758
S......	32	0	44	*Pr*..	140	2490	1732
Cl......	35	0	101	*Nd*....	144	2762	2540
K......	39	18	61	*Sa*..	150	1670	1085
Ca......	40	114	84	*Eu*...	151	857	1508
Sc......	44	342	204	*Gd*.	156	1687	1411
Ti......	48	1132	1705	*Tb*	159	2487	1409
V......	51	1642	2837	*Nh*..		1482	1222
Cr.....	52	1697	1806	*Dy*....	163	3312	1464
Mn.....	55	865	1216	*Er*..	166	2321	1785
Fe......	56	2302	1838	*Tm*..	171	1007	667
Co......	59	1830	1360	*Ad*.	173	905	795
Ni....	59	976	623	*Cp*.	174	164	236
Cu.....	64	368	328	*Ta*	183	1285	1560
Zn.....	65	35	134	*Wo*..	184	3254	3012
Ga....	70	14	14	*Os*...	191	1340	867
Ge....	72	27	62	*Ir*..	193	806	1400
As....	75	18	69	*Pt*....	195	401	618
Se....	79	0	63	*Au*...	197	55	370
Br....	80	0	153	*Hg*.....	200	78	99
Rb...	85	19	62	*Tl*....	204	22	18
Sr.....	88	146	89	*Pb*	207	46	84
Y.....	89	684	430	*Bi*..	209	48	121
Zr.....	91	1070	1529	*Ra*.	226	50	10
Nb....	94	1770	2086	*Th*...	232	2316	2298
Mo...	96	3390	3248	*U*.....	240	4940	5655
Ru.....	102	1948	1650				

reviewer found their wave-lengths on the whole to be accurate to about 0.03 or 0.04 A. with individual discrepancies of double that amount. He found the wave-lengths in practically all cases thoroughly reliable. He found the spectra almost without impurities, and he wishes hereby to record his deep obligation and gratitude for these tables, without which the identifications could not possibly have been made as complete as they were.

S. A. Mitchell

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXXIX MAY 1914 NUMBER 4

A DETERMINATION OF THE SUN'S TEMPERATURE

By GLENN A. SHOOK

INTRODUCTION

In 1906 Moissan carried out a number of experiments upon the vaporization of metals.[1] He placed the temperature of his furnace at 3500° C. and made the statement that all known elements volatilize at that temperature. Now it is thought by Schulz that the temperature of the furnace must have been considerably above 3500° C. and probably as high as the sun's photosphere which he sets at 5400° C.[2] He argued that owing to the large current used by Moissan there was an enormous amount of energy which had no adequate escape by conduction or radiation and which therefore must have raised the temperature of the furnace up to the point where it was checked by the melting and evaporization of the limestone of which it was constructed. He moreover asserts that the volatilization of the metals is not to be regarded as complete.

We also find the following remarks in regard to molybdenum and tungsten:

Molybdenum.—The 150 grams were not fused by a current of 500 amperes and 110 volts. After applying 700 amperes and 110 volts for seven minutes,

[1] *Annales de chemie et de physique*, **8**, 151, 1906.

[2] *Astrophysical Journal*, **29**, 33, 1909.

the metal was fused but nothing evaporated. After twenty minutes 56 grams were distilled.

Tungsten.—After applying 500 amperes and 110 volts for 5 minutes the metal was not yet fused. After applying 800 amperes and 110 volts for twenty minutes, boiling commenced but only 25 grams distilled.

It thus appears that the volatilization is partly a question of time, and when we remember that the sun's photosphere is probably at a temperature of 8000° C. or 9000° C. and that such a temperature has existed for years and not minutes, we must conclude that all elements in the sun are necessarily in the gaseous state.

The following hypothesis which has been advanced by a number of investigators[1] is confirmed by the present research.

In the first place the material of the sun is "gaseous," that is, it follows the extended law for gases.

Secondly, the radiation that reaches us comes from the reversing layer alone or at least only from the superficial layers of the photosphere.

Thirdly, there is a relatively large drop in the temperature at the reversing layer.

If there is considerable scattering of light due to the gases of the reversing layer, then the light that reaches us comes from a small depth only. Moreover, the scattering is greater for blue light than for red, consequently the blue part of the spectrum must be relatively weaker than the red part. Hence, if the temperature falls off rapidly as we move outward radially through the reversing layer we should expect the temperature for blue light to be less than that determined for red light. Also as we move across the sun's disk, we should expect the apparent temperature to fall off rapidly as we approach the limb and we should, moreover, expect the temperature gradient for blue to be greater than that for red. This is precisely what the writer finds. The sharp boundary of the photosphere is additional proof of the gaseous scattering.

That the scattering prevents us from seeing beyond a shallow depth of the reversing layer may be shown by a rough calculation.

[1] Secchi, *Le soleil*, 1, Book III, chap. iv, p. 207; 2, Book VII, chap i, p. 299; Schwarzschild, "Ueber das Gleichgewicht der Sonnenatmosphäre," *Gottingen Nachr., Math. Phys. Kl.*, 1906, pp. 1–13; Abbot, *The Sun*, p. 236.

The law of molecular absorption is expressed by the following formula:

$$I = I_0 e^{-kh}$$

or

$$\log \frac{I}{I_0} = -kh$$

where $I_0 =$ the intensity of light incident upon the absorbing medium; $I =$ the intensity of the transmitted light; $k =$ the fraction of light absorbed by unit thickness of the medium; and $h =$ the thickness or height of the absorbing layer.

Using Abbot's[1] values for the transmission of the atmosphere above Mount Wilson we have:

Wave-length in μ	0.4	0.5	0.6	0.7
Percentage of transmission. . . .	76	89	95	97

Taking the Mount Wilson atmosphere, which is about 10 miles, as our unit thickness, the length of a column of gas for an extinction of 99 per cent or a transmission of 1 per cent for a wave-length of $0.4\ \mu$ becomes

$$\frac{\log 0.01}{0.4343} = -0.24h$$

or

$$h = 18.5 ,$$

that is, the column would have to be 185 miles if the gas had the same density as the Mount Wilson atmosphere.

The relative densities of the photosphere and the Mount Wilson atmosphere may be determined by means of Boyle's Law as follows:

Let the pressure, volume, and absolute temperature of the former be p', v', and T', and the corresponding quantities for the latter be p, v, and T. We may now write:

$$pv = RT$$

and

$$p'v' = RT'$$

hence

$$\frac{pv}{p'v'} = \frac{T}{T'} .$$

[1] *Nature*, **81**, 97, 1909.

Assuming that the pressure of the reversing layer is about 5 atmospheres, that its mean temperature is 7000° A., and that the temperature of the earth's atmosphere is 250° A., we obtain the relation:

$$\frac{1\times v}{5\times v'}=\frac{250}{7000}.$$

Writing d_s for the density of the reversing layer and d_e for the density of the earth's atmosphere, the above equation becomes:

$$\frac{d_s}{d_e}=\frac{250\times 5}{7000}.$$

Hence a column of gas on the sun sufficient to produce an extinction of 99 per cent at wave-length 0.4 μ would have to be

$$18.5\times 10\times\frac{7000}{250\times 5}=1000 \text{ miles high}.$$

In this manner Table I was constructed

TABLE I

Wave-Length	Miles
0.4 μ	1000
0.5	2400
0.6	5200
0.7	8600

Since the radius of the sun is 435,000 miles, it is readily seen that the radiation which we are utilizing for the estimation of temperature comes from only the outermost solar layers. It is also observed that for short wave-lengths the depth to which we are able to penetrate is smaller than that which obtains for the longer wave-lengths.

EXPERIMENTAL METHOD FOR THE DIRECT DETERMINATION OF THE SUN'S APPARENT TEMPERATURE

The method employed by the writer for the determination of the sun's apparent temperature is an application of Planck's formula for the visible spectrum. In this method the brightness of the sun's disk is compared photometrically with the brightness of the filament of a miniature incandescent lamp for three different colors. To carry out these observations the Department of

Astronomy of this university kindly permitted the use of their small observatory, which is equipped with a six-inch equatorial telescope. A new eyepiece was constructed, providing a receptacle for the lamp between the eye-lens and the field-lens. A new finder, provided with a micrometer scale and parallel hairs, was also attached to the telescope.

An image of the sun is formed by the objective in the focal plane of the eyepiece. The incandescent lamp is adjusted until its filament lies in the plane of the image of the sun's disk.

If one looks through the telescope when it is directed toward the sun he sees the image of the lamp-filament superimposed upon the image of the sun's disk. Now by varying the current through the lamp the filament can be made to disappear against the bright background of the sun's image. When this condition obtains, the temperature of the filament is equal to the apparent black-body temperature of the image, and by means of Planck's formula the apparent black-body temperature of the sun's disk can be estimated if the temperature of the filament is known as a function of the current through the lamp.

In the present investigation the lamps used were calibrated by the Bureau of Standards. The eyepiece that was used in the equatorial and which contained a lamp receptacle was fitted into a small telescope and this arrangement was used by the bureau in calibrating the lamps by means of their standard black body. They furnished for each lamp a table containing a series of temperatures and the corresponding currents through the lamp. The error which might be caused by reflections from the lamp globe and eyepiece lenses was thus eliminated.

As a matter of fact, the entire filament will never disappear since all parts are not of the same intensity, but one always uses the central portion of the tip and this is practically uniform in intensity.

The filament (Fig. 2) may be moved about easily to any point on the disk, which is represented by the dotted line, by means of the right ascension and the declination screws. Fig. 1 shows the reticle of the finder with the scale and parallel spider lines. These parallel lines are adjusted so that their distance apart is equal to

the diameter of the sun's image, and they are, moreover, always parallel to the ecliptic.

The axis of the lamp is generally maintained perpendicular to the ecliptic. The lamp is connected in series to a few storage cells, an adjustable resistance, and a milliammeter (Fig. 3).

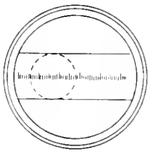


FIG. 1

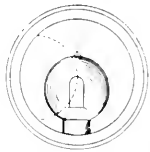


FIG. 2

This arrangement of lamp and eyepiece, which is the result of some experimenting, was found to be the most satisfactory. With an equatorial as small as this one the image is only about 1 cm in diameter, and in order to investigate the intensity along any radius, i.e., along a distance of 0.5 cm, with any accuracy it is necessary

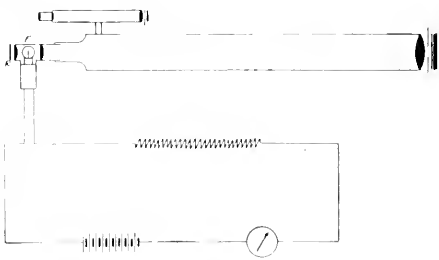


FIG. 3

to have a rather large magnification. In order to obtain a clear image the field-lens is also indispensable. Again, with the present arrangement the globe of the lamp just about fills up the space between the two lenses and therefore it is not in focus; consequently when one looks at the tip of the filament the contour of the globe is scarcely noticed. If an eye-lens of longer focal length were used, the globe would cause a distortion of the image.

The problem of diminishing the intensity of the sun's image to that of an incandescent lamp-filament presents no small difficulty. The intensity may be partly diminished by diaphragming down the objective, but one cannot resort solely to this method without seriously impairing the definition of the image. When the aperture is made as small as is permissible, an absorption glass may be used, but it is almost necessary to use three or more if the absorption coefficient of the arrangement is required in any calculation. The density of a single glass required to make the necessary reduction in intensity is so great that it is impossible to measure its absorption coefficient with any accuracy. Since the absorption of these glasses is never absolutely general, i.e., non-selective, and since they differ slightly among themselves, it is necessary to measure the absorption factor of each glass for each wave-length used.

Moreover, the optical properties of these glasses must be almost as good as those of the telescope objective; otherwise aberrations would result. It is for this reason that it is practically impossible to use a large telescope since the absorption glasses would have to be made with as much care as the objective of the telescope.

In order to determine the best arrangement for the six-inch equatorial used in this investigation a number of observations were carried out upon the moon's disk. The most conspicuous craters were carefully studied with a full objective and then with a number of diaphragms having apertures of different size. In this manner it was found that an aperture of about 1.5 cm still produced good definition. In addition to this diminishing of the aperture, three absorption glasses were also used. The objective of the finder was also stopped down and in addition an absorption glass was used.

Monochromatic light was produced by placing colored glasses directly before the eye-lens R (Fig. 3). It is practically impossible to obtain a single colored glass which is even approximately monochromatic. Four colors of Jena glass were obtained from Petitdidier, Chicago—namely, red, yellow, green, and blue. The red is remarkably good, transmitting only a red band, and that rather narrow. The yellow, which appeared monochromatic to the

unaided eye, was found to transmit almost the entire spectrum. The green contains a faint band in the yellow but it is free from blue. The blue glass transmits a band in the red, as is usually the case with blue glasses, and also faint lines in the green.

While, according to our information, these are the best glasses that can be obtained, it is readily seen that they were unsuitable without some modifications.

A detailed study of monochromatism of various kinds of glass was then undertaken. A quantity of different kinds of colored glass was obtained and these glasses were all examined separately by means of a spectroscope, and then different combinations were tried until the best arrangement was obtained. The Jena glasses were found to be superior to any examined but a combination of three different glasses was found to give the best results.

For example, some green glass transmits blue light but no red, while nearly all blue glass transmits some red; consequently a combination of the two is practically free from red without any perceptible reduction of the blue light.

In this manner it was possible to obtain combinations for red, green, and blue light, all of which are practically monochromatic. The search for monochromatic yellow was, however, futile. It seems almost impossible to obtain a glass or a combination of glasses which produces yellow and excludes all the other colors in even a moderate degree.

The fact that a glass for a particular color may contain a faint band of another color is often of no consequence, providing that consistent readings may be made, and a very narrow band is not always necessary if the band contains only one color. For instance, we may have a rather wide red band, but so long as there is no orange included in the band a good photometric balance can always be made, and the wave-length used would always be the central part of the band. The difference between this wave-length and the true optical center of gravity is too insignificant to consider in this particular problem.

There are other methods for producing monochromatic light, but none is very well suited to this particular problem. The spec-

troscopic eyepiece designed by Mendenhall[1] for pyrometers using the disappearing-filament principle is best adapted to this particular apparatus, but it was rejected for several reasons. In Mendenhall's pyrometer a short horizontal section of the lamp-filament and the superimposed image are focused upon the slit of an auxiliary direct-vision spectroscope. The slit of the spectroscope is vertical so that the field is crossed by three spectra, the middle one corresponding to the lamp-filament. A diaphragm is so placed in the focal plane of the eyepiece of the spectroscope that only the desired region of the spectrum is transmitted to the eye. In order that this central band may be wide enough to make a photometric comparison it is necessary to use a very thick lamp-filament, and this is impossible when a large magnification of the image is required as in the present investigation, for then all parts of the filament would not be in focus. Even with a fine filament there is some distortion of the image.

Furthermore, any such spectroscopic method diminishes the intensity of the light considerably, making it necessary in the blue and violet region to open both slits of the instrument very wide in order to get sufficient light to make a balance. If this is done, we have no longer strictly monochromatic light, and we may as well employ colored glasses. With a colored glass one sees the filament and sun's image directly so that he always knows just what part of the disk he is on, but with the spectroscopic eyepiece this of course is not the case, and he must depend entirely upon the finder.

It has been shown by a number of experimenters that the disappearing-filament principle is by far the most sensitive photometric scheme that we have, and it is particularly adapted to this problem, since one can move the filament about to any point on the sun's image, and make a temperature measurement at that point.

The wave-lengths of the monochromatic glasses were determined by means of a Lummer-Brodhun spectrophotometer made by Schmidt and Hensch. The same instrument was also used to determine the absorption coefficients of the absorption glasses.

[1] *Physical Review*, **33**, 1, 1911.

DATA AND RESULTS

1. *Wave-lengths of the monochromatic glasses.*—The readings of the arbitrary scale of the Lummer-Brodhun spectrophotometer for the three colored glasses used are given in the following tables:

TABLE II

OCULAR SLIT=0 05 CM SLIT NO. 1, 50

Red Glass		Green Glass		Blue Glass	
Blue End	Red End	Blue End	Red End	Blue End	Red End
616	546	734	654	994	750
618	546	732	654	994	748
614	544	734	652	990	750
618	546	736	654	994	748
616	542	736	652	994	752
616	546	734	656	1,000	750
616	544	734	656	1,002	748
616	546	736	654	996	750
614	546	736	654	994	748
614	544	734	654	996	754
Mean	580	Mean	694	Mean	868

The blue light was very faint, hence the readings are not quite so consistent as in the case of the red and green.

The wave-lengths in μ, corresponding to these arbitrary scale readings, are as follows:

TABLE III

	Lummer-Brodhun Scale Reading	Wave Length in μ
Red glass	580	0 661
Green glass	694	0 537
Blue glass	868	0 446

2. *Absorption factors.* In determining the absorption factor R for any particular glass the zero reading of the Lummer-Brodhun spectrophotometer was taken before and after the observations with the glass.

The standard lamps were connected in parallel to the same mains and the voltage was controlled by a rheostat.

TABLE IV

Red Light

L.-B. No. 580
Ocular Slit = 0 05 cm

Zero Reading	Volts	Glass No. 1	Volts
Slit No. 1, 50.0		Slit No. 1, 100	
Slit No. 2, 51 0	107.0	Slit No. 2, 8 4	107 0
51 9		8 3	
50 6		8 4	
50 9	107 0	8 3	107 0
51 0		8 3	
51 9		8 4	
51 0		8.4	107 0
51 4		8 4	
52 0		8 3	
51 3	107 0	8 3	107 0
Mean . . 51 30		Mean 8 35	

Glass No. 1	Volts	Glass No. 1	Volts
8 3	107.0	8 3	107 0
8 5		8.5	
8 4		8 3	
8.4		8 4	
8 3		8 3	
8 5		8 2	
8 5		8 3	
8.5	107.0	8 3	107 0
8 2		8 5	
8 3		8 3	
Mean . . . 8.39		Mean . . . 8 34 Mean . . . 30 observations 8 36	

REDETERMINATION OF ZERO READING OF INSTRUMENT

	Volts		
Slit No. 1, 50.0	107 0	Slit No. 2, 52.3	Slit No. 2, 51 3
Slit No. 2, 52 6		52 3	51 3
51 3		52 3	50 3
52 0		51 3	50 5
51 5		52 2	50 3
52.2		53 0	51 6
50 3		52.7	51 1
52 3		51 8	50 3
52.4		51 9	51 2
53 2		52 3	50 0
53 0		51 9	

Mean of 30 observations	51 36
First reading	51 30
Mean	51 33

Calculation of the absorption factors for red light, 0.661 μ.

Let the reading of slit No. 1 be S_1 and slit No. 2 be S_2 when no glass is interposed between lamp and slit No. 2. Also let S_1' and S_2' be the corresponding readings when a glass is inserted; then the absorption factor R is

$$R=\frac{S_1'}{S_2'}\cdot\frac{S_2}{S_1}=\frac{S_1'}{S_1}\cdot\frac{S_2}{S_2'}$$

Glass No. 1 . . . . $R_1=\frac{100}{50}\cdot\frac{51.3}{8.36}=12.3$

Glass No. 2 . . . . $R_2=11.8$

Glass No. 3 . . . . $R_3=11.9$

whence

$$R=R_1R_2R_3=1725 \text{ (red)} .$$

The absorption factors for the green and blue glasses, obtained in the same manner, are 340 and 656 respectively.

The lamps used for estimating the sun's temperature were calibrated in a small telescope of 2.68 cm aperture. The distance from the filament to the aperture was 59.8 cm. In the observatory telescope the distance from filament to aperture was 157.5 cm and the aperture was 1.49 cm in diameter.

The ratio of the two solid angles gives the reduction factor for the telescope. We therefore obtain:

$$R'=\frac{\pi}{4}\cdot\frac{(2.68)^2}{(59.8)^2}\div\frac{\pi}{4}\cdot\frac{(1.49)^2}{(157.5)^2}=22.3 .$$

The resultant reduction factors for the three colors then become:

$$\text{For } 0.661\ \mu \quad R=22.3\times1725=38,500 \tag{1}$$

$$0.537\ \mu \quad R=22.3\times 340=7580 \tag{2}$$

$$0.446\ \mu \quad R=22.3\times 656=14,610 \tag{3}$$

3. *Temperature measurement of the sun's disk.*—The distance across the sun's disk was measured by means of a micrometer scale in the finder of the telescope, but the number corresponding to the center of the disk would of course change if the lamp were raised or lowered. For the observations carried out for the red and green light 54 corresponded to the center of the disk and 69 to the extreme edge or limb.

The radius of the disk is thus equal to 15 divisions on the scale of the finder. In the following tables the readings of the ammeter are given for various distances from the center of the disk. When the filament was adjusted to the desired point on the disk the current through the filament was varied continuously until the tip had the same intrinsic intensity as the region surrounding it or until it disappeared against the disk.

The following (Table V) is a sample of the data obtained for the variation of the temperature with distance from the edge to the center of the disk.

TABLE V

AMMETER READINGS FOR GREEN LIGHT

$\lambda = 0.537\,\mu$

69	68	67	64
80.0	84.0	86.0	91.0
82.0	84.5	87.5	91.5
79.5	83.5	87.0	89.5
81.0	83.5	86.5	89.0
81.0	83.5	88.5	91.0
81.0	83.0	87.5	91.5
81.0	85.0	87.5	89.5
82.0	82.5	86.5	90.5
81.5	82.5	87.5	91.5
81.0	82.5	85.5	90.0
Mean....81.1	Mean 83.5	Mean ...87.0	Mean 91.0

60	54	69	69	69
92.0	93.5	81.5	80.0	80.0
93.0	92.5	82.0	80.5	80.0
91.5	93.5	81.5	81.5	81.5
91.5	93.0	82.0	80.5	80.0
93.0	91.0	81.5	80.0	80.5
93.0	92.0	81.5	80.0	82.0
92.5	92.0	81.0	82.0	80.0
91.5	92.0	82.0	80.0	80.5
92.5	91.5	81.5	81.0	81.0
91.0	91.5	81.5	81.5	80.0
Mean 92.2	Mean 92.3	Mean of 30 observations.		81.0

In this case the observation on the edge, i.e., 69, was repeated and it is seen that the agreement is better than might be expected considering the uncertainties of such measurements.

Instead of reducing these readings to temperatures of the sun's disk, a curve was plotted for each color, co-ordinating ammeter readings and distances from center of disk. For any particular distance, the corresponding ammeter reading may be obtained directly from the curve. This gives a better average of all the values taken across the disk.

4. *Reduction of an observation.*—Since it is somewhat easier to use Wien's formula for the reduction of these temperatures, that formula will be used for all the calculations. A temperature estimation will also be made by means of Planck's formula to show the difference in the two results.

We shall consider in detail only the data obtained for red light, as the same method applies equally well to green and blue.

Let T' equal the black-body temperature of the sun's disk, E' the intensity of radiation incident upon the objective of the telescope. Also let T be the apparent temperature of the sun's image and E the intensity of the energy transmitted by the absorbing media of the telescope.

Wien's formula may now be written for the two cases as follows:

$$\log E' = k_1 - k_2 \frac{1}{T'} \tag{4}$$

and

$$\log E = k_1 - k_2 \frac{1}{T} . \tag{5}$$

Subtracting (5) from (4) we obtain:

$$\log \frac{E'}{E} = k_2 \left(\frac{1}{T} - \frac{1}{T'} \right) .$$

But

$$\frac{E'}{E} = R$$

where R is the reduction factor of the telescope.

Whence

$$\frac{1}{T} - \frac{1}{T'} = \frac{\log R}{k_2} = k = \frac{\log R \lambda}{14.500 \times 0.4343} .$$

From (1):

$$R = 38,500$$

and

$$\lambda = 0.061 .$$

Therefore

$$k=\frac{\log 38{,}500\times 0.661}{14{,}500\times 0.4343}=0.000482$$

whence

$$\frac{1}{T'}=\frac{1}{T}-0.000482\,. \tag{6}$$

Now consider curve 1, Fig. 4, for scale division 54, i.e., the center of the sun's image; the ammeter reading is 67.2, and this corresponds to a temperature of 1317° C. or 1590° A. If we substitute this value for T in equation (6) we obtain for T' the

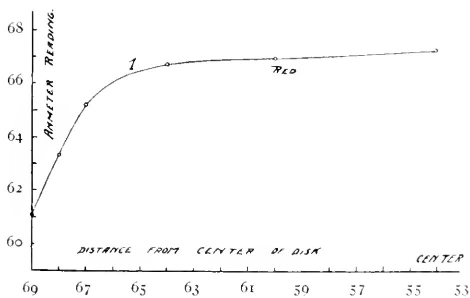


FIG. 4

temperature of the sun's disk, a value of 6803° A. In this manner data were obtained for curves 2, 3, and 4, Fig. 5.

As we move from the center of the disk toward the limb, the temperature falls off more rapidly for the shorter wave-lengths, but near the limb it falls off less rapidly.

There has always been considerable discussion as to the best value of the constant C_2. The value used by Lummer, Pringsheim, Paschen, and Wanner is 14,500. Our own Bureau of Standards[1] also accepts the same value but the value determined by

[1] *Bulletin Bureau of Standards*, 3, No. 2.

Holborn and Valentiner is much lower 14.200.[1] Again, Nernst and Wartenberg use the value 14,600.[2] To show the effect of the

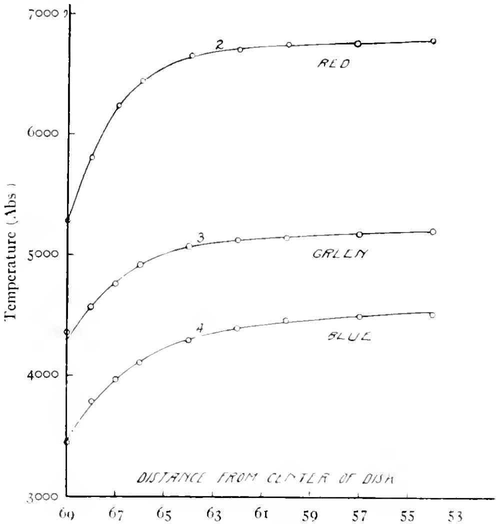


FIG. 5

variation of this constant on the value of the temperature of the sun, the following values were calculated for red and blue light:

TABLE VI

C_2	Red 0.66 μ T'	Blue 0.436 μ T'
14,000 . .	6135	4310
14,100. . .	6211	4348
14,200.	6320	4386
14,300 .	6404	4425
14,400 .	6607	4464
14,500	6803	4505
14,600. .	6944	4545
14,700 .	7143	4587
14,800 . .	7260	4630

[1] *Ann. der Physik*, **22**, 1, 1907.

[2] *Verh. der deutschen phys. Ges.*, **8**, 48, 1906.

We shall now determine the value of the temperature for red light $\lambda = 0.661\ \mu$ by means of Planck's formula in order to see what error results by using Wien's formula.

Using the same notation, we may write Planck's formula for the two temperatures as follows:

$$E' = C_1\lambda^{-5}\frac{1}{\left(e^{\frac{c_2}{\lambda T'}} - 1\right)} \tag{7}$$

and

$$E = C_1\lambda^{-5}\frac{1}{\left(e^{\frac{c_2}{\lambda T}} - 1\right)}. \tag{8}$$

Dividing (8) by (7) we obtain:

$$\frac{E'}{E} = \frac{\left(e^{\frac{c_2}{\lambda T}} - 1\right)}{\left(e^{\frac{c_2}{\lambda T'}} - 1\right)} = R\,. \tag{9}$$

Writing (9) in the form:

$$\left(e^{\frac{c_2}{\lambda T}} - 1\right)\frac{1}{R} + 1 = e^{\frac{c_2}{\lambda T'}}$$

we finally obtain:

$$T' = \frac{\frac{c_2}{\lambda}}{\log\left[\frac{1}{R}\left(e^{\frac{c_2}{\lambda T}} - 1\right) + 1\right]}. \tag{10}$$

Now let

$$\frac{C_2}{\lambda}\log\ e = k_2$$

whence

$$T' = \frac{k_2}{\log\left[\frac{1}{R}\left(e^{\frac{k_2}{\lambda T}} - 1\right) + 1\right]}. \tag{11}$$

To evaluate $e^{\frac{k_2}{\lambda T}}$ let

$$\frac{k_2}{\lambda T} = \log x$$

whence

$$\log x = \frac{14{,}500 \times 0\ 4343}{0\ 661 \times 1590} = 5\ 99$$

and

$$x = 977{,}000\,.$$

Since

$$R = 38{,}500$$

$$\log\left[\frac{977{,}000}{38{,}500}+1\right] = \log\,(25.3+1) = 1.420\,.$$

The constant k_2 is 9520 and the real temperature now becomes:

$$T' = \frac{9520}{1.420} = 6700^\circ\ \text{A}.$$

If we neglect the 1 in the expression $\log(25.3+1)$, then the expression reduces to Wien's formula in which case

$$\log 25.3 = 1.403$$

whence

$$T = \frac{9520}{1.403} = 6800^\circ\ \text{A}.$$

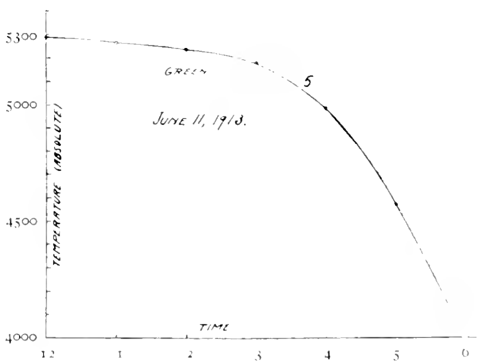


FIG. 6

The variation of the apparent temperature with the sun's zenith distance was determined for green light and the results are shown in curve 5, Fig. 6.

5. *Correction for atmospheric absorption.*—The ratio of the intensity of the sun, at the boundary of the atmosphere and at the surface of the earth, is given by the well known formula:

$$\frac{E_s}{E_e} = \frac{1}{A^{\sec Z}} \tag{12}$$

where

E_s = intensity of sun at the boundary of the atmosphere
E_e = intensity of sun at the telescope
A = the transmission coefficient
Z = the zenith distance

If the intensity is known for two different hours, say 12:00 and 4:00 o'clock, then A may be determined from the relation:

$$\log A = \frac{\log \frac{E_{12}}{E_4}}{\sec Z_{12} - \sec Z_4}. \tag{13}$$

By means of curve 5, the apparent temperature of the sun for these two hours may be determined and by means of Wien's formula the ratio of the corresponding intensities may be determined. Writing Wien's formula for the two cases we obtain:

$$\log E_{12} = k_1 - k_2 \frac{1}{T_{12}}$$

and

$$\log E_4 = k_1 - k_2 \frac{1}{T_4}$$

whence

$$\log \frac{E_{12}}{E_4} = k_2 \left(\frac{1}{T_4} - \frac{1}{T_{12}} \right). \tag{14}$$

By means of (14), using the data obtained from curve 5, we obtain:

$$\log \frac{E_{12}}{E_4} = 0.14$$

and by means of tables

$$Z_{12} = 17.1 \text{ and } Z_4 = 54.3$$

whence

$$\log A = \frac{0.14}{\sec 17.1 - \sec 54.3} = 0.803 - 1$$

and

$$A = 0.63 .$$

The mean of 5 values of A determined for 2:00, 3:00, 4:00, 5:00, and 6:00 o'clock was found to be 0.64.

Using the values of the transmission coefficients obtained for Washington and Mount Wilson[1] the following values were found by interpolation for red and blue light:

For red light, 0.661 μ, $A = 0.74$, and for blue light, 0.440 μ, $A = 0.50$.

The absorption factor already determined for the telescope will now be determined for red light, 0.661 μ.

Equation (12) may be written in the form

$$\log \frac{E_s}{E_e} = -\sec Z \log A .$$

The time of observation of the temperature for red light was 2:00 P.M., September 1, 1912, and the zenith distance for this hour is 42°7. We therefore obtain:

$$\log \frac{E_s}{E_e} = -\sec 42.7 \times \log 0.74 = 0.178$$

and

$$\frac{E_s}{E_e} = 1.505 .$$

The reduction factor for red light, 0.661 μ, corrected for atmospheric absorption now becomes:

$$R' = 38,500 \times 1.505 = 58,000$$

and the new constant k' is

$$k' = \frac{\log 58,000 \times 0.661}{14,500 \times 0.4343} = 0.000498 .$$

[1] Abbot, *The Sun*, p. 242.

Equation (6) therefore becomes:

$$\frac{1}{T'}=\frac{1}{T}-0.000408$$

and the corrected temperature for the center of the disk is 7580° A. In a similar manner the reduction factors for the other colors were found to be 15,300 for green and 37,000 for blue light. The data for green light were determined on September 22, 1912, at 2:00 P.M., and that for blue light on April 15, 1913, at 2:00 P.M.

The sun's temperature for the three colors is therefore as follows:

For red light, 0.661 μ: 7580° A.
For green light, 0.537 μ: 5990° A.
For blue light, 0.446 μ: 5230° A.

Without the absorption of the light through the atmosphere of the earth we find for the three colors the following values:

For red light 6803° A.
For green light 5208° A.
For blue light 4505° A.

There is possibly a small error in the determination of the transmission coefficients of the atmosphere, due to the fact that these coefficients were not determined at the same time and therefore possibly not under exactly the same conditions of the atmosphere as those under which the radiation of the sun was measured.

The last two series of values indicate clearly that the sun is not a black body, because for a black body we should find the same temperature for each wave-length. This fact is also demonstrated directly by the actual distribution of the energy of radiation through the spectrum and through the results of the three general methods which may be used for the determination of the sun's temperature based upon the following laws:

Stefan-Boltzman Radiation Law:

$$E=76.8\times10^{-12}T^4,$$

Wien's Displacement Law:

$$\lambda_m T=2930.$$

Planck's Distribution Law:

$$E=c_1\lambda^{-5}\frac{1}{\left(e^{\frac{c_2}{\lambda T}}-1\right)}.$$

The first method gives a temperature of 5830° A. if we use Abbot's value of the solar constant 1.922. In the second method, if we take the wave-length of the maximum energy as 0.470 μ[1] we get a temperature of 6230° A. The third method, as has just been shown by the present investigation, gives a temperature of 7580° A. All the determinations of the temperature of the sun by means of radiation give therefore only approximative results, and the deviations of the temperatures for different colors and for different methods indicate how far the sun's radiation differs from that of a black body. Another example taken from laboratory practice may illustrate the difference between the thermodynamic temperature of a radiating body and the temperature obtained by radiation methods. If we attempt to measure the temperature of a piece of iron at about 1000° A., by means of the total radiation emitted we obtain a temperature which is about 400° lower than the true temperature, but if we utilize the radiation corresponding to a single wave-length, say 0.6 μ, we obtain a temperature which is about 150° lower.

SUMMARY

1. The temperature of the sun has been measured by a new method based on Planck's and Wien's laws of radiation for three different wave-lengths.

2. The variation of the radiation of the sun from the center to the limb has been measured for three different colors.

3. The absorption of green light in the atmosphere of the earth has been measured.

In conclusion I wish to thank Professor A. P. Carman and Professor J. Kunz for their many helpful suggestions during the investigation of the above problem.

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October 1913

[1] Abbot, *The Sun*, p. 69.

ON THE THEORETICAL PHOTOMETRY OF DIFFUSE REFLECTION

By L. GRABOWSKI

By diffuse reflection is understood the property of a body whereby in contrast to bodies with a polished surface it acts under the influence of radiation as if each element of its surface would send out light to *all* directions of external space; in so doing the intensity J of this apparent luminosity toward the different directions of external space (different directions of emanation) follows a law of the form

$$J=\delta \cdot F(i, \epsilon, \eta) \, . \tag{1}$$

Here δ signifies the intensity of the incident luminous radiation at the point of the surface under consideration (spatial density of the luminous energy), and i denotes the angle of incidence of these rays. ϵ denotes the angle of emanation, and η the azimuth of this direction reckoned in the tangential plane from the plane of incidence (azimuth of emanation). F is a function peculiar to the body.

In what follows we shall denote this phenomenon, which has been hitherto called diffuse scattering or diffuse reflection, as briefly "diffection." *Per contra*, we shall use the simple term "reflection" to denote the phenomenon ordinarily called regular reflection. This is characterized by the fact that J differs from zero only for a single direction of emanation, namely, for $\eta=180^\circ$ and $\epsilon=i$. This value is $J=\delta \cdot f(i)$, where f is a function peculiar to the body. According to Fresnel's law of reflection (the correctness of which we shall not presuppose in what follows) the function f has, as is well known, for all polished bodies the following form:

$$f(i)=\tfrac{1}{2}\left[\frac{\tan^2(i-r)}{\tan^2(i+r)}+\frac{\sin^2(i-r)}{\sin^2(i+r)}\right], \qquad \sin r=m \sin i$$

where m is a constant of the body (the reciprocal of the index of refraction).

The existing theories of "diffection" concern themselves almost exclusively with the case where the function of "diffection" F in

equation (1) does not contain the azimuth of emanation η, so that it is reduced to $F(i, \epsilon)$. The apparent emission of light from an element of surface of such a body is therefore equally strong for all directions of emanation which form a circular cone about the normal. We shall say in this case that the diffiection is circular. Several different expressions have been proposed for the form of the function F, of which the most important for theoretical investigations are the law of Lambert and that of Lommel-Seeliger. The first of these constitutes merely an assumption not thoroughly founded, the latter is analytically derived from certain plausible conceptions of the cause of the phenomenon of diffiection. The first contains one undetermined constant, the second two. Experimental investigations on different diffiecting substances have, however, shown that in the first place many of them do not "diffiect" in a circular manner at all, and, second, that even in circular diffiection neither the Lambert, nor the Lommel-Seeliger, nor any other law has unlimited validity, but rather that different substances follow different laws of diffiection.

Bouguer formed a conception of the physical cause of diffiection which unquestionably must appear at the first glance as a thoroughly plausible explanation of the phenomenon. He assumes that each element of surface consists of countless infinitely small mirrors which are pointed toward all possible directions. H. von Seeliger has more recently tested Bouguer's hypothesis analytically and established the fact that it is impossible to determine the frequency function of the orientations of the mirrors, and the law of reflection f which holds for the mirrors so that there shall result a diffiection according to Lambert's or according to the Lommel-Seeliger law.

It will be proved in what follows that the phenomenon of circular diffiection cannot be explained by the Bouguer hypothesis, *whatever may be the special law of diffiection of the given body* (excluding the law $F(i, \epsilon) = \text{constant}$).

Imagine around the position (P) of an element of surface df a sphere constructed with any selected large radius and mark on this spherical surface by the points N, S, O (Fig. 1, view from above), the directions: of the external normal of the element, the direction

meeting the incident rays, and the direction toward the point of observation. If PM is the direction which falls just in the middle between the two last directions, then it is clear that of all the countless little mirrors of which df is constituted, only those can send light to the point of observation whose normals have just the direction PM. If we designate the number of little mirrors of this orientation

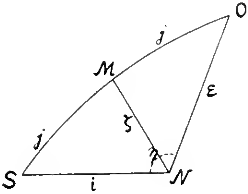

FIG. 1

contained in the element df by ndf, then we shall have to assume that n is a function (unknown) of the zenith distance ζ of the direction PM, but is independent of the azimuth of this direction. If the average area of a mirror is σ, then we may place for the total surface $ndf \cdot \sigma$ of mirrors oriented toward M:

$$\phi(\zeta) \cdot df \, .$$

ζ may be expressed according to formulae of spherical trigonometry by the angle of incidence i of the rays with the normal of df, the angle of emergence ϵ, and the azimuth of emergence η. We have:

$$\cos 2j = \cos i \cos \epsilon + \sin i \sin \epsilon \cos \eta \tag{2}$$

and (from the formulae for $\cos i$ and $\cos \epsilon$ from the two triangles):

$$\cos \zeta = \frac{\cos i + \cos \epsilon}{2 \cos j} = \frac{\cos i + \cos \epsilon}{\sqrt{2}\sqrt{1 + \cos i \cos \epsilon + \sin i \sin \epsilon \cos \eta}} \, . \tag{3}$$

The angle of incidence of the rays in respect to the normal to the effective mirrors is j. If we designate the function of reflection by the symbol f, and if we assume Fresnel's law, we should put

$$f(j) \equiv \tfrac{1}{2} \left\{ \frac{\tan^2 [j - \text{arc sin}\,(m \sin j)]}{\tan^2 [j + \text{arc sin}\,(m \sin j)]} + \frac{\sin^2 [j - \text{arc sin}\,(m \sin j)]}{\sin^2 [j + \text{arc sin}\,(m \sin j)]} \right\}$$

then the intensity of the apparent luminosity of the element in the direction PO will be given by the expression

$$J_{\epsilon,\eta}=\delta\cdot\phi(\zeta)\cdot f(j)\,, \qquad (4)$$

where ϕ is an unknown function, while ζ and j are expressed by equations (3) and (2) in terms of i, ϵ, η.[1]

If the phenomenon of pure circular diffection should arise, expression (4) would remain unchanged, if O goes in a circle about N. Hence the following relation must exist between the functions ϕ and f,

$$\phi(\zeta)=\frac{\phi(0)}{f(0)}f(\zeta)\,,$$

which is found by comparison of the two values of J [$\delta\cdot\phi(0)\cdot f(i)$ and $\delta\cdot\phi(i)\cdot f(0)$] for $\eta=180°$, $\epsilon=i$, and for $\eta=0$, and $\epsilon=i$. Expression (4) therefore is transformed into:

$$J_{\epsilon,\eta}=\frac{\phi(0)}{f(0)}\delta\cdot f(\zeta)\cdot f(j)\,. \qquad (5)$$

The necessary and sufficient condition for pure circular diffection in respect to the reflection function reads, *the reflection function f must be so constituted that the expression*

$$\frac{\phi(0)}{f(0)}\cdot f(\zeta)\cdot f(j) \qquad (6)$$

is a function of ϵ alone for every given $i<90°$, but is independent of η (ζ and j being expressed in terms of i, ϵ, η, by means of equations (3) and (2)). (In this theorem the case of $i=90°$ is excluded because the relation $(\phi)0\cdot f(i)=\phi(i)\cdot f(0)$, which was used in the derivation of this theorem, resulted from the consideration that for a point of observation symmetrically opposed to S with respect to N, only those mirrors are effective whose normals are directed toward N; but this is not correct in the case $i=90°$, for then all those mirrors send light to the point of observation, the normals of which are directed to points of the vertical circle perpendicular to NS.)

In our further discussion we shall limit ourselves, as is sufficient for our negative proof, to the consideration of the case that the

[1] The formulae developed to this point have already been given by von Seeliger. Their derivation is reproduced here for the convenience of the reader.

point O goes around N on the circle *passing through* S, whence $\epsilon = i$. ζ and j will then on account of their dependence on the azimuth of emanation vary as indicated by the following equation in which s denotes $|\sin i|$, and instead of the azimuth of emanation the new variable has been introduced, $\alpha = \left|\sin \frac{\eta}{2}\right|$:

$$\left.\begin{aligned} \sin j &= s\alpha\,, \\ \sin \zeta &= \frac{s\sqrt{1-\alpha^2}}{\sqrt{1-s^2\cdot\alpha^2}}\,. \end{aligned}\right\} \qquad (7)$$

In order that the intensity of illumination shall be the same in all directions which have the same inclination to the normal of the element as the direction of the incident ray, the function f must have the property that, when (7) is introduced in the expression (6), α is canceled; hence $f(\zeta)f(j)$ must then become a function of s alone. This function, however, as the consideration of the special case $\alpha = 1$ (O symmetrical to S) teaches, is nothing other than $f(0)f(i)$. Since we are concerned in every reflection function $f(i)$ only with values of the argument between 0 and $\frac{\pi}{2}$, we may regard each given reflection function as also a function of sin i, by setting $f(i) \equiv U(\sin i)$. Our condition therefore reads that the reflection function must have the property

$$U\left(\frac{s\sqrt{1-\alpha^2}}{\sqrt{1-s^2\cdot\alpha^2}}\right)\cdot U(s\alpha) = U(0)\cdot U(s)\,, \qquad (8)$$

and this identity must be fulfilled for all values of $s (0 \leqq s < 1)$ and for all values of $\alpha (0 \leqq \alpha \leqq 1)$.

We shall now seek for the general solution of this functional equation. If we differentiate (8) partially, first with respect to s and then with respect to α, and if we subtract the second equation after multiplication by $\frac{\alpha}{s}$ from the first equation, we obtain:

$$U'(q_{s,\alpha})\left[\frac{\partial q_{s,\alpha}}{\partial s} - \frac{\alpha}{s}\,\frac{\partial q_{s,\alpha}}{\partial \alpha}\right]\cdot U(s\alpha) = U(0)\cdot U'(s)$$

in which we have temporarily abbreviated by placing

$$\frac{s\sqrt{1-\alpha^2}}{\sqrt{1-s^2\alpha^2}} = q_{s,\alpha}\,.$$

If we introduce here for $U(s\alpha)$ its value from (8), and carry out the differentiation indicated in the square brackets, we get:

$$\frac{U'(q_{s,\alpha})}{U(q_{s,\alpha})}\cdot\frac{1}{\sqrt{(1-s^2\alpha^2)(1-\alpha^2)}}=\frac{U'(s)}{U(s)}\,.$$

If we now introduce a new functional symbol Ω defined by

$$x\frac{d\log U(x)}{dx}=\Omega(x)$$

we shall finally obtain:

$$\Omega\left(\frac{s\sqrt{1-\alpha^2}}{\sqrt{1-s^2\alpha^2}}\right)=(1-\alpha^2)\cdot\Omega(s) \tag{9}$$

as the condition which the sought-for function Ω shall satisfy for all values of $s(0\leqq s<1)$ and all values of $\alpha(0\leqq\alpha\leqq 1)$.

This equation has in fact a solution for the function Ω.[1] In order to find this function we differentiate (9) logarithmically, once partially with respect to s, the next time partially with respect to α. Thus we find, if we temporarily set $\log\Omega(x)=W(x)$, that the function W must satisfy *both* conditions (wherein $q_{s,\alpha}$ is the same abbreviation as before): first,

$$q_{s,\alpha}W'(q_{s,\alpha})=sW'(s)\cdot(1-s^2\alpha^2)\,;$$

and on the other hand

$$W'(q_{s,\alpha})=\frac{2}{q_{s,\alpha}(1-q^2_{s,\alpha})}\,;$$

or, in general,

$$W'(x)=\frac{2}{x(1-x^2)}\,,$$

where x may represent any value whatever between 0 and 1 $(0\leqq x<1)$. From the last equation it follows that $W(x)$ must equal

$$\log C\left(\frac{x^2}{1-x^2}\right)$$

[1] It may be incidentally remarked that a functional equation of this sort does not necessarily have a solution, but on the contrary it will generally not have one. Thus equation (9) would be impossible if in the right-hand member under the symbol Ω instead of s, for instance s^2, or some other definite function of s (multiples of s excepted), should occur.

(C being an undetermined constant); whereby the first condition is fulfilled. The solution of equation (9) is therefore:

$$\Omega(x) = \frac{Cx^2}{1 - x^2} .$$

Now since

$$\log U(x) \equiv \int \frac{\Omega(x)}{x} dx ,$$

we finally obtain, as the general solution of the functional equation (8)

$$U(x) = \frac{A}{(1 - x^2)^n} ,$$

where A, n are two undetermined constants.[1]

To return now to the reflection function f, we must place $x = \sin i$, and we obtain as the law of reflection

$$f(i) = A \sec^{2n} i = f(0) \cdot \sec^{2n} i . \qquad (10)$$

This law of reflection not only disagrees with that of Fresnel, but it is a priori impossible, at least if we disregard the special case $n = 0$, or $f(i) = \text{constant}$; and if we further demand that the reflection function shall be an increasing one. This last condition is justified by the consideration that the reflection function must take the

[1] The solution of the functional equation (8) can be accomplished in a somewhat shorter way, as Professor von Seeliger kindly indicated to me in a letter after reading this paper. Putting in general $U(x) = F(1 - x^2)$, and $1 - s^2 = u$, $1 - s^2\alpha^2 = v$, (8) becomes

$$F\left(\frac{u}{v}\right) = \frac{F(u)}{F(v)} \cdot F(1) . \qquad (a)$$

By logarithmic differentiation of this equation with respect to u and the application of the equation thus obtained to the special case of $\alpha = 1$, that is, $u = v$, it is easily found that

$$u \frac{F'(u)}{F(u)} = \frac{F'(1)}{F(1)} ;$$

therefore if we place

$$\frac{F'(1)}{F(1)} = -n ,$$

it follows that

$$F(u) = A \cdot u^{-n} ; \qquad (b)$$

A and n are arbitrary, as will be seen by introducing (b) in (a). The expression for function U found in the text follows from (b) immediately.

value 1 for $i=\frac{\pi}{2}$, while, on the other hand, it may not be larger than 1 for any value of i. If the function (10) shall be an increasing one, then the constant n must evidently be positive. But then with increasing i, the right-hand member increases *without limit* and must therefore with a definite value of i become greater than 1. A mirror would, therefore, with sufficiently oblique incidence reflect more light than it received; and the intensity could be indefinitely increased by inclining the mirror, an obviously impossible conception.

But if we regard as valid the law of reflection $f(i)=$ constant, then equation (4) does show that in this case circular diffection can occur; and as may readily be seen by applying (4) with $f(j)=$ constant to the motion of a point O about any circle around N, it will occur always and only if the $\phi(\zeta)=$ constant, that is, if an equal number of normals to the mirror point to each external direction of space. Equation (5) further shows that the circular difflection arising in this case is equal in all directions, and, moreover, that it is independent of the angle of incidence of the radiation illuminating the element with the normal to the element.

We may therefore say in summarizing: *No given law of circular difflection can be explained on the hypothesis of countless small mirrors*, with the exception of the law of "difflection" $J=\delta\cdot$ constant. This special case requires, however, for such an explanation, the assumption that the reflection at a mirror is independent of the angle of incidence with the normal to the mirror, and, further, that the surface of the illuminated element has such a structure that an equal number of normals to the mirror point to every direction of external space.

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PHOTOGRAPHIC PHOTOMETRY WITH THE 60-INCH REFLECTOR OF THE MOUNT WILSON SOLAR OBSERVATORY[1]

By FREDERICK H. SEARES

1. INTRODUCTION

The following is an account of the methods used for investigations in photographic photometry with the 60-inch reflector of the Mount Wilson Solar Observatory. It includes a statement of the underlying principles and the processes used to test their reliability, a description of the procedure for the measurement and reduction of the photographs, and a general indication of the precision of the results.

The size of the instrument fixes its field of greatest usefulness among the fainter stars; the large ratio of aperture to focal length (1 to 5) and the consequently limited field restrict its application to isolated regions or to relatively small areas of the sky. It is possible to secure photometric results in such abundance as to be of service for statistical investigations, but anything approaching a photometric survey would be inadvisable.

One of the most important applications of the instrument, and that with which this paper is principally concerned, is the determination of faint standards of magnitude. This imposes the condition that the methods employed shall be such as to provide reliable determinations of the scale. For the faint stars modification is required according as the exposures are moderate or long, and the desirability of comparing the scale for the fainter objects with that of the bright stars leads to further adaptation. We have thus to deal with three classes of objects: bright stars, whose lower limit is at the 10th magnitude; intermediate stars, including those between magnitudes 10 and 18; and faint stars, from the 18th magnitude to the faintest that may be registered with long exposure.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 80.

All the methods involve successive exposures upon the same plate. For the bright and intermediate stars these are of the same duration, one being made with the full aperture, the other with the intensity reduced by a known amount. The comparison of the images for the full and the reduced intensities leads, in the case of the intermediate stars, to the relation connecting size of image and magnitude which fixes the scale. For the bright stars it gives at once an extension of a scale assumed to have been previously established for the intermediate group. For the third class—the faint stars—the method involves again an extension of a known scale; but in this case the results are based on an application of the law of photographic action.

2. THE METHODS

a) Intermediate stars.—These are observable with what may be regarded as a normal arrangement of the program. Neither the intensity reduction nor the exposure required is extreme. Errors resulting from the use of large reduction constants are thus avoided, and, at the same time, the maximum exposure, which may be set at 30 or 40 minutes, is short enough to insure reasonable freedom from atmospheric disturbances. As an extension to either the brighter or the fainter objects presupposes a knowledge of the scale for the intermediate stars, the methods applicable to this class are considered first.

If I and I' represent the maximum and the reduced intensities, the reduction in magnitudes is

$$\Delta m = 2.5 \log \frac{I}{I'}. \tag{1}$$

Let $i_1, i_2, i_3 \ldots$ denote the full aperture images arranged in order of decreasing size; and, similarly, let $i'_1, i'_2, i'_3 \ldots$ be the corresponding reduced-intensity images. The differences

$$i_1 - i'_1, \quad i_2 - i'_2, \quad \ldots \; i_n - i'_n, \quad \ldots \; i_r - i'_r, \quad \ldots$$

expressed in some convenient unit, correspond to the magnitude interval Δm. We now make the important assumption that S_n, a star Δm magnitudes fainter than S_1, produces during the maximum-intensity exposure the same photographic effect as S_1

during the reduced-intensity exposure. By this assumption, $i_n = i'_1$. Similarly, if the star S_r be Δm magnitudes fainter than S_n, we have $i_r = i'_n$. In practice, the matter is reversed. If we observe the equalities of images indicated, we conclude that S_1 and S_n, S_n and S_r differ by Δm magnitudes. Let the brightness of S_1 be M. The magnitudes of S_n and S_r are therefore $M+\Delta m$ and $M+2\Delta m$, respectively, and we have established a scale for the stars in question. The scale thus derived is absolute, for the relation between star-intensity and magnitude is independent of assumed magnitudes. Usually the equality of images indicated is not exact; but the inclusion of this circumstance and the extension of the scale to other stars afford no difficulty. Both are accomplished by an interpolation process described in a later section.

To reduce the results to the International System, they must be referred to the standard zero point. This is defined by the Harvard magnitudes of stars of spectrum A0 whose brightness falls within 5.5 and 6.5 of the Harvard scale. The reference involves, in some form, a comparison of the calculated magnitudes with others already based on the International System. This may be variously accomplished, but the principles involved are so simple that they require no description.

We now consider the fundamental assumption: if the primary image i_n of a faint star equals the secondary image i'_1 of a brighter star, the two differ by Δm magnitudes, Δm being defined by equation (1), in which I and I' are the intensities active during the primary and secondary exposures, respectively.

As far as photographic phenomena are concerned, there appears no reason for doubt. The assumption is that equal photographic effects produced during equal exposure times must be caused by equal light-intensities. Disregarding local variations in sensitiveness, it is difficult to find an objection, although a question is perhaps raised by the phenomenon described below. The fact that the exposure times are equal avoids difficulties which would otherwise enter.

There are, however, at least two circumstances which may invalidate in some degree the fundamental assumption. One arises from the fact that most methods of reducing the intensity

involve a change in the aperture of the instrument; the other has its origin in atmospheric disturbances.

Any change in the aperture entails a corresponding modification in the diffraction pattern of the optical image. If the aperture be reduced by one-half, the diffraction disk and its surrounding rings will be doubled in diameter; and it is not certain that equal quantities of light distributed over unequal diffraction images produce the same photographic effect. Nor, conversely, is it possible to infer from the equality of the images i_1' and i_n that they have been produced by equal quantities of light. The question is one requiring special investigation before any method which involves a change in the diffraction pattern can be accepted as unobjectionable. It is discussed in the section on the diffraction effect.

The relation of atmospheric conditions to the fundamental assumption is obvious. Changes in transparency during either or both of the exposures modify the relative size of primary and secondary images. A similar disturbance may enter through irregularities in seeing. If these be greater during one exposure than the other, systematic differences between the images produced by equal quantities of light will occur. Both factors vitiate the determination of the scale by an amount which may be large or small according to circumstances. The difficulties are inherent in any method requiring successive exposures, but with a powerful instrument they are minimized by the relatively short exposures. With the 60-inch reflector, 10 or 15 minutes reaches the 17th photographic magnitude, and the trouble is not serious. Moreover, a repetition of the exposures in the reverse order favors the elimination of minor disturbances, such as those arising from changing zenith distance, and affords a test which reveals the presence of irregularities that are excessive.

Aside from the diffraction effect and the influence of atmospheric conditions, there is a third source of error which may affect results derived from successive exposures. It relates to a phenomenon noted by Pickering.[1] When several exposures are impressed

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 64, p. 7; *Astrophysical Journal*, 36, 374, 1912.

on the same plate, the first and last being short and equal, the images of the first are often systematically brighter than those of the last, the differences amounting on the average to a quarter of a magnitude. The cause is obscure, though apparently photographic in origin. Although measures at the Harvard Observatory revealed the effect on some of the earlier Mount Wilson photographs, it does not appear to be appreciable upon others subsequently obtained.[1] Here again, a symmetrical arrangement of the exposures presumably would eliminate the greater part of the error, and, in any event, would serve as a valuable control.

Scale errors arising from the diffraction effect are constant for any given method of reducing the intensity. For wire gauze screens they are zero, since the central diffraction disk is the same with as without the screen. For the diaphragms, they are negligible, as will appear later. Errors caused by atmospheric disturbances, changing zenith distance, and the photographic phenomenon just described are systematic for a plate. The use of short exposures, symmetrically arranged, reduces them to a minimum. As some duplication of plates is necessary, they tend, in the final result, to become accidental.

b) *Faint stars*.—Atmospheric disturbances set a limit to the exposure times that may be used when successive exposures are employed. The limit depends upon the character of the night. Fifteen or twenty minutes may be used upon any night suitable for photometric work. Little difficulty has been experienced in prolonging the exposures to forty minutes, and upon good nights they may be extended to an hour. But even the maximum limit excludes the possibility of extending the scale to the faintest stars. To meet this difficulty the method now to be described is available.

A knowledge of the scale for the intermediate stars in the same region is presupposed. To extend this to fainter objects, plates of long exposure, preceded or followed by a single short exposure, both with the full aperture, are used. The method involves a change in the energy acting on the plate by a variation in the exposure instead of the intensity. The change is known, but cannot

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 64, p. 12; *Astrophysical Journal*, **36**, 370, 1912.

easily be utilized on account of the complexity of photographic action with different exposures. The reduction constant, therefore, enters as an unknown whose value is to be determined from the stars of known brightness. But the question arises whether, for these conditions, the constant has a specified value. In order that such may be the case, the magnitude difference in the photographic effects produced by the light energies It and It' must be constant, t and t' being the exposure times.

This implies that

$$\Delta m = f(It') - f(It) \tag{2}$$

is independent of I. Let I' be an intensity such that during the second exposure it produces a photographic effect equal to that produced by I during the first. Then

$$f(It) = f(I't') ,$$

and by (2)

$$\Delta m = f(It') - f(I't')$$

whence

$$\Delta m = 2.5 \log \frac{I'}{I} . \tag{3}$$

Hence, the constancy of Δm also implies that the ratio of intensities producing the same effects in the two exposures shall be independent of the intensity.

The condition for constant photographic effect is[1]

$$t I \cdot 10^{\alpha} = \text{const.} \tag{4}$$

in which

$$\alpha = -a \sqrt{\log^2 \frac{I}{I_0} + 1} .$$

From (4)

$$\frac{I'}{I} = \frac{t}{t'} 10^{\alpha - \alpha'} \tag{5}$$

in which α' refers to the second exposure. The form of (5) is such that the ratio I'/I cannot be expressed independently of I. Consequently Δm is not constant, but has the form

$$\Delta m = 2.5 \left(\log \frac{t}{t'} + \alpha - \alpha'\right) .$$

[1] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **48**, 120, 1913.

If, however, I and I' are small as compared with I_0, we may write, taking the negative sign of the radical in order that $\alpha-\alpha'$ may be negative (a is assumed to be positive),

$$\alpha = a \log \frac{I}{I_0}, \quad \alpha' = a \log \frac{I'}{I_0}$$

whence

$$\alpha - \alpha' = a \log \frac{I}{I'}.$$

For this case, the condition for constant photographic effect (4) reduces to

$$tI^{1+\alpha} = tI^q = \text{const.} \tag{6}$$

and

$$\Delta m = \frac{2.5}{q} \log \frac{t}{t'}. \tag{7}$$

For small values of I and I', the correction to (7) is approximately

$$+\frac{a}{2q} \frac{\Delta m}{\log \frac{I}{I_0} \log \frac{I'}{I_0}}$$

and decreases as I and I' approach zero. Δm therefore decreases as fainter and fainter stars are considered and approaches (7) as a limit.

Now let

$$\begin{array}{l} \ldots\, i_0,\ i_1,\ i_2\ \ldots\ldots\ i_n,\ i_{n+1}\ \ldots\ldots\ i_r \\ \ldots\, i'_0,\ i'_1,\ i'_2\ \ldots\ldots\ i'_n \end{array}$$

represent the primary and secondary series of images arranged in the order of decreasing brightness, those with the same subscript being of the same star. The problem is to find the magnitudes for the faint images $i_{n+1} \ldots i_r$. For simplicity, suppose that, when written in the form,

$$\begin{array}{lllll} \ldots\, i_0,\ i_1,\ i_2\ \ldots\ldots & i_{n-1}, & i_n, & i_{n+1} & \ldots\ldots\ i_r \\ \ldots\ldots & i'_0, & i'_1, & i'_2 & \ldots\ldots\ i'_n \end{array}$$

the images in the same vertical line are of the same size. The magnitude difference of any pair of images thus aligned is, by (3), Δm. The absolute magnitudes of the stars o to n are assumed to be known. Denoting them by m with corresponding subscripts,

we obtain from the pairs to the left of the vertical line in above arrangement

$$. \quad . \quad . \quad . \quad \Delta m_0 = m_{n-1} - m_0 , \quad \Delta m_1 = m_n - m_1 . \tag{8}$$

For the faint stars with which we are concerned, (7) is approximately satisfied; the change in Δm is slow and nearly linear, and it is possible to extrapolate for the intensities $i_{n-1} \ldots . i_r$ with some precision. To find the magnitudes for stars $n+1$ to r, we have only to add the extrapolated values of Δm to the magnitudes of stars 2 to n. The method is more precise than an extrapolation of the relation between scale reading and magnitude for the change in m as s increases is much less regular than the variation of Δm.

Since the error in (7) is proportional to the reduction constant Δm, the scale extension should not be too great. The shorter the interval the greater will be the reliability; but if too greatly restricted, the attainment of the lowest limit will become laborious. Two magnitudes, corresponding to a ratio of about 10 to 1 in the exposure times, is a satisfactory value.

c) Bright stars.—It is not often necessary to use a powerful instrument for the determination of the magnitudes of bright stars. But for the connection of intermediate stars with the brighter objects it is desirable that a method for an upward extension of the scale should be available. Modification of that described above is necessary, since the images of bright stars are unmeasurable even with very short exposures. Further difficulty is introduced by the fact that the field of the reflector limits the number of bright objects that may be photographed with a single exposure. The required modifications are described in "The Photographic Magnitude Scale of the North Polar Sequence."[1] The method is most conveniently applied when the scale has already been established for an adjacent group of intermediate stars. An exposure is made upon the bright stars with an intensity reduction sufficient to produce images of an apparent brightness falling within the region of known magnitudes. A second exposure, with the full aperture, is then made upon the intermediate stars, and finally, the first exposure is repeated. The subtraction of the reduction constant

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 70; *Astrophysical Journal*, **38**, 241, 1913.

from the apparent magnitudes of the bright stars, found by comparison with the fainter objects, gives the required result. Although the exposures are short (two minutes) the method is tedious on account of the small field of view; but it has been found useful in establishing the scale over a long interval, and has the advantage of giving results that are homogeneous with the adopted scale for the intermediate stars.

It will be seen that some of the difficulties encountered affect all methods involving successive exposures. Although these can be avoided in part by a proper arrangement of the observations, there remains the objection that such methods are wasteful of time. Were it possible to secure the full and reduced intensity exposures simultaneously, the observing time could be reduced by one-half. Such a possibility is afforded by the method of prismatic companions, proposed and used by Pickering.[1] A trial has not been possible, owing to the lack of a suitable prism, which, with the reflector, would have a minimum diameter of 14 inches. It is desirable, however, that the method be employed, both as a control upon processes now in use and because of the saving in time. Its usefulness for the determination of very faint magnitudes is obvious.

Another method with important advantages, especially in the elimination of atmospheric disturbances, involves the use of a screen covering one-half of the plate.[2] Since there is no known absorbing substance which is neutral in tint, wire gauze a little in front of the plate is employed.[3] The method cannot, however, be used advantageously with the reflector. With a grating covering one-half the plate, and at such a distance as to separate the central image from the spectra, the penumbral region is so large as to include a considerable fraction of the area of good definition.

3. MEANS USED FOR REDUCING THE INTENSITY

The clear aperture of the mirror is 59.5 inches (151.1 cm), but the entire area is not available on account of the Newtonian flat and its mounting. In order that the boundary of the shadow

[1] *Harvard Circular*, Nos. 150, 170.

[2] Kapteyn, *Plan of Selected Areas*, p. 27, 1906.

[3] Schwarzschild, *Astronomische Nachrichten*, **183**, 297, 1910.

may be regular, a central stop 23 inches (58.4 cm) in diameter is placed over the end of the tube. The free area is therefore an annulus with diameters of 59.5 and 23.0 inches.

The means used for the reduction of the intensity are mainly circular diaphragms and wire gauze screens. Trial plates were also made with a sector-shaped diaphragm having four 15° openings, with a screen of wire gauze placed in the cone of rays 30 cm from the focus, and with a 60° rotating sector mounted close to the plate. The sector diaphragm was used in studying the diffraction effect because of its radically different pattern as compared with circular apertures. The rotating sector was intended to afford further control of the diffraction effect, as the momentary and very minute disturbance of the full-aperture pattern by the edges of the sector is negligible. But the intermittent exposure introduces doubt. Schwarzschild[1] has shown that the cumulative photographic effect of a series of exposures is not the same as that produced by a single equivalent exposure. The question, however, is this: Is the photographic effect of n exposures of duration kt/n to an intensity I the same as that produced by a single exposure t to an intensity kI? If so, the reduction produced by the sector is equal to the ratio of the sum of the intermittent exposures to the total exposure, and this in turn to the angular aperture of the sector divided by 360°. A law strictly analogous to that of Talbot for visual observations would then hold. The question has been investigated by Weber at Munich, who finds that apparently such a law does hold, at least within certain limits.

Three scales thus established are in agreement with those found with the diaphragms. We may interpret the results in two ways: we may assume the validity of Talbot's law and conclude that the diffraction effect (sensibly zero for the rotating sector) is negligible for the diaphragms; or we may regard the negligibility of the diffraction effect as established by independent methods, and consider the results a confirmation of Talbot's law in its special application.

Table I gives a list of the various devices used for reducing the

[1] *Photographische Korrespondenz*, **36**, 171, 1899; *Astrophysical Journal*, **11**, 92, 1910.

intensity. Owing to the 23-inch central stop, which is always in position during photometric observations, the aperture with the 40- and 32-inch diaphragms are annuli, as is the case with the full aperture.

TABLE I

LIST OF DIAPHRAGMS, SCREENS, ETC.

Designation	Description	Reduction Constant
		Magnitudes
40	40-inch circular diaphragm	1.12
32	32-inch circular diaphragm	1.96
14	14-inch circular diaphragm	2.97
9	9-inch circular diaphragm	3.93
8	8-inch circular diaphragm	4.16
6	6-inch circular diaphragm	4.81
G	60-inch single thickness wire gauze screen	3.06
G_2	60-inch double thickness wire gauze screen	6.12
G'	14-inch single thickness wire gauze screen	3.10
G''	14-inch single thickness wire gauze screen	3.08
g	Small wire gauze screen 30 cm in front of plate.	3.06
S	60° sector diaphragm, 60 inches in diameter	1.95
RS	60° rotating sector mounted in front of plate	1.95

The reduction constants in the last column were calculated for the diaphragms from their areas relative to that of the full aperture. The methods used for determining the absorption of the screens are described in the following section. The 6-, 9-, and 14-inch diaphragms are necessarily placed to one side of the central stop. They are shifted from quadrant to quadrant between the exposures in order that different parts of the mirror may be used. The threads of the two thicknesses of G_2 make an angle of 45° with each other. G' and G'' are small screens used, either singly or together, with the 14-, 9-, and 6-inch diaphragms. The resulting reduction constants are approximately 6, 7, and 8 mags., or 9, 10, and 11 mags., according as one or both of the screens are employed. The corresponding designations are $14+G'$, $14+G''$, etc. Other combinations are also possible, for example, $40+G$, $32+G$, etc., which have approximately the same reduction constants as the 9- and 6-inch diaphragms. The possibility of obtaining the same constant by quite different means affords a control which has been useful in investigating the diffraction effect and is of importance in actual observations.

4. ABSORPTION CONSTANTS OF THE SCREENS

The screens are of bronze wire 0.2 mm in diameter, and have openings 0.16×0.28 mm. Although the material was the most uniform that could be found in the market, it was clear that something more than mere calculation would be required for the determination of the absorption. A laboratory investigation was accordingly undertaken with the results summarized below. As the large screens G and G_2 were in use on Mount Wilson, the investigation was based upon a number of pieces of the material from which they were made.

Ten of these, I, II, ... X, averaging 55×65 mm, came from the edges of the original strip. Three others, about 50 cm in diameter, were cut from the centers of the large screens in order that they might clear the mounting of the Newtonian flat when attached to the end of the tube. In each of these, three areas, A, B, C; D, E, F; G, H, I, each 12.5 cm in diameter, were investigated. The first two groups relate to the 14-inch screens G' and G'', which were subsequently constructed from two of the central pieces. Finally, J, a 12.5-cm area from the edge, was added, making 20 in all that were examined.

A screen of wire gauze acts as a pair of crossed gratings. When placed in front of an objective, the transmitted light is distributed over a complicated diffraction pattern, consisting of a central diffraction disk and a large number of spectra. For light emitted by a point source, the details of this pattern are visible. If the source be a luminous surface, the image is the resultant of an infinite number of overlapping patterns of the type described—a luminous surface also, but less intense than the source. The former case corresponds to the conditions of observation, but the latter can also be utilized for the determination of the absorption.

Expressed in magnitudes, the theoretical absorption for surfaces is

$$A_s = -2.5 \log p p_1 , \tag{9}$$

and for point sources

$$A_p = -2.5 \log p^2 p_1^2 = 2 A_s , \tag{10}$$

in which p and p_1 are the transmission coefficients of the crossed gratings. In other words,

$$p=\frac{b}{a}, \quad p_1=\frac{b_1}{a_1} \tag{11}$$

in which a and a_1 are the grating constants, and b and b_1 the spaces.

Equation (9) is rigorous, but (10) refers to ideal conditions not realized in practice. A_s is more easily measured in the laboratory than A_p. Were it not for the approximation of (10), A_p could be found by doubling the observed value of A_s. The deviations of observed values of A_p from those calculated by (10) have been investigated by du Bois and Rubens;[1] but their results are not immediately applicable. It was therefore decided to measure both A_s and A_p, as the deviation of the observed values from

$$A_p=2A_s\ ,$$

compared with an extrapolation of the results of du Bois and Rubens, would afford an excellent control upon systematic errors.

The surface absorption of pieces I to X was measured with a Lummer-Brodhun photometer. The point absorption was determined for all the pieces with an apparatus presently to be described. As a further control, the values of A_s for pieces I and V were calculated by (9) and (11) from micrometric measures of the constants and spaces.

The photometer results for A_s are in Table II, the weighted means being given in the last line. To guard against systematic errors, different arrangements of the bench were used. Series 5 and 6 were with a bench-length of 1084 mm; Series 8 with 2030 mm. The respective means of the 10 pieces for the two sets of conditions were 1.506 and 1.497 mags. Other variations in the conditions were also introduced without, however, any sensible variation in the results.

The bench photometer was also used to determine the values of A_s for two thicknesses of the wire gauze (threads at 45°). The results are in Table III. The calculated values in the third column

[1] *Annalen der Physik*, **49**, 593, 1893.

were obtained by adding the means for I and II, III and IV, etc., in Table II.

TABLE II

VALUES OF A_s DETERMINED WITH THE LUMMER-BRODHUN PHOTOMETER

Series	I	II	III	IV	V	VI	VII	VIII	IX	X
1	1.518	1.458								
2	1.526									
3	1.540	1.394								
4	1.515	1.428								
5	1.527	1.482	1.591	1.518	1.393	1.572	1.518	1.500	1.527	1.482
6	1.522	1.463	1.562	1.512	1.404	1.562	1.526	1.468	1.526	1.463
7	1.528									
8	1.532	1.422	1.564	1.530	1.374	1.559	1.537	1.460	1.526	1.465
Means	1.527	1.441	1.572	1.520	1.390	1.564	1.527	1.476	1.526	1.470

The values of A_s calculated from the micrometric measures were 1.518 (I) and 1.420 (V) mags. The observed values for the same pieces were 1.527 and 1.390 mags.

TABLE III

A_s FOR DOUBLE THICKNESS OF WIRE GAUZE

Pieces	A_s Observed	A_s Calculated	O−C
	Mags	Mags	Mags
I and II	2.987	2.968	+0.019
III and IV	3.050	3.092	−0.042
V and VI	3.000	2.954	+0.046
VII and VIII	2.987	3.003	−0.016
IX and X	3.000	2.996	+0.004
Means	3.005	3.003	+0.002

The internal agreement of the results for a single piece is satisfactory, but the deviations in the means for the different pieces are large. It is evident, however, that these are the result of irregularities in the mesh of the screen. That they show so clearly is due to the size of the areas measured, which were limited by the construction of the photometer to a diameter of about 20 mm.

The arrangement used for measuring the absorption for point sources is shown in Fig. 1. L_1 and L_2 are lamps fed from the same

circuit; D_1 and D_2 are small diaphragms placed at the principal foci of the objectives O_1 and O_2, whose apertures and focal lengths are 12.5 and 396 cm, respectively; C is a Lummer-Brodhun cube; E, an eyepiece; and S_1 and S_2, diffusing surfaces. The screen to be investigated is placed in the parallel beam at S. The dimensions are such that the central diffraction image of D_1 formed upon D_2 is a surface of appreciable diameter, but entirely free from the spectra. Light from the latter is obstructed by the diaphragm D_2, so that the central spot of the cube is illuminated only by light from the diffraction disk. The outer area of the cube is filled with light from L_2, whose distance d from the diffusing surface S_2 may be varied until the field of E is of uniform intensity. Comparison of the positions of L_2 for uniform illumination with and without the screen S, and the application of the inverse square law lead at once to the value of A_p.

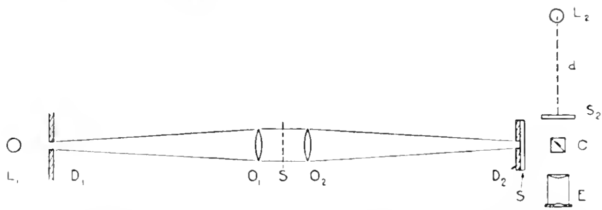


FIG. 1.—Arrangement for measurement of absorption of wire gauze screen for point sources.

The results obtained are in Table IV. For comparison, the values of $2A_s$ obtained by doubling the means in Table II are given in the fourth column.

The difference of 0.033 mag. between the means of A_p and $2A$ for I to X is in the direction and of the order required by the results of du Bois and Rubens. The difference between the mean A_p for I to X and that for A to J suggests a systematic difference between the edges and the middle of the strip from which the screens were constructed. The mean of the two results, namely 3.06 mags., was adopted for the 60-inch screen G, and twice this, 6.12 mags., for G_2. For G' we have the mean of A to C, 3.10 mags.; and

for G'', that of D to F, namely 3.08 mags. These are the values given in the last column of Table I along with the other reduction

TABLE IV

ABSORPTION FOR POINT SOURCES

Piece	A_p		2A from Table II	Piece	A_p	
	Series 1	Series 2			Series 3	Series 4
	Mags.	Mags.	Mags.		Mags.	Mags.
I	3.114	3.108	3.045*	A	3.106	3.082
II	2.952	2.906	2.882	B	3.104	3.096
III	3.096	3.060	3.144	C	3.121	3.101
IV	2.999	2.988	3.040	D	3.121	3.102
V	2.960	2.916	2.810*	E	3.083	3.052
VI	3.206	3.181	3.128	F	3.086	3.060
VII	3.131	3.134	3.054	G	3.030	3.074
VIII	2.976	3.002	2.952	H	3.100	3.104
IX	3.096	3.074	3.052	I	2.961	3.002
X	2.922	2.933	2.940	J	3.086	3.096
Means	3.045	3.030			3.081	3.077
	3.038		3.005		3.079	

* Mean of photometric and micrometric measures.

constants. The results for A_s serve only as a control and do not enter into the adopted constants.

5. ARRANGEMENT OF OBSERVATIONS

Photometric observations are made only during nights of uniform transparency and at least reasonably good steadiness. Bad seeing is not necessarily dangerous, unless irregular, although it increases the difficulty of measurement and prevents the registration of faint stars. Observations are discontinued when the seeing drops below 2 on a scale of 10.

The diaphragms and screens, with the exception previously noted, are attached to the end of the telescope. The practice of a symmetrical arrangement of the exposures is uniformly followed for reasons already described. For stars of intermediate brightness, the means commonly used for reducing the intensity are diaphragms of 32 and 14 inches, and the single-thickness screen G. The constant for the 32-inch diaphragm is approximately two magnitudes; that of the smaller diaphragm and the screen, three

magnitudes. For regular determinations of the scale, larger reduction constants are not practicable on account of the impossibility of properly establishing the relation between scale reading and magnitude. On the other hand, with reduction constants less than two magnitudes the scale is more seriously affected by errors in the constants. All things considered, a value of two magnitudes is a satisfactory compromise, though, as a control, it is important to include observations with other constants as well.

The order of the exposures is 60, 32, 32, 60; 60, 14, 14, 60; or 60, *G*, *G*, 60. Usually the inverse series is made upon the same plate as the direct, but occasionally, in the case of rich regions, a fresh plate is taken. A second plate is also used when the individual exposures exceed half an hour. In this case the photometric exposures on each plate are preceded and followed by short exposures of one or two minutes. The equality of the latter affords a partial test of the constancy of the atmospheric conditions. When the exposures are short, six or eight are often made on the same plate, thus: 60, 32, 14, 14, 32, 60; 60, *G*, 32, 9, 9, 32, *G*, 60; and other similar combinations. The 9-inch diaphragm included in the last arrangement is frequently useful in reaching stars near the upper limit of brightness, the main scale determination, however, being based upon the other secondary exposures.

For faint stars the arrangement, as already indicated, is very simple. It includes two exposures with the full aperture, one as long as necessary to reach the desired limiting magnitude, and the other short. The ratio of the exposure times may be taken as 10 to 1. It is desirable to have the shorter of sufficient length to register the faintest intermediate stars of known magnitude. On the other hand, the principal exposure should be such that the scale extension will not exceed two magnitudes.

To connect a very bright star with the known magnitudes of intermediate stars, greater reductions in intensity are required than can ordinarily be used. Combinations of diaphragms by means of which this may be accomplished are indicated in Section 3. The important point is that the reduced intensity of the bright star produce an image comparable in size with the full-aperture images of stars of known brightness. The determination is

strengthened by using combinations of diaphragms and screens having different reduction constants. For a fifth-magnitude star the following would be a convenient arrangement:

Bright star: G_2, $9+G'$, $6+G'$,
Intermediate stars: 60, 60, Exposure time, 2^m
Bright star: $6+G'$, $9+G'$, G_2,

Since the approximate reduction constants are, respectively, 6, 7, and 8 mags., the apparent brightness of the reduced intensity images will be in the neighborhood of 11, 12, and 13 mags., which fall well within the limits of brightness of the intermediate stars whose magnitudes are assumed to be known. The advantage of such a program lies in the possibility of using very different means for the reduction of the intensity, and in the fact that the calculated magnitude is made to depend upon different parts of the adopted scale for the intermediate stars.

The shifting of the plate between exposures is accomplished by a screw with a graduated head, which may be moved without changing the relative position of the guiding microscope and the guiding star. One revolution displaces the plate by 1 mm, which is the shift usually employed. Except in the case of bright objects, the position of the telescope with respect to the stars remains unchanged. The distance of a given star from the axis of the mirror is therefore the same for all exposures, which simplifies the correction for distance.

The range of adjustment for the guiding microscope is such that with reduction constants of three magnitudes or less there is rarely any difficulty in finding a star bright enough for guiding. Larger constants are used only with very short exposures of two or three minutes, and if a bright star be not available, the driving of the clock is usually of sufficient uniformity.

The plates used are Seed's "Gilt Edge, No. 27." The development is with Rodinal, 1 to 25, usually for 10 minutes at 20° C. To secure uniformity and save time, a tank is used, 12 plates being developed at a time.

It is of course obvious that with an appropriate plate and filter

the methods described may be employed for the derivation of magnitudes on the visual scale (photovisual magnitudes).

6. MEASUREMENT OF THE PLATES

The images are measured with a scale placed in the common focus of the ocular and objective of a low power microscope. The plate itself is mounted upon a movable stage which can be shifted to bring any given image to the center of the field. The scale was made by exposing a plate successively to the same star, the exposure times forming a geometrical progression whose ratio was 4/3. The images are numbered from 0 to 18 in the order of decreasing size. For the law of photographic action expressed by equation (6) the magnitude difference of two successive images is

$$\Delta m = \frac{2.5}{q} \log \frac{4}{3} = \frac{0.30}{q} . \qquad (12)$$

Since Δm depends only upon the ratio of the exposure times and the quantity q, which never differs greatly from unity, the relation between scale reading and magnitude is nearly linear, with one interval corresponding to about 0.30 mag.

The measures are made by bringing a star to the center of the field and shifting the scale until the images nearest in size stand on opposite sides of the star. The reading is the number of the larger image plus the tenths of a scale interval corresponding to the difference in size of the star and the larger image. A single estimate constitutes an observation, but all plates are measured twice, usually by different observers. For the faintest images, which show little or no difference in diameter, the photographic density is the determining factor in the estimate. It is a peculiar advantage of the method that objects at the limit of visibility can be utilized, which is impossible when the images are measured micrometrically; further, the effects of poor seeing seem to be less disturbing than with the micrometric method.

The stars are identified by means of an enlarged chart of the field. In the case of rich fields, omissions and confusion are avoided by subdividing the chart into small areas by concentric circles and lines radiating from the center.

Owing to the large and irregular correction for distance from the axis of the mirror, full aperture images at distances exceeding 25 mm (11′.3) are not measured. The distances themselves are expressed in units and tenths of a 5-mm interval, and are read either from the identification chart or directly from the plate by means of a translucent scale of concentric circles.

7. THE DISTANCE CORRECTION

The determination of the corrections which reduce the observed scale readings to the center of the field has given more trouble than any other part of the investigation. An average correction curve has been determined, but irregular deviations occur and are responsible for a large part of the error affecting the final results. Fortunately these are accidental in character; the determination of the scale does not appear to be influenced, and the main disadvantage is a slightly lower precision in the final magnitudes.

For a reflecting telescope the theoretical aberration in the optical image of a point source is[1]

$$\gamma = \tfrac{3}{16} D'' R^2$$

in which γ is the maximum diameter of the cone of rays in seconds of arc at its intersection with the focal surface for parallel rays. R is the ratio of the aperture to the focal length, and D'' the angular distance of the object from the axis in seconds.

For $R=1/5$ and $D''=680''$, the maximum distance at which full aperture images are measured, $\gamma = 5''.1 = 0.19$ mm; in the focal plane it is about 10 per cent larger still. As the diameter of the diffraction disk for a star on the axis is only $0''.075$, it is clear that the correction for distance will be large.

These figures do not, however, give the variation in the photographic image, as photographic irradiation, unsteadiness, and the distribution of intensity within the cone modify the result. Irradiation is greatest on the axis and tends to equalize the difference between axial and extra-axial images, although for bright stars the latter are always the larger. The intensity distribution is

[1] Poor, *Astrophysical Journal*, 7, 114, 1898.

such that most of the light is concentrated in a small section of the cone adjacent to the axis of the mirror. For faint stars the intensity in other parts of the cone lies below the threshold value; some of the light is thus lost, while on the axis the entire amount contributes to the photographic effect.

The dependence of the correction upon distance and brightness was determined by photographing successively, with constant exposure, the same field, the telescope being shifted slightly between the exposures. The images were measured with the photometric scale, and the readings for each star plotted against distance. The individual curves are sensibly rectilinear, but differ in slope. In accordance with the above, the inclinations for bright stars correspond to an increase in image size with increasing distance, while for faint objects the reverse is the case. For a certain intermediate brightness there is no variation, and the curves are parallel to the axis. The factor determining the slope is the size of image and depends, therefore, upon both the exposure and the brightness of the star.

To reduce the results to a useful form, the values of the correction (*D.C.*) corresponding to $D=5.0$ (unit = 5 mm) were read from the curves for a large number of plates. The corrections were expressed in scale intervals and arranged in the order of the readings of the images to which they refer. The means of all values within certain limits of scale reading gave the relation between brightness and correction, which is also nearly linear. The results for $D=5.0$ are shown in Fig. 2. They range from $+2.5$ to -2.0 scale intervals, or from approximately $+0.75$ to -0.60 mag. For any other distance the correction may be found by an application of the linear relation for distance with the condition that for $D=0$, $D.C.=0$. In practice the values are read from a table with the distance and the scale reading as arguments and applied directly to the observed scale readings.

Although it was presumed that photographs with the 60-inch screens would require the same correction as those without screens, the matter was specially investigated for the single thickness screen *G*. The result was a curve practically identical with that of Fig. 2. The 32-inch diaphragm was similarly investigated. In this case

the corrections are so small that usually they may be disregarded. With the smaller diaphragms a different procedure showed that the corrections are sensibly zero.

The irregularities to which reference has been made could not at first be traced to a definite cause. The influence of errors in focus and of variations in the energy, temperature, and duration of development were successively studied, without finding an adequate explanation. Finally, it was noted that the deviations for different photographs of the same region were similar, and correlated with the direction as well as with the distance from the axis. This suggested temperature deformations of the mirror as the source of the difficulty. As a decisive test, the instrument was exposed to

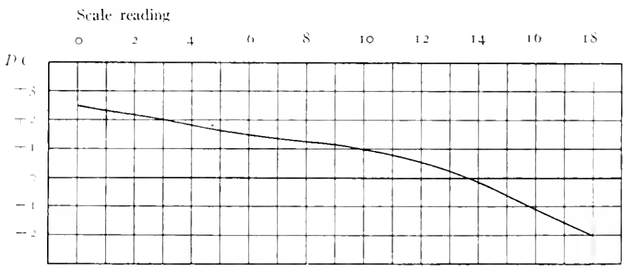


FIG. 2.—Distance correction in scale intervals for $D=5=25$ mm $=11'.3$

abnormal temperature conditions, and during the following night distance correction plates were made at intervals until normal figure had been regained. As a control upon the latter, extra-focal knife-edge photographs of the mirror were made by the method of Hartmann. The earlier distance correction plates showed the characteristic irregularities in an exaggerated form. With improvement in the figure of the mirror these decreased, and with normal figure the curve was approximately that given above. The location of the difficulty has made it possible to reduce the uncertainty; but it cannot be wholly removed, for small deformations, apparently without influence upon other kinds of observations, are appreciable in photometric work.

8. REDUCTION OF THE MEASURES

a) Intermediate stars.—After the measurement of the plates with the photometric scale, the mean scale reading is formed for the images of each star made with the same aperture. Thus, for the series of exposures represented by 60, 32, 32, 60, the two readings on each of the 60-inch images are combined. A similar combination gives the mean for the diaphragm images. Results requiring correction for distance are treated in the manner described in the preceding section. The difference in the corrected readings

$$s'-s=\Delta s \tag{13}$$

corresponds to the reduction constant Δm of the secondary exposure. Each of the brighter stars gives a relation of the form of (13), and the problem is to find

$$m=f(s) \tag{14}$$

subject to the condition

$$m+\Delta m=f(s+\Delta s)=f(s')\ . \tag{15}$$

An analytical solution has been given by Schwarzschild.[1] Usually Δs is a function of s, but the fact that the relation between magnitude and scale reading is nearly linear reduces the variation in Δs to a minimum. Not infrequently it may be regarded as constant. In this case, for full aperture

$$m=\alpha+\beta s \tag{16}$$

and for reduced intensity,

$$m=\alpha+\beta s'-\Delta m\ , \tag{17}$$

in which α is a constant depending on the zero point, and

$$\beta=\frac{\Delta m}{\Delta s}\ . \tag{18}$$

In any case, Δs is plotted with s as abscissa. If the curve shows Δs to be sensibly constant, the magnitudes are calculated at once by equations (16) to (18). Otherwise, the scale is calibrated by a process based upon the Schwarzschild solution. This is illustrated

[1] *Astronomische Nachrichten*, **172**, 65, 1906.

by the example in Table V, which relates to a plate with the 14-inch diaphragm and is chosen because of the unusually large variation in Δs.

TABLE V

Calibration of the Scale

Plate 808 P, Apertures 60 and 14, $\Delta m = 2.07$ mags.

s	Δs	s'	m_{60}		m	m_{14}
0	10.5	10.5	0.00		2.07	−2.07
1	10.5	11.5	0.28	28	3.25	2.69
2	10.4	12.4	0.57	29	3.54	2.40
3	10.3	13.3	0.85	28	3.82	2.12
4	10.2	14.2	1.13	28	4.10	1.84
5	10.0	15.0	1.42	29	4.39	1.55
6 . . .	9.7	15.7	1.70	28	4.67	1.27
7	9.3	16.3	1.98	28	4.95	0.99
8	9.0	17.0	2.26	28	5.23	0.71
9 .	8.6	17.6	2.55	29	5.52	0.42
10 .	8.2	18.2	2.83	28	5.80	−0.14
11	7.8	18.8	3.11	28	6.08	+0.14
12			3.41	30		0.44
13 .			3.73	32		0.76
14			4.06	33		1.09
15			4.42	36		1.45
16			4.81	39		1.84
17			5.23	42		2.26
18			5.71	48		+2.74

The process is begun by writing in the first column the scale numbers from 0 to 18. Opposite these are inserted the corresponding values of Δs read from the curve. The horizontal summation of the first two columns gives the third. Values of s and s' standing opposite each other correspond to the magnitude difference

Δm, which is the condition used for the derivation of the magnitudes in the fourth and fifth columns relating, respectively, to s and s'.

For small values of s, the Δs curve, in this case, is parallel to the axis. This implies that for s less than

$$s+\Delta s=1.0+10.5=11.5\,,$$

the relation of magnitude to scale reading is linear, with the proportionality factor β, by (18), equal to 0.283 mag. ($\Delta m=2.97$ mags., $\Delta s=10.5$). The products of β into the scale numbers 0 to 11 are therefore formed and inserted in the fourth column. To these are added the reduction constant 2.97, the results being entered in the fifth column. By the condition (14), the latter series of magnitudes corresponds to the scale readings in the third column, and it is now possible to find by interpolation the values of m for the integral values of s from 12 to 18. The results are entered in the fourth column, small arbitrary changes being made at two or three points in order to secure a greater regularity in successive differences.

With the data in the first and fourth columns, the magnitudes of all the stars on the plate may be obtained. For the diaphragm exposures it is necessary to subtract the reduction constant from the results of the interpolation in order that they may be referred to the zero point of the magnitudes given by the full aperture images. This is best done by subtracting once for all the constant from the tabular values of m_{60}. The results are in the last column under the heading m_{14}, and are to be used in interpolating the magnitudes for the secondary exposures. The zero point is fixed by the method used for finding the values of m_{60}. Its relation to the true zero point must be found by a comparison with stars of known magnitude.

The process described is readily adapted to such variations in the curve for Δs as have been found to occur in practice. The assumption of a linear relation between s and m can almost always be made for some part of the scale. Should this prove impossible, more general relations proposed by Schwarzschild may be employed.

In case more than one set of diaphragm exposures is made upon

the same plate, as for example with the arrangement 60, 32, 14, 14, 32, 60, two separate determinations of the scale are made, using the combinations 60–32 and 60–14. When the 9-inch diaphragm is also added, there are not ordinarily sufficient data to permit a separate determination; but the images thus secured are easily reduced by means of the scales established with the other diaphragms, and strengthen the results for the brighter stars which are always somewhat uncertain.

b) *Faint stars.*—The images of the two exposures are measured, corrected for distance error, and plotted with the known magnitudes as ordinates. The ordinates of the two curves thus obtained differ by Δm. Were the law of photographic action expressed by equation (6) rigorously true Δm would be constant. We have already seen, however, that the value of Δm decreases with decreasing intensity, and experience shows that its change is slow and nearly linear.[1] Its values are read from the sheet and extrapolated over the intensity interval of the faint stars registered by the long exposure. The addition of the extrapolated values to the ordinates of the short-exposure curve in the region of faint intensity gives a series of points through which the long-exposure curve may be produced, thus establishing the relation between magnitude and scale reading for the faint stars. The magnitudes of the latter are then read directly from the extended curve.

The following series of values from two plates indicate the character of the change in Δm:

Scale Reading	Pl 1332 P	Pl 1360 P
2	...	2.22
4	2.35	2.17
6	2.20	2.11
8	2.12	2.07
10	2.03	2.03
12	1.92	1.98
14	1.80	1.90

With this degree of uniformity, it is probable that the extrapolation over 5 or 6 scale intervals can be made with some accuracy, and the agreement of the results from different plates seems to justify this opinion.

c) *Bright stars.*—The process to be employed for the reduction is sufficiently indicated by the account of the method already given. Details and illustrative results may be found in "The Photographic Magnitude Scale of the North Polar Sequence."[1]

9. THE DIFFRACTION EFFECT

The relatively unimportant influence of a change in the diffraction pattern was established in two ways: first, by determining the increase in the diameter of star images produced by small errors in focus, and, second, by examining the accordance of the magnitude scales for the same group of stars established by the different methods of reducing the intensity.

a) *Errors of focus.*—When the aperture is decreased by a circular diaphragm, the diameter of the diffraction disk is increased in the inverse ratio of the apertures. The intensity is, at the same time, reduced by the equivalent of Δm magnitudes. Will the reduced intensity of a star S, distributed over the enlarged diffraction disk, produce the same photographic effect as the equivalent intensity of a star Δm fainter than S photographed with the full aperture? It is a question as to the equality of photographic effects when equal quantities of light act upon areas of the sensitive film differing in size. The matter may be studied by measuring the diameters of focal and slightly extra-focal images of the same star photographed with the full aperture and constant exposure.

Two series of photographs, 18 plates in all, were made with different focal settings, the true focus being determined with the knife-edge. Five or six two-minute exposures were made on each plate, and upon each, 15 stars, on the average, were measured with the photometric scale. The mean results for an image of average brightness are in Table VI. The first column contains the deviations from the true focus, and the second and third, the corresponding change in the image expressed in magnitudes. In forming the means in the fourth column, the second series was given double weight. The quantities in the last column are the means for equal positive and negative focal deviations.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 70; *Astrophysical Journal*, **38**, 241, 1913.

With the full aperture, the diameter of the cone of rays 0.005 inch from the focus is very nearly ten times that of the diffraction disk. The circular aperture giving a diffraction disk of this diameter is one-tenth the full aperture, or 6 inches. From Table VI it appears that the diffraction effect for such a diaphragm should be of the order of 0.03 mag. As the corresponding reduction constant is nearly five magnitudes, the resulting error in the scale must be very small.

TABLE VI

RELATION BETWEEN FOCUS AND SIZE OF IMAGE

ΔF (INCHES)	Δm Series I	Δm Series II	Δm Mean	MEAN EX-F. AND IN-F
+0.015	−0.19		−0.19	−0.15
+0.010	−0.19	−0.06	−0.10	−0.08
+0.005	−0.08	0.00	−0.03	−0.03
0.000	0.00	0.00	0.00	0.00
−0.005	0.00	−0.04	−0.03	. .
−0.010	−0.04	−0.09	−0.07	
−0.015	−0.09	−0.14	−0.12	

Strictly speaking, the matter should have been investigated separately for stars of different brightness, but the material available, when thus divided, is insufficient for a precise determination of the minute quantities involved.

b) *Accordance of magnitude scales.*—The best test of the diffraction effect is afforded by a comparison of results found with the wire gauze screens and the diaphragms. With screens the central diffraction disk is unchanged, and the results are free from the error in question. Assuming the screen constant to be known with precision, the comparison indicates at once the errors to be expected in using the diaphragms.

A series of photographs was made specially for an investigation of the question shortly after photometric work with the reflector was begun. At that time the importance of a rigid temperature control of the instrument was not realized and, as a consequence, difficulties arose in the correction of the observations for distance. Two separate reductions were made by entirely different methods. In the first, the distance error entered as an unknown. In the

second, it was found by comparing the full aperture images with the results of a preliminary reduction based upon exposures made wholly with diaphragms of 32, 14, 8, and 6 inches, for all of which the field of view is sensibly normal.

Although the mean scales from the two reductions differ considerably, the deviations of the results of the different methods of intensity reduction from the two means are small. The former difference arises from the uncertainty in the distance correction, and is of no great importance for the moment. The deviations from the separate means represent the systematic differences between the different methods for establishing the scale. These are shown in Tables VII and VIII, the unit being 0.01 mag.

TABLE VII

SYSTEMATIC DIFFERENCES—FIRST REDUCTION

Mean Mag.	60–G	60–32	60–14	60–8,60–6	32–8,32–6	No. Stars
0.61	−5	+8	−7	+ 8	+ 1	5
1.55	+6	+7	−2	− 5	−11	5
2.44	−1	+1	+6	+ 2	−12	10
3.58	+2	+1	+5	−13	+ 2	8
4.24	0	+1	−1	+ 2	− 2	9
4.74	−3	−2	0	− 2	+9	15
5.21	−1	0	−2	+ 4		12
5.53	+2	−3	0	0		12
5.98	+8	−4	−4	+ 2		7
No. scale determinations.	10	10	11	7	6	83 44

TABLE VIII

SYSTEMATIC DIFFERENCES—SECOND REDUCTION

Mean Mag.	60–G	60–32	60–14	60–8 60–6	32–14	32–8 32–6	60–*S*	60–*RS*	No. Stars	Average Deviation
0.38	+7	−6	−5	+1	+ 3	+1	+6	−3	9	±13
1.77	+4	−3	+5	−1	+ 6	−8	−4	−3	10	8
2.05	+2	+1	+1	−9	− 1	+1	−2	+1	10	10
3.77	+3	0	+3	+3	−11	−5	−6	−3	10	10
4.11	−2	+1	0	−2	+ 3	+9	−5	−6	10	11
4.45	−4	+3	−1	+3			−1	0	10	14
4.72	−3	+2	−1	+1			+3	+3	10	11
5.13	−4	+2	−4	+3			+9	+9	10	±15
No. scale determinations	10	10	11	7	5	7	2	3	79 55	$\pm 11^5$ ± 0.007 mag. P.E.)

The zero points of the mean magnitudes given in the first columns are arbitrary. The interval actually covered was from 9.5 or 10 to about 15. The column headings indicate the apertures used for establishing the scale whose deviations are given in the body of the table. These are generally small, and in only a few cases are they progressive. Even here the result is apparently due quite as much to the uncertainty in the distance correction as to other sources of systematic error, for it will be noted that the divergences for the 60–32 combination of apertures are in opposite directions in the two reductions.

Although the results for the sector diaphragm (*S*) and the rotating sector (*RS*) are of low weight, it is of interest to note that their deviations are not appreciably greater than the others. The diffraction pattern for the former is very different from that of a circular or annular aperture, and that there is no marked divergence may be regarded as significant. The interpretation of the results for the rotating sector has already been mentioned.

As a further test of the accordance of scales established by different methods of reducing the intensity, results from an unpublished investigation of the Polar Sequence are given in Table IX. In this instance the distance correction used was that found by the method of Section 7. The magnitudes were calculated by the interpolation process given in Section 8. The mean scale, at least as far as 15.5, is probably close to the true value, as the differences between the Mount Wilson and Harvard results shown in the second column are small.

As before, small progressive deviations are in evidence; but these are perhaps caused by errors peculiar to the plates, rather than by systematic differences inherent in the methods of establishing the scale. If, for example, all the deviations in Tables VII–IX relating to combinations of the full aperture with diaphragms of 32, 14, 8, and 6 inches be formed into one set of means, and those derived with 60 *G* and 60 *g* into another, we find the results of Table X, in which there remains little trace of progressive divergence.

The results in Tables VII–X relate to stars of intermediate brightness. For the internal agreement of scales found for the

brighter stars, reference may be made to "The Photographic Scale of the North Polar Sequence."[1] The mean scale for these objects is in less satisfactory agreement with that of the Harvard Observatory, but there is no evidence of appreciable systematic difference between the results found by the different methods of reducing the intensity.

TABLE IX

SYSTEMATIC DIFFERENCES—POLAR SEQUENCE STARS

Mean Mag.	Mt W. −H.*	60−*G*	60−*g*	60−32	60−14	60−8	60−*S*	No. Stars	Average Deviation
10.78	−2	−12	−6	+3	−2	+18	−12	7	±16
12.27	−4	0	−7	+3	−3	−6	−4	7	11
13.21	+3	−2	−5	−1	+7	+10	−1	10	13
13.91	0	+3	+5	−3	+5	−8	+14	11	14
14.44	+4	+6	+4	−2	+1	+4	+2	11	12
15.05	−5	+12	+5	−4	−4	−10	+9	11	12
16.07	−21	+19	+9	−6	−4	−21	+25	5	20
16.79	−35	+14	+14	−11	+1	−16	.	5	±13
No. scale determinations	. . .	5	2	13	4	2	3	67 29	±13.0 ±0.110 mag. (P.E.)

* Since the publication in *Contribution* No. 70 of a similar comparison with the Harvard results, the calculations have been revised and small corrections to the zero points of the separate scales have been applied. This and a different grouping of the stars accounts for the difference between the above results and those originally given. The number of stars in the next to the last column does not apply to the differences Mt.W.−H.

TABLE X

MEAN SYSTEMATIC DIFFERENCES BETWEEN DIAPHRAGMS AND SCREENS

Mean Mag.	Diaphragms	Screens	*D*−*S*	Mean Mag.	Diaphragms	Screens	*D*−*S*
0.52	+2	−4	+6	4.27	+1	0	+1
1.80	+1	+2	−1	4.77	0	0	0
2.80	+2	−2	+4	5.17	0	+1	−1
3.65	0	+2	−2	5.78	−2	+5	−7

In all of the foregoing comparisons it is, of course, to be understood that the errors of the reduction constants, as well as a possible diffraction effect, are included in the residuals. It seems to be reasonably clear that neither of these can be of any great importance.

As far as the diffraction effect is concerned, the result is perhaps what might have been anticipated. If we consider the ideal case

[1] *Loc. cit.*

in which the light is assumed to act only upon a region of the plate of the same size as the diffraction disk, we shall in all cases be dealing with areas smaller than the smallest star image. In other words, the greater part of the image is the result of photographic irradiation or other causes. This being the case, the actual size of the diffraction disk, within the limits considered, would presumably be a matter of minor importance. Actually, however, the area of the plate affected is larger than the diffraction disk, because of the oscillations of the image produced by atmospheric inequalities, and this tends to minimize the differences which might otherwise exist between large and small apertures.

10. PRECISION OF THE RESULTS

The time has scarcely arrived for a final estimate of the precision of results derived by the methods described. Nevertheless, as an indication, the quantities in the last columns of Tables VIII and IX are given. In Table IX these are the average deviations of a single magnitude, based upon the mean of two images on the same plate, from the mean established by 29 separate determinations of the scale. Fifteen plates were available with a possible maximum of 30 scale determinations, but the rejection of one set of diaphragm exposures reduced the number to 29. Over 1500 magnitudes are involved, and of this number, none were excluded in forming the means. There is some tendency for the deviation to increase as the ends of the scale are approached. The average for the entire series is 0.130 mag., corresponding to a probable error of 0.110 mag. The result includes not only the accidental errors, but also the systematic deviations of the individual scales from the mean for a range of six magnitudes.

To obtain an estimate of the latter, the differences between the magnitudes derived for each star from the full- and reduced-intensity exposures on each plate were formed whenever possible. These differences are free from scale-error, and their mean value without regard to sign is a measure of the remaining errors. From 400 such differences for the Polar Sequence stars a mean of 0.142 mag. was found. The probable error in a single magnitude, exclusive of scale error, is therefore 0.085 mag. The difference

between this and the entire probable error of 0.110 mag. is the part contributed by errors of scale. The probable error corresponding to the latter is 0.070 mag. This result is not unsatisfactory when the scale interval covered is considered, but the remaining error represented by 0.085 mag. is large. As already pointed out, much of this is doubtless due to temperature deformations of the mirror; the accordance of the scale readings is such that the error of estimate, which besides distance error is the only source of any considerable uncertainty, must contribute something less than one-half of the outstanding 0.085 mag.

Similar results follow from Table VIII. Here, however, the deviations were calculated for the mean of the magnitudes found with full and reduced intensity whenever the secondary image was registered, which occurs for about one-third of the cases. Taking this into account, the probable errors from Tables VIII and IX are in good agreement.

The Polar Sequence plates, as well as those for the study of the diffraction effect, were made under unfavorable conditions of temperature. Latterly the instrument has been as carefully protected as possible, and the results are more precise. As an illustration, there may be noted the probable errors, including scale divergence, for each of six selected areas. These are 0.082, 0.079, 0.069, 0.090, 0.109, and 0.111 mag., respectively. The last two are the same as that found above, but the others are smaller, and the mean of 0.085 mag. shows an appreciable gain in precision.

Each of these values is based upon an average of 300 magnitudes. Four plates were used for each area, and the deviations are referred to the mean of six or more determinations of the scale.

No detailed examination has yet been made of the relative precision afforded by the different methods of reducing the intensity. It seems likely, however, that there is not much preference in this respect between the different circular diaphragms and the screens, at least for reduction constants not exceeding three magnitudes. The smaller number of values of Δs obtained with large reduction constants is compensated for by the size of the constant itself. But beyond the limit mentioned, there is difficulty in calibrating the

photometric scale. The sector diaphragm is not to be recommended, for although accordant scales have been established by its use, it affects the definition, and the accidental errors are large. The rotating sector is probably not to be trusted without assurance that for the special conditions involved the equivalent of Talbot's law may be applied.

Mount Wilson Solar Observatory
November 25, 1913

THE RADIAL VELOCITIES OF ONE HUNDRED STARS WITH MEASURED PARALLAXES[1]

By WALTER S. ADAMS and ARNOLD KOHLSCHÜTTER

A part of the observing program of the 60-inch reflector for radial velocity determinations during the past three years has consisted of stars fainter than magnitude 5.5 on the visual scale for which observations of parallax are available. The measurement of the radial velocity completes the list of fundamental constants for such stars, and makes it possible to calculate their real motions in space. This fact is of especial interest since these stars are in general those nearest to the solar system, and may be expected, in view of their large proper motions, to show high average radial velocities. A knowledge of the spectral types of these stars is also of much importance, since their absolute magnitudes may be calculated with the aid of the parallax determinations.

The instrument employed for all of the observations has been the Cassegrain spectrograph adapted for use with one prism. Two cameras have been used: first, a lens with a focal length of 102 cm for stars between magnitudes 5.5 and 6.5; and second, a lens of 46 cm focal length for stars fainter than 6.5. The first is a special triplet made by Brashear, and the second a Cooke lens of the "Astrophotographic" type. Both lenses give excellent definition, but the field of the second lens is considerably the wider. As a result the measures which, in the case of the larger scale photographs, are limited to the region between H_γ and H_β, are extended for the smaller scale plates to the region between λ 4200 and H_β. The approximate linear scale of the photographs at H_γ is as follows:

102-cm camera............	1 mm = 16 angstroms
46-cm camera............	1 mm = 36 angstroms

The exposure times with the two cameras are nearly in the ratio of 5 to 1, allowing for a well-measurable width of spectrum in

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 70.

both cases. To illustrate the efficiency of the large reflector and spectrograph under good conditions of transparency and definition the following list of data for a few photographs obtained with the short camera is taken from the observing book. The definition is on a scale of 5.

TABLE I

Star	Mag.	Spectrum	Exposure Time	Slit-Width	Definition
			Min	mm.	
Lal. 27742	6.8	G2	27	0.038	3
W.B. 15^h720	6.8	G0	30	0.038	3
Groom. 1855	7.4	G8	61	0.038	3–4
OΣ 208	7.0	K0	75	0.038	3–4
Lal. 5761	8.0	A3p	77	0.038	3
Groom. 34	8.2	Ma	122	0.038	2–3
Lal. 21185	7.0	Ma	92	0.033	4

In the course of measurement we have found it desirable to select somewhat different stellar lines on the photographs taken with the two cameras. On the larger scale plates most of the lines used are of moderate intensity, and relatively few of them are direct comparisons with the iron lines of the comparison spectrum. With the small scale given by the short camera, however, the strong lines become so narrow that they are well measurable, and a considerable proportion of the lines used are direct comparisons with the stronger arc lines such as λ 4260, λ 4383, λ 4404, etc. Our experience has shown that in spectra of types F to K5 the use of such strong lines avoids many of the difficulties arising from uncertainties in the wave-lengths of blended lines. In stars of types K5 to Mb, on the other hand, the relative intensities of the stellar lines as compared with the sun are so greatly modified that even the strong iron lines may be affected by neighboring lines which blend with them, and much care has to be exercised in the selection of those to be used. The number of lines measured is usually from ten to fifteen.

In the tabulation of our results for the radial velocities of 100 of these stars we have adopted the plan of giving the values for the individual plates rounded off to the nearest kilometer. The mean

values for the long camera photographs are given to fractional parts of a kilometer, while for the small scale plates they are left in even kilometers. The fractional parts, however, are used in the formation of the mean values in both cases so that no error enters into the mean from this source. With the small linear scale of the spectrograms taken with the short camera it is of course evident that a fairly large range is to be expected among the separate determinations of radial velocity. The occasional discrepancies found among the long camera results are due in part to the character of some of the stellar spectra, and in part may represent actual variations of velocity. We do not, however, consider such variations as established from these results.

The spectral classifications given in Table II are made from our own photographs, the data already published for these stars being somewhat scanty. Occasionally large differences are found between our values and such as are given by the Harvard observers, and in these cases we have taken especial care to check the results. The Harvard system of classification is used throughout.

In the accompanying table (II) the right ascension and declination given for each star are for the epoch 1910. The magnitudes are taken from *Publications of the Astronomical Laboratory at Groningen*, No. 24, and are on the Harvard visual scale. The column v is the mean observed radial velocity, and v' is this velocity corrected for the sun's motion, assuming the apex at $\alpha = 270°$, $\delta = +30°$, and a solar motion of 20 km. The quantities μ, ψ, and π denote as usual the star's proper motion, the position angle of the proper motion, and the absolute parallax, and are taken from the Groningen publication already referred to. The absolute magnitudes given in the last column are also taken from this publication. The remaining three columns of the table denoted by A, Δ, and V give the right ascension and declination of the star's apex and its absolute motion in space. These quantities were computed with the aid of the convenient formulae derived by Dr. Wilson and published in *Lick Observatory Bulletin*, No. 214, where the values for a large number of the brighter stars with great proper motions are given. The formulae were adapted for logarithmic computation, and the

TABLE II

RADIAL VELOCITIES OF STARS WITH MEASURED PARALLAXES

Star	R.A. 1910	Dec. 1910	Mag.	Spec.	v from Separate Plates	Mean v	v′	μ	ψ	π	A	Δ	V	Abs. Mag.
					km	km	km						km	
Pi. 23^h267	$0^h\ 0^m.1$	+34°10′	6.2	G0	+ 0 + 4 + 2	+ 3.0	+ 0.5	0″.77	84°	+0″.01				
Brad. 3212	0 1.0	+28 32	6.2	K0	− 8 − 6 − 9	− 7.7	− 3.0	0.43	110	+0.15	232°	+10°	7	7.1
Groom. 34	0 13.0	+43 30	8.2	Ma	+ 4 + 1 + 3	+ 3	+ 9	2.85	82	+0.28	100	+29	35	10.4
26 *Androm.*	0 13.9	+43 17	6.0	B9	+ 9 + 7 + 6	+ 7.1	+ 13.2	0.04	63	−0.02				
Pi. 0^h137	0 34.5	+ 2 37	7.4	G3	− 28 − 29 − 30	− 29	− 31	0.79	68	+0.02	111	+26	170	
Lal. 1045	0 35.8	+30 43	7.5	K1	− 60 − 65 − 61	− 62	− 58	0.81	152	+0.02	50	−53	185	
Lal. 1299	0 43.6	+ 4 40	5.8	K2	− 12 − 11 − 12	− 11.8	− 14.2	1.34	148	+0.16	172	−62	28	6.9
Lal. 1966	1 4.1	+61 1	7.9	F3	−326 −325 −323	−325	−310	0.69	90	+0.08	188	−59	320	7.4
Fed. 263	1 34.8	+66 28	7.6	G4	+ 19 + 13 + 15	+ 15	+ 21	0.75	108	+0.04	94	+10	74	5.4
Pi. 1^h150	1 41.2	+63 25	5.7	K1	+ 5 + 3 − 2	+ 1.9	+ 7.5	0.62	113	+0.06	96	+ 6	31	4.7
Lal. 4141	2 10.2	+23 53	6.9	G2	+ 3 − 50 − 3 − 1	− 1	− 6	0.60	106	+0.03	123	−11	78	
Pi. 2^h123	2 31.1	+ 6 28	5.9	K2	+ 22 + 25 + 23 + 20	+ 22.5	+ 13.1	2.31	51	+0.14	120	+52	78	6.7
Lal. 4855	2 33.3	+30 28	7.2	G0	− 95 −103 − 99	− 99	−103	0.60	228	+0.02	287	−44	177	
Lal. 5496 6	2 56.7	+61 23	6.7	G1	− 9 − 7 − 6	− 7	− 4	1.00	134	+0.06	98	−23	66	5.4
Lal. 5761	3 3.1	+26 0	8.0	A3p	−143 −144 −144	−144	−151	0.90	102	+0.04	245	−55	170	6.0
Groom. 864	4 35.2	+41 58	7.3	G0	+101 +104 +100	+105	+100	0.73	127	+0.03	107	+12	141	..
Groom. 884	4 45.1	+45 42	7.1	F0	+ 24 + 26 + 27	+ 26	+ 22	0.68	145	+0.09	104	+17	29	6.9
Lal. 9091	4 46.2	+10 55	7.0	F5	+ 35 + 41 + 39	+ 38	+ 24	0.10	107	−0.02				
Lal. 9100	4 46.8	+13 30	6.7	F8	+ 35 + 36 + 43	+ 38	+ 24	0.13	117	+0.01				
Lal. 11196	5 50.9	+13 56	6.5	G1	− 4 + 1 − 2	− 2	− 16	0.69	137	+0.06	106	−38	49	5.3
W.B. 6^h1500	6 51.8	+ 1 17	7.7	G4	− 11 − 12 − 14	− 12	− 29	0.57	170	+0.01				
Pi. 6^h305	6 57.8	+29 30	6.0	F9	+ 22 + 20 + 19	+ 20.3	+ 10.6	0.82	168	+0.07	134	−40	42	5.1
Lal. 13840	7 4.8	+21 24	6.5	G7	− 11 − 17 − 15	− 14	− 26	0.52	197	+0.11	201	−35	27	6.6
Groom. 1281	7 9.2	+47 24	5.6	F9	+ 89 + 89 + 86	+ 88.0	+ 84.2	0.18	171	+0.07	107	+46	83	4.7
Lal. 15290	7 47.8	+30 54	8.2	F7	−247 −241 −239	−242	−250	1.07	158	+0.04	82	+62	316	6.5
Lal. 15565	7 54.9	+29 29	6.9	G4	+ 15 + 13 + 13	+ 14	+ 6	1.19	187	+0.09	117	−55	46	6.7
Pi. 7^h321	8 6.1	+32 45	7.0	G2	+ 26 + 30 + 25	+ 27	+ 20	0.82	213	+0.05	84	−29	62	5.4
Lal. 16304	8 14.1	−12 20	6.0	G0	+ 31 + 33 + 28	+ 31	+ 15	1.03	167	+0.13	55	−59	38	6.6
32 *Lyncis*	8 27.5	+36 44	6.1	F2	+ 2 + 2 + 1	+ 1.7	− 3.4	0.14	274	+0.09	294	+40	18	5.9
55 ρ^1 *Cancri*	8 47.2	+28 41	6.1	K1	+ 30 + 26 + 28 + 27	+ 27.6	+ 21.1	0.53	243	+0.08	90	+24	27	5.7

Lal. 18115 *Pr*	9 8 3	+53 5	7 9	K8	+ 8	+ 11	+ 13		+ 11	+ 12	1 69	248	+0 16	63	+12	36	9 0
Lal. 18115 *Fol*	9 8 3	+53 5	7 9	K8	+ 11	+ 13	+ 9	+ 8	+ 10	+ 11	1 69	248	+0 16	62	+11	36	9 0
Lal. 18286..	9 12 6	+28 58	7 3	G0	− 16	− 11	− 20		− 16	− 21	0 52	172	+0 01				
11 *Leo Min.*	9 30 3	+36 13	5 5	G7	+ 13	+ 9	+ 14		+ 12 2	+ 9 6	0 74	250	+0 04	66	− 6	72	3 4
20 *Leo Min.*	9 55 9	+32 23	5 6	G1	+ 55	+ 57	+ 56		+ 55 9	+ 53 7	0 68	230	+0 07	128	+15	60	4 9
Groom. 1618	10 5 1	+49 55	6 8	K7	− 32	− 30	− 28		− 30	− 28	1 45	249	+0 18	279	−38	33	8.1
39 *Leonis.*	10 12 4	+23 34	5 8	F5	+ 38	+ 39	+ 39	+ 40	+ 38 8	+ 35 6	0 44	258	+0 09	139	+34	38	5 7
Brad. 1433.	10 16 9	+41 41	5 9	F7	− 7	− 9	− 6	− 6 − 9	− 7 3	− 6 3	0 10	224	+0 11	328	+ 6	9	6 1
Groom. 1646	10 22 5	+49 18	6 5	F9	− 11	− 7	− 9	− 5	− 8	− 5	0 90	173	+0 08	194	−41	47	6 1
Pi. 10h96..	10 28 3	+49 39	7 6	F7	− 24	− 28	− 27		− 26	− 23	0 30	60	+0 04	285	+ 2	60	5 8
Groom. 1697	10 47 5	+70 20	6 1	K0	+ 19	+ 14	+ 18		+ 17 1	+ 24 7	0 42	260	−0 01				
Lal. 21185..	10 58 4	+36 35	7 6	Ma	− 91	− 84	− 85		− 87	− 85	4 77	186	+0 40	330	−66	96	0 6
51 *Leo Min.*	11 0 4	+25 42	7 5	G0	− 10	− 8	− 7		− 9	− 9	0 45	260	+0 04	63	− 3	37	5 4
Groom. 1757	11 11 6	+49 58	6 0	K0	− 3	0	− 2	+ 1	− 1	+ 4	0 09	256	−0 08				
Groom. 1774	11 19 0	+37 43	6 9	A0	− 12	− 14	− 12		− 13	− 9	0 06	259	+0 03	317	+ 5	14	4 3
Groom. 1812	11 34 0	+45 37	6 3	F9	− 19	− 15	− 19		− 17 6	− 11 8	0 63	273	+0 04	67	+ 1	60	3 2
Pi. 11h218..	11 57 9	+43 37	6 8	G4	− 13	− 15	− 11		− 13	−	0 67	213	−0 03				
Groom. 1855	12 5 1	+40 45	7 4	G8	− 3	− 3	− 3		− 3	+ 4	0 34	260	+0 06	103	+20	8	6 4
33 *Virginis.*	12 41 8	+10 3	5 8	K1	+ 51	+ 51	+ 51		+ 51 0	+ 55 8	0 54	148	−0 11				
Lal. 24414 6.	13 4 3	+ 5 43	6 9	G3	+ 21	+ 25	+ 21		+ 22	+ 28	0 72	175	+0 01				
Lal. 25224..	13 35 2	+11 12	5 5	A4	− 25	− 23	− 22		− 23 4	− 14 6	0 08	300	+0 26	341	+16	22	7 6
Lal. 26289.	14 18 6	+ 1 40	6 3	G0	− 14	− 21	− 16	− 19	− 17 8	− 7 7	0 55	156	−0 01				
Pi. 14h212.	14 52 2	−21 1	5 8	K6	+ 20	+ 20	+ 21		+ 20 1	+ 27 5	2 07	150	+0 17	296	−44	62	6 9
Lal. 27742 *Br*	15 8 8	+19 38	6 8	G2	− 34	− 33	− 39		− 35	− 20	0 68	296	0.00				
Lal. 27744.	15 9 3	− 1 0	6 7	G8	− 72	− 71	− 67	− 70	− 70	− 58	1 38	247	+0 13	79	− 7	67	7 3
ο *Serpentis.*	15 16 5	+ 1 2	5 5	K4	+ 11	+ 10	+ 11		+ 10 5	+ 23 8	0 12	209	+0 13	250	+14	27	6 1
Lal. 28358.	15 26 8	+57 45	6 9	F3	− 31	− 34	− 32		− 32	− 16	0 29	301	+0 03	101	− 8	38	
OΣ 298.	15 32 8	+40 8	7 9	K0	− 71	− 73	− 70		− 72	− 55	0 48	279	+0 05	90	−28	65	6 2
W.B. 15h720	15 32 9	+40 6	6 8	G9	− 66	− 75	− 68	− 73	− 71	− 54	0 48	279	+0 05	91	−28	64	5 1
Lal. 28607..	15 38 3	−10 39	7 3	A2p	−166	−168	−172	−175	−170	−158	1 17	253	+0 03	100	− 3	233	
Groom. 2305	16 1 8	+39 24	6 8	G5	− 60	− 60	− 59		− 60	− 42	0 56	275	+0 07	103	−29	50	6 1
Lal. 29437 8.	16 4 8	+ 6 39	6 6	K1	− 1	− 5	− 1	− 2	− 2 3	+ 13 9	0 79	162	+0 01				
Lal. 30024 6.	16 25 0	+18 36	7 0	G7	− 36	− 39	− 34		− 36	− 18	0 49	322	+0 09	85	+33	32	6 8
Lal. 30044 5.	16 26 1	+ 4 25	7 3	F6	− 49	− 54	− 54		− 52	− 35	1 46	198	+0 01	(161	−72	683)	
Lal. 30699	16 42 9	+68 15	7 8	G4	+ 7	+ 3	+ 11		+ 7	+ 22	0 47	328	+0 05	119	+52	39	6 2
Lal. 30604.	16 48 4	+ 0 10	6 8	G5	+ 38	+ 45	+ 41		+ 41	+ 57	1 66	206	+0 07	215	−51	117	5 9
Lal. 31132.	17 0 1	+47 12	6 7	G7	− 44	− 47	− 46		− 46	− 27	0 83	9	+0 14	61	− 8	36	7 4

TABLE II—*Continued*

Star	R. A. 1910	Dec. 1910	Mag	Spec	*v* from Separate Plates				Mean *v*	*v*′	μ	ψ	π	*A*	Δ	*V*	Abs. Mag.
					km				km	km						km	
Brad. 2179	17^h10^m7	−26 25	6 7	K3	− 7	− 12	− 13		− 11	0	1″20	204°	+0″30	158°	−22°	5	6 1
72 *w Herculis*	17 17 3	+32 35	5 4	G0	− 80	− 77	− 77	− 80	− 78 4	− 58 7	1 05	174	+0 12	63	−68	72	5 8
Fed. 2805.	17 25 3	+67 24	6 3	K1	− 41	− 38	− 35		− 38 3	− 22 5	0 53	273	+0 05	171	−27	54	4 7
Brad. 2388.	18 53 7	+32 48	5 2	G1	− 41	− 44	− 46		− 43 4	− 23 8	0 23	135	+0 02	11	−53	57	
Groom. 2789 N	19 9 8	+40 40	6 8	G0	− 44	− 40	− 34	− 38	− 39	− 21	0 62	348	+0 02	124	+31	144	
Groom. 2789 S.	19 9 8	+40 40	6 6	G0	− 43	− 40	− 39	− 41	− 41	− 23	0 62	348	+0 02	124	+30	144	
31 *b Aquilae*	19 20 7	+11 45	5 2	G7	− 96	− 97	−100		− 97 6	− 79 6	0 96	49	+0 06	80	+21	110	4 3
3 *Cygni*	19 21 7	+24 45	6 2	F4	− 4	− 7	− 5		− 5 0	+ 13 9	0 06	196	+0 07	271	−40	47	5.3
Groom. 2875	19 20 7	+55 24	6 7	K0	+ 10	+ 14	+ 11		+ 12	+ 30	0 66	232	−0 06	...			
Lal. 37120-1.	19 30 1	+33 0	6 6	G2	−162	−162	−164	−161	−162	−143	0 48	295	+0 06	310	+26	150	5.4
16 *Cygni Pr*	19 30 4	+50 10	6 3	G3	− 28	− 27	− 24		− 26 1	− 8 4	0 22	228	+0 16	216	−46	17	7 3
Lal. 38287	19 58 6	+15 23	7 2	G6	+ 9	+ 11	+ 15		+ 12	+ 29	0 60	198	+0 09	272	−22	42	6 9
Lal. 38380.	19 50 9	+29 40	5 7	K0	− 47	− 44	− 48		− 46 4	− 28 4	0 86	128	+0 04	25	−43	98	3 5
15 *Sagittae*	20 0 0	+16 50	5 9	G0	+ 2	+ 5	+ 3		+ 3 1	+ 20 3	0 56	227	+0 10	250	−10	37	6 0
Lal. 38383.	20 0 1	+23 7	7 2	G8	+ 3	− 7	− 3		− 2	+ 16	1 39	227	+0 03	232	−34	224	
Groom. 3042	20 3 8	+52 53	5 7	F5	− 43	− 45	− 37	− 39	− 41	− 24	0 33	42	+0 06	102	−23	29	4 5
Pi. 20^h23.	20 7 0	+15 56	7 3	G7	− 40	− 52	− 51		− 50	− 33	0 55	311	+0 08	161	+18	54	6 7
Lac. 8381.	20 9 7	−27 18	5 7	K6	− 58	− 57	− 60		− 58	− 50	1 27	101	+0 01	(41	− 6	592)	
Groom. 3150	20 16 5	+66 33	6 1	F8	− 11	− 4	− 4		− 6 1	+ 8 8	0 57	57	+0 12	32	+43	14	6 4
Groom. 3215	20 29 8	+41 34	7 0	G8	− 13	− 13	− 8		− 11	+ 6	0 48	340	+0 01				
Groom. 3263	20 38 4	+60 11	6 0	F0	− 10	− 11	− 14		− 11 6	+ 3 7	0 10	3	+0 02				
Groom. 3357	20 56 5	+30 53	6 6	F6	− 35	− 36	− 41		− 37	− 21	0 33	47	+0 08	129	− 8	25	6 1
H. B. 21^h07	21 7 9	+17 23	7 3	F5	− 48	− 46	− 43		− 46	− 32	0 91	188	−0 12				
Lac. 8777.	21 14 6	−26 44	6 5	G5	− 46	− 40	− 44		− 43	− 37	0 68	238	−0 10				
Brad. 2792.	21 22 1	+46 20	5 5	A2	− 5	+ 5	− 2	+ 50	+ 1	+ 16	0 20	72	+0 09	300	+52	16	5 2
24 *Aquarii*.	21 34 9	− 0 27	6 8	F8	− 20	− 16	− 16	− 17	− 17	− 7	0 24	82	−0 03				
Lal. 42883 5	21 54 8	+29 24	6 9	F7	+ 6	+ 11	+ 5	+ 5	+ 7	+ 20	0 56	223	+0 03	270	−25	98	
Lal. 43402	22 12 7	+12 26	6 9	G0	− 28	− 35	− 29		− 31	− 21	0 84	84	+0 02	171	+ 7	186	...
Pi. 22^h214.	22 41 5	+29 58	6 5	K0	− 5	+ 5	− 1	− 2	− 1	+ 9	0 47	214	−0 02				
Fed. 4371.	23 1 5	+67 56	7 5	G2	− 23	− 22	− 20		− 21	− 10	0 66	75	+0 06	102	− 8	33	6.2
Brad. 3077.	23 8 9	+56 40	5 6	K4	− 20	− 18	− 25	− 19	− 20 4	− 9 9	2 11	82	+0 16	275	− 2	47	6 6
Lal. 45755.	23 17 3	+43 34	7 6	G7	+ 2	+ 3	0		+ 2	+ 11	0 67	71	+0 04	92	+27	67	5 5
Pi. 23^h164	23 38 9	+57 33	7.0	F8	− 65	− 69	− 71		− 68	− 59	0 62	38	−0 08	...	...	...	...

values obtained have been checked by means of the following equations which are due to Professor Kapteyn:

$$x = v \cos\delta \cos\alpha - 4.74\,\frac{\mu \sin\psi}{\pi} \sin\alpha - 4.74\,\frac{\mu \cos\psi}{\pi} \sin\delta \cos\alpha$$

$$y = -17.3 + v \cos\delta \sin\alpha + 4.74\,\frac{\mu \sin\psi}{\pi} \cos\alpha - 4.74\,\frac{\mu \cos\psi}{\pi} \sin\delta \sin\alpha$$

$$z = +10.0 + v \sin\delta + 4.74\,\frac{\mu \cos\psi}{\pi} \cos\delta$$

$$V^2 = x^2 + y^2 + z^2. \quad \tan A = \frac{y}{x}. \quad \sin\Delta = \frac{z}{V}.$$

(sin A has the same sign as y)

The uncertainty in the value of the parallax of many of these stars of course affects very greatly the determination of their motions in space, especially when the parallax is small. Accordingly, when the direction and amount of motion of a star with a parallax of but a very few hundredths of a second are given, they are to be considered rather as representing what the values would be in case the observed parallax is assumed to be correct, and not necessarily the actual motion of the star.

The number of measures of each plate by different observers varies between one and five, the great majority being either two or three. All of the plates without exception have been measured by Miss Lasby, and a large proportion by Miss Ensign and by Adams. Miss Burwell has measured many of the more recent spectrograms, and some of the earlier results are due to Miss Waterman.

The computations have been made almost entirely by Miss Lasby, to whom we wish to express our hearty appreciation.

In addition to the values given in Table II the following determinations of the radial velocities of stars observed elsewhere are added for comparison:

TABLE III

Star	Mag	Spec	Separate Plates	Mean	Campbell
Groom. 1830..	6.5	G5	−09 −06 −06	−07.1	−07
61¹ *Cygni*...	5.6	K8	−65 −66 −65 −65	−65.2	−63
61² *Cygni*	6.3	K8	−69 −68 −62 −61	−65	

Some of the principal conclusions to be drawn from an inspection of the results shown in Table II are as follows:

1. A few stars show enormous radial velocities. Chief among these are *Lal.* 1966 and *Lal.* 15290, with velocities of −325 and −242 km. The first of these shows the highest radial velocity so far observed among any of the stars. In addition to these there are four stars showing velocities of over 100 km, and several with velocities between 75 and 100 km.

2. A peculiar fact is the great preponderance of large negative over large positive velocities. If we use for this comparison the velocities corrected for the sun's motion, given as v' in the table, and set 50 km as a limit, we obtain the following:

Positive (5)			Negative (15)					
Groom. 864	Go	+100	*Lal.* 1045	K1	−58	*W.B.* 15^h720	Go	−54
Groom. 1281	Fo	84	*Lal.* 1966	F3	319	*Lal.* 28607	A2p	158
20 *Leo Min.*	G1	54	*Lal.* 4855	Go	103	72 *w Herculis*	Go	59
33 *Virginis*	K1	56	*Lal.* 5761	A3p	151	31 *b Aquilae*	G7	80
Lal. 30604	G5	+57	*Lal.* 15290	F7	250	*Lal.* 37120–1	G2	143
			Lal. 21185	Ma	85	*Lac.* 8381	K6	50
			Lal. 27744	G9	58	*Pi* 23^h104	F8	−59
			OΣ 208	Ko	−55			

Accordingly no less than 75 per cent of the large velocities observed are negative.

3. An examination of the spectral types of the stars with great velocities shows that nearly all classes are represented among them. An interesting fact, however, is that the two stars with the largest velocities of all are of types F3 and F7, and the two stars succeeding these are of the A type. The spectra of these last stars, *Lal.* 5761 and *Lal.* 28607, are both peculiar in that the magnesium line λ 4481 is either absent or extremely faint; while in other respects the spectra are of the normal A2 or A3 type. The remarkably high and nearly equal velocities for two stars of nearly identical but peculiar spectra form a singular coincidence. The stars are widely apart in the sky, and the apices of their motions are quite different. The existence of high radial velocities among stars having what is generally considered an early type of spectrum is shown by these results, although there can be no doubt that such cases are rare. Professor Campbell found no certain case of a star

with a constant velocity exceeding 40 km in an investigation of 337 stars having spectra between types B and F4,[1] and in the course of our observations of about 350 stars of types B and A we have found but one star with a very large constant velocity. This star is 7 *Sextantis*, *Boss* 2647, and four determinations of its velocity give values of +97, +100, +93, and +94 km, with a mean value of +96 km. Its spectrum is A3 and normal in all respects. The proper motion is exceptionally large as well.

4. Two interesting stars in the list are *Groom.* 34 and *Lal.* 21185, both of which have very large proper motions and parallaxes. The motion of *Groom.* 34 is almost entirely across the line of sight, but *Lal.* 21185 shows the high radial velocity of −87 km. These stars are among the faintest stars known, having absolute magnitudes of 10.4 and 10.6 respectively, when the unit of distance is taken for a parallax of 0″01, or the sun with a magnitude of 5.5. Their spectra are both of type Ma.

5. The physical connection of two pairs of double stars in the list is almost certainly established by the radial velocities. These are *Lal.* 18115, where the components have a separation of 20″, and *Groom.* 2789, with a separation of 10″. The spectra of the two components are identical in each case. These stars belong to what is frequently known as the 61 *Cygni* type of double stars. A peculiar case is that of OΣ 298 and *W.B.* 15^h720, which are distant over 2′ from each other, but show common proper motions and equal radial velocities of −71 km. By a singular coincidence the direction of motion of these stars in space is almost exactly opposite to that of the sun. Of the other stars in the list only one, 32 *Lyncis*, has a direction of motion which is approximately parallel to that of the sun.

MOUNT WILSON SOLAR OBSERVATORY
December 8, 1913

[1] *Stellar Motions*, p. 198.

SOME PYRHELIOMETRIC OBSERVATIONS ON MOUNT WHITNEY

By A. K. Ångström and E. H. Kennard

In the summer of 1913 an expedition supported by a grant from the Smithsonian Institution proceeded to California in order to study the nocturnal radiation under different atmospheric conditions. In connection with these investigations we had an opportunity to measure the intensity of the solar radiation during seven clear days on the summit of Mount Whitney (4410 m). These measurements were made for different air masses and include observations of the total radiation and of the radiation in a special part of the spectrum, selected by means of an absorbing screen, as has been proposed by K. Ångström.[1] Our paper will present the results of the observations and a computation from them of the solar constant.

INSTRUMENTS

The observations were made with Ångström's pyrheliometer No. 158. With this instrument the energy of the radiation falling upon the exposed strip is given in calories per square centimeter per minute by the relation $I = kC^2$, where C is the compensating current sent through the shadowed strip, and k is a constant which was determined for this instrument at the solar observatory of the Physical Institute in Upsala and found to be 13.58.[2] The compensating current was furnished by 4 dry cells, which proved entirely suited to the purpose. It was measured by a Siemens and Halske milliammeter. For further details of the instrument and the method of using it, we refer to the original paper.[3]

The absorbing screen, used in order to study a limited part of the spectrum, was composed of a water cell, in which the water layer had a thickness of 1 cm, and a colored glass plate, Schott

[1] *Nova Acta Reg. Soc. Sc. Upsal.*, Ser. IV, 1, No. 7.

[2] A comparison made at the Smithsonian Institution in Washington showed that the readings of this instrument are 4.57 per cent lower than the Smithsonian scale.

[3] *Astrophysical Journal* 9, 332, 1899.

and Genossen, 436[III], the thickness of which was 2.53 mm. The transmission of the combination for different wave-lengths as previously determined at Upsala by Mr. A. K. Ångström is given in Fig. 1. The maximum of transmission occurs at wave-length 0.526 μ, and 85 per cent of the transmitted light is included between 0.484 μ and 0.576 μ.

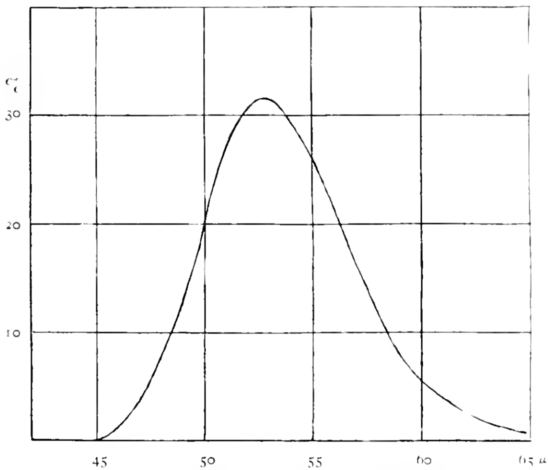


FIG. 1.—Transmission curve of absorbing screen

The local time of each observation, from which the sun's zenith angle and finally the corresponding air mass was computed, was determined from the readings of three watches. Before and after the expedition to Mount Whitney, the watches were compared with the daily telegraphic time signal at Claremont, California. The time is probably accurate within half a minute.

RESULTS

The results are given in Tables I and II. Table I refers to the measurements of the total radiation and contains: (1) the date, (2) the local apparent time, (t), (3) the computed air mass, (m),

(4) readings of the milliammeter, (s), (5) the total radiation computed from the readings. Table II contains the same quantities relating to measurements taken with the absorbing screen.

Bemporad's[1] expression for the air mass in terms of the apparent zenith angle was employed. His values for 60°, 70°, 80°, and 85° were available in a short table given by F. Lindholm.[2] The differences between these values and the secant of the angle give

TABLE I

Measurements of Total Radiation

	t	m	s Milliamp $\times \frac{1}{3}$	Q_m cal/cm² min
August 2	6h 34m.2	3.337	100.1	1.224
	6 49.2	2.872	102.5	1.287
	7 30.7	2.088	106.3	1.381
	8 13.2	1.657	108.8	1.446
	9 20.7	1.299	111.3	1.514
August 4	6 28.3	3.630	99.4	1.202
	6 58.8	2.672	104.0	1.322
	7 6.8	2.501	104.1	1.325
	8 4.3	1.741	108.6	1.441
	9 6.8	1.359	110.5	1.493
	11 0.3	1.089	111.7	1.520
	11 8.8	1.081	112.0	1.533
August 5, A.M.	6 29.5	3.608	97.8	1.169
	7 2.0	2.616	103.0	1.206
	7 48.0	1.906	107.0	1.309
	8 59.0	1.397	110.6	1.405
	10 0.5	1.190	111.1	1.508
August 5, P.M.	2 0.3	1.103	111.2	1.511
	3 3.3	1.410	109.5	1.465
	4 4.3	1.830	106.3	1.381
	4 33.8	2.185	104.2	1.320
	5 4.8	2.783	100.1	1.224
	5 24.3	3.377	96.6	1.141
August 10	6 33.0	3.630	95.6	1.117
	7 3.0	2.681	100.5	1.235
	7 56.5	1.857	105.5	1.360
August 11	6 27.1	3.952	96.9	1.147
	6 54.6	2.914	101.9	1.269
	7 40.1	2.053	106.0	1.373
	8 41.6	1.514	109.3	1.460
	10 13.1	1.177	111.7	1.525
August 12	6 26.6	4.018	98.1	1.176
	6 59.1	2.817	103.6	1.312
	7 55.1	1.889	108.5	1.430
	8 57.1	1.435	111.1	1.509
	10 43.6	1.127	113.0	1.561

[1] *Mitteilungen der Grossherzoglichen Sternwarte zu Heidelberg*, No. 4, 1904.

[2] *Nova Acta Reg. Soc. Sc. Upsal.*, Ser. IV, 3, No. 6.

TABLE II

MEASUREMENTS WITH ABSORBING SCREEN

	t	m	Milliamp. × 2	Q_m $\frac{\text{cal}}{\text{cm}^2 \text{ min}}$
August 2	6h 18m.2	4.044	104.5	0.0371
	6 54.7	2.733	114.1	0.0442
	7 25.7	2.158	122.0	0.0505
	8 22.7	1.589	125.4	0.0534
	8 31.7	1.530	126.8	0.0546
	9 15.2	1.319	128.8	0.0562
August 4	6 16.8	4.204	103.1	0.0361
	6 36.3	3.316	112.1	0.0426
	7 11.8	2.406	118.9	0.0480
	8 0.3	1.699	125.3	0.0533
	9 19.3	1.311	128.0	0.0556
	11 13.3	1.077	129.9	0.0573
August 5, A.M.	6 17.0	4.237	101.8	0.0352
	6 36.0	3.352	108.8	0.0402
	8 3.5	1.755	123.1	0.0515
	9 5.5	1.368	127.9	0.0554
	10 7.0	1.175	129.4	0.0568
August 5, P.M.	2 6.8	1.209	129.3	0.0567
	3 12.8	1.457	126.7	0.0545
	4 11.8	1.907	122.4	0.0509
	4 40.3	2.287	118.3	0.0475
	5 10.3	2.928	114.1	0.0441
	5 30.3	3.615	106.6	0.0386
August 9	6 14.4	4.607	96.0	0.0313
	6 33.9	3.559	103.4	0.0363
	11 38.9	1.126	128.8	0.0563
August 10	6 21.5	4.211	100.6	0.0344
	6 38.0	3.428	106.7	0.0387
	7 8.0	2.570	113.8	0.0439
	8 2.0	1.804	122.4	0.0508
	8 6.0	1.767	122.0	0.0505
August 11	6 14.6	4.716	102.5	0.0356
	6 33.6	3.641	107.9	0.0395
	7 0.1	2.770	114.6	0.0445
	7 45.1	1.992	122.1	0.0507
	8 51.1	1.462	127.1	0.0549
	10 18.6	1.166	129.9	0.0573
August 12	6 13.1	4.805	99.1	0.0333
	6 34.1	3.656	108.0	0.0397
	7 5.1	2.671	116.4	0.0459
	8 3.6	1.804	123.5	0.0517
	9 2.6	1.409	128.1	0.0557
	10 52.6	1.115	131.5	0.0587

the (negative) corrections to be applied to the secants of these angles. Through these values of the correction an algebraic curve of 4 terms was passed and the correction was then calculated for other angles. In obtaining the apparent zenith angle, allowance was made for refraction.

GENERAL DISCUSSION OF THE EMPIRICAL METHODS FOR COMPUTING THE SOLAR CONSTANT

Empirical methods for determining the solar constant from pyrheliometric measurements alone have been proposed by K. Ångström[1] and by Fowle.[2] Both these methods are based upon results obtained from spectrobolometric observations. Ångström's method assumes that from Abbot and Fowle's observations we know both the distribution of energy in the sun's spectrum and the general transmission of the atmosphere for all wave-lengths in terms of its value for any given wave-length. It assumes further that the absorption caused by the water-vapor is a known function of the water-vapor pressure at the earth's surface; for this Ångström proposed an empirical formula based upon his spectrobolometric curves. The influence of diffusion and absorption can then be calculated if the transmission for some chosen wave-length is known from pyrheliometric observations on a limited part of the spectrum.

Fowle's method is much briefer. He plots the logarithms of the observations against the air masses and extrapolates to air-mass zero by means of the straight line that best fits the points. To the "apparent solar constant" thus obtained he applies an empirical correction depending upon the locality, and derived from local spectrobolometric observations.

Since these methods are founded upon the spectrobolometric method, one may ask, what is the justification for using them instead of the latter? Can they be expected to give something more than the method upon which they are founded? To the first question one may reply that the justification lies in their simplicity, which makes it possible to apply them under a wide range of conditions where the more cumbersome bolometric method could never be used. A spectrobolometric investigation, like that of Abbot on Mount Whitney in 1910, will probably always be a rare event. But especially in regard to the question of solar variability it is desirable that the number of simultaneous observations be large and extended to as high altitudes as possible.

[1] *Nova Acta Reg. Soc., Sc. Upsal.*, Ser. IV, 1, No. 7.

[2] *Annals of the Astrophysical Observatory, Smithsonian Inst.*, 2, 114.

The second question, whether the abridged methods can ever deserve the same confidence, or even in rare cases give greater accuracy then the spectrobolometric observations, is one that must be answered rather through experimental results than through general considerations. Here, however, two points may be noted.

The first is, that the spectrobolometric method, which under ideal conditions is naturally superior to any abridged method, is in all practical cases a method involving a large number of precautions, some of which are very difficult to take. The abridged methods, founded as they are upon mean values, may possibly under special conditions avoid accidental errors to which single spectrobolometric series are subjected.

Secondly, it may be noted, that even in the analytical method of bolometry, there arises some uncertainty in regard to the ordinates of the bolometric curve, corrected for absorption, at the points where absorption bands are situated. This causes an uncertainty in the water-vapor correction in this method as well as in the abridged methods founded upon it.

The methods just discussed lead to a numerical value for the solar constant. But the measurements in a selected part of the spectrum lead also to a direct test of solar variability, which seems likely to be especially valuable because these observations are not affected by aqueous absorption.

MEASUREMENTS WITH ABSORBING SCREEN

We may put:

$$I = I_0 e^{-\gamma m}$$

where I_0 is the energy transmitted through the absorbing screen at the limit of the atmosphere, I is its value after passing through the air mass m, and γ is a constant dependent upon the scattering power of the atmosphere. If now we plot $\log I$ against m, the points should lie on a straight line, whose ordinate for $m = 0$ is $\log I_0$.

The values of I_0 thus obtained from our observations are given under the heading I_0 in Table III. The straight lines were run by the method of least squares, not so much because the presuppositions of this method seemed here to be satisfied, as because thereby

all personal bias was eliminated. The "probable error" e of each value of I_0 is appended as a rough indication of its reliability, and the weighted mean I_0 is given at the bottom of the table. A comparison between the different values of I_0 shows that they all differ by less than 2 per cent; half of them by less than $\frac{1}{2}$ per cent, from the mean value. The deviation falls as a rule within the limits of the probable error.

This result thus fails to support the variability of the sun inferred by Abbot from simultaneous observations at Bassour and Mount Wilson. We cannot, however, with entire safety draw any conclusions about the total radiation from measurements in a limited part of the spectrum. All that can be said with certainty is that *a change of the energy in the green part of the solar spectrum exceeding 2 per cent during the period of our observations is improbable.*

If we, from this, are inclined to infer that the total solar radiation during the same period was constant, this inclination rests upon a statement by Abbot[1] himself to this effect: "So far as the observations[2] may be trusted, then, they show that a decrease of the sun's emission of radiation reduces the intensity of all wave-lengths; but the fractional decrease is much more rapid for short wave-lengths than for long."

Yet unpublished measurements by Mr. A. K. Ångström, in Algeria at 1160 m altitude, give a mean value for I_0 equal to 0.0708 which is in close agreement with the value 0.0702 given above. On the former occasion Mr. Abbot's spectrobolometric observations gave a mean value for the solar constant of 1.945. If we assume the energy transmitted by our green glass on Mount Whitney to bear the same ratio to the total energy, the Mount Whitney observations give a value for the solar constant reduced to mean solar distance equal to 1.929, which differs by less than 1 per cent from the former value.

MEASUREMENTS OF THE TOTAL RADIATION

The general basis of the Ångström-Kimball method of calculation has already been described. It is here convenient to make

[1] *Annals of the Astrophysical Observatory, Smithsonian Institution*, 3, 133, 1913.

[2] *Observations at Bassour and Mount Wilson*, 1911–1912.

use of the spectrum of constant energy introduced by Langley, where the abscissa represents the energy included between an extreme (ultra-violet) wave-length and the wave-length corresponding to the abscissa; the energy-density plotted as ordinate would then be constant. A table giving wave-lengths and corresponding abscissae is given by Kimball.[1]

Referred to such a spectrum, the atmospheric transmission y_λ for any wave-length is well represented by the empirical formula

$$y_x = p^{m\delta} x^{nm\phi(\delta)} \tag{1}$$

where x is the abscissa, m the air mass, and δ a quantity dependent upon the scattering power of the atmosphere. Ångström made the natural assumption $\phi(\delta)=\delta$. Kimball[2] finds that $\phi(\delta)=\sqrt{\delta}$ better fits the observations at Washington and Mount Wilson. In the latter case we have,

$$p=0.93, \qquad n=0.18$$

Making these substitutions in (1) and integrating,

$$Q_m = Q_0 \int_0^1 0.93^{m\delta} x^{0.18m\sqrt{\delta}} dx$$

or

$$Q_m = Q_0 \frac{0.93^{m\delta}}{1+0.18m\sqrt{\delta}}.$$

Kimball then adds an empirical correction for the absorption due to water-vapor, based upon bolometric measurements at Washington and at Mount Wilson, and finally obtains

$$Q_0 = \frac{Q_m}{\frac{0.93^{m\delta}}{1+0.18m\sqrt{\delta}} - [0.061 - 0.008\delta + 0.012E_0 m]} \tag{2}$$

where E_0 represents the depth in millimeters to which the earth's surface would be covered by water if all the aqueous vapor were precipitated. We have adopted this expression, *but instead of attempting to determine E_0 from humidity measurements at the*

[1] *Bulletin of the Mount Weather Observatory*, 1, Parts 2 and 4.

[2] *Ibid.*

earth's surface we have eliminated it between two equations such as (2) involving different air masses.

Kimball eliminates δ between two such equations. We have, however, followed the original method of K. Ångström and have determined δ for each day from our measurements with the green glass. The energy maximum of the light transmitted by it lies at $0.526\,\mu$ (see Fig. 1) to which corresponds the abscissa 0.27 in the constant energy spectrum. Hence for the transmitted green light

$$I_m = I_0\, 0.93^{m\delta}\, 0.27^{0.18m\delta}$$

from which δ can be computed. The values of δ thus obtained are given in Table III.

In order to compute Q_0, a smooth curve was drawn through the observations and values of Q_m for $m=1$, 2, and 3 were read off from the curve. These values and the value of δ for the day were inserted in (2) and E_0 then eliminated between the first and second and the first and third of the equations thus obtained. The results are given in Table III under the headings Q_{12}, Q_{13}; the mean of these for each day is given under $Q_{K.\mathring{A}}$ and represents the solar constant as obtained for that day by the Ångström-Kimball method.

The mean value of all the measurements, reduced to mean solar distance, is $1.931\ \frac{\text{cal}}{\text{cm}^2\,\text{min.}}$ (Ångström scale) or 2.019 (Smithsonian scale). The maximum deviation from the mean is 3 per cent.

Finally, Fowle's abridged method was applied to the same observations. Sufficient observations are not available for the elaboration of a special correction suited to Mount Whitney. But from the values of δ, it appears that the transmission over Mount Whitney was about the same as over Mount Wilson, where the average value of δ is 0.25; and the water-vapor pressure, the most uncertain factor, was low (2–4 mm). Hence it may not be devoid of interest to apply here Fowle's rule as elaborated for Mount Wilson, which is: To the "apparent solar constant" obtained by straight-line extrapolation add 2.7 per cent and as many per cent as there are millimeters in the water-vapor pressure. The results thus obtained are given in Table III under the heading Q_f; the mean water-vapor pressure is given under p.

TABLE III

FINAL RESULTS

	P mm	δ	I_o $\frac{\text{cal}}{\text{cm}^2\ \text{min}}$	ϵ per cent	Q_o $\frac{\text{cal}}{\text{cm}^2\ \text{min}}$	Q_o $\frac{\text{cal}}{\text{cm}^2\ \text{min}}$	$Q_{KÅ}$ $\frac{\text{cal.}}{\text{cm}^2\ \text{min}}$	Q_F $\frac{\text{cal.}}{\text{cm}^2\ \text{min}}$
August 2	(3 0?)	0 30	0 0680	0 0	1 904	1 886	1 895	(1 820)
August 4	3 0	0 28	0 0678	0 9	1 847	1 829	1 838	1 703
August 5, A.M.	2 5	0 32	0 0683	0 3	1 871	1 874	1 873	1 832
August 5, P.M.	2 9	0 32	0 0684	0 8	1 887	1 900	1 894	1 878
August 9		(0 39)	(0 0688)					
August 10	3 4	0 33	0 670	0 7		1 826	(1 826)	(1 770)
August 11	2 2	0 30	0 685	0 5	1 877	1 870	1 874	1 703
August 12	2 0	0 29	0.685	0 5	1 896	1 888	1 892	1 802

$$\text{Weighted mean } I_o = 0\ 0683\ \frac{\text{cal.}}{\text{cm}^2\ \text{min.}}\ ,$$

$$\text{reduced to mean solar distance } I_o = 0\ 0702\ \frac{\text{cal.}}{\text{cm}^2\ \text{min.}}$$

(Ångström scale)

Mean reduced to mean solar distance: $Q_{KÅ} = 1\ 931(\text{Å.})$,

$$= 2\ 019(\text{Sm.})\frac{\text{cal.}}{\text{cm}^2\ \text{min.}}$$

$$Q_F = 1\ 872(\text{Å.})\ ,$$

$$= 1\ 960(\text{Sm.})\frac{\text{cal.}}{\text{cm}^2\ \text{min.}}$$

SUMMARY

Our pyrheliometric observations on the top of Mount Whitney, extending from August 2 to August 12, 1913, have led to the following results:

1. A variation in the solar constant amounting to more than 2 per cent during this time is improbable.

2. The solar constant computed from the measurements in a selected part of the spectrum, reduced to mean solar distance, came out 1.929 $\frac{\text{cal.}}{\text{cm}^2\ \text{min}}$ (Smithsonian scale), with a possible error of 1.5 per cent. This value is obtained on the assumption that the energy included between 0.484 μ and 0.576 μ is a constant known fraction of the total energy in the solar spectrum.

3. The solar constant computed by the Ångström-Kimball method was found to be 2.019 $\frac{\text{cal}}{\text{cm}^2\ \text{min}}$ (Smithsonian).

4. The solar constant computed according to Fowle's method comes out 1.960 $\frac{\text{cal}}{\text{cm}^2 \text{ min.}}$ (Smithsonian).

The value of the solar constant given in (2) is in close agreement with Abbot's mean value of 1.932 obtained from several series of observations made during the years 1902–1912 at much lower altitudes (e.g., at 1160 m in Algeria). The value given in (3) is also in close agreement with the solar constant computed by Kimball according to the same method from measurements at Washington. Consequently our observations give no support to a value of the solar constant greatly exceeding 2 $\frac{\text{cal}}{\text{cm}^2 \text{ min.}}$.

Because of their bearing upon the question of solar variability, it seems desirable that the observations in selected parts of the spectrum by means of absorbing screens should be extended to different localities, and that if possible simultaneous measurements at elevated stations should be undertaken.

Cornell University
December 1913

THE COLOR OF THE FAINT STARS[1]

BY FREDERICK H. SEARES

During the last two or three years evidence that the faint stars are appreciably redder than the brighter objects has gradually been accumulating. Kapteyn has found, for example, that certain clusters are distinctly redder than bright stars whose spectra are the same as the average spectrum of the clusters with which they were compared. A similar result, although complicated with photographic phenomena, is indicated by Mount Wilson photographs of faint stars made with red and blue filters. Perhaps the most recent contribution is that of Hertzsprung,[2] who has photographed numerous regions with a large grating attached to the tube of the 60-inch reflector. The results for *N.G.C.* 1647 are striking. From the ninth magnitude on there is a gradual increase in the effective wave-length. At magnitude 14.5 its minimum value is λ 4320. If this result is interpreted in terms of spectral type, it means that for the region considered there are no stars of this magnitude whose spectra are earlier than F0. Other considerations which need not be discussed here all point in the same direction.

It is important that the question should be investigated further, for other regions and for still fainter stars, and preferably by independent methods. A simple method of attack is the direct formation of color indices by comparing visual and photographic magnitudes. If the faint stars are redder than the bright stars, this fact must immediately be revealed by a difference in the average index for the two groups. Should blue or white stars be rare or altogether lacking among the fainter objects, we shall find no negative and possibly no small positive color indices.

Although simple in principle, the method is exacting in its demands, for both visual and photographic magnitudes must be determined independently in accordance with an absolute scale.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 81. Read at the sixteenth meeting of the Astronomical and Astrophysical Society of America, Atlanta, January 1914.

[2] "Annual Report of the Director of the Mount Wilson Solar Observatory," *Year Book of the Carnegie Institution of Washington*, No. 12.

Moreover, careful attention must be given to the zero points, as an error in either of those enters directly into the color index. So far as a change in color with increasing magnitude is concerned, such an error is of no consequence. But to determine the actual degree of color, the zero points must be established in a definite relation to each other—preferably that already fixed by international convention, which requires that the photographic and visual magnitudes of stars of spectrum A0 between 5.5 and 6.5 on the Harvard scale shall be equal. Since we desire color indices for very faint stars—let us assign temporarily magnitude 17.5 as a limit—we must establish absolute photographic and visual scales over an interval of at least twelve magnitudes.

A natural point of beginning is the North Polar Sequence, for which two determinations of the photographic scale are already available—one made under the direction of Professor Pickering at the Harvard Observatory, the other from observations at Mount Wilson. With the exception of a divergence of about 0.4 mag. between the sixth and the tenth magnitudes, the agreement is very satisfactory. In addition, Pickering has determined the visual magnitudes of the stars brighter than about thirteen. The essential thing remaining, therefore, is the extension of the visual scale to the fainter stars.

This has been done photographically with the 60-inch reflector, using isochromatic plates and a yellow filter. The scale has been established by diaphragms, the methods of observation and reduction being the same as those previously employed for the determination of the photographic scale.[1] The zero point was determined from nine stars between 9.84 and 13.94 for which visual magnitudes are given in *H.C.*, No. 170. As a control, the bright stars of the sequence between the fifth and seventh magnitudes were directly connected with fainter stars between magnitudes twelve and thirteen. Here the method was the same as that used in deriving the photographic magnitudes for the brighter stars described in *Contribution* No. 70. The results of the control observations were most satisfactory, and confirm the relation of

[1] *Contributions from the Mount Wilson Solar Observatory*, Nos. 70, 80; *Astrophysical Journal*, **38**, 241, 1913; **39**, 307, 1914.

the Harvard visual magnitudes between twelve and thirteen to those near the sixth magnitude. There can scarcely be any question, therefore, as to the substantial accuracy of the visual scale as far as the thirteenth magnitude. Beyond this point, as already stated, the same methods were used as had been successfully employed for the photographic scale of the fainter stars. The separate determinations, of which there are five, are all closely accordant, and aside from the relatively small amount of observational data, there seems to be no more reason to fear systematic errors here than there.

Briefly, the question of scales stands thus, reference throughout being to the international zero point. For the brighter stars there is a linear divergence between the Harvard and Mount Wilson photographic scales, which at the tenth magnitude amounts to 0.37 mag. From 10 to 15.5 the scales are parallel, the Harvard magnitudes being brighter than the Mount Wilson values. Beyond 15.5 there is again a small divergence, unimportant for the moment, as it does not affect the general character of the result. The Harvard visual magnitudes from 12 to 13 are confirmed in their relation to the international zero point and the Mount Wilson extension of the visual scale to the seventeenth magnitude is presumably reliable, as it has been established by tested methods. Any change in the average calculated color index between the tenth and seventeenth magnitudes should therefore be reliable to the same degree, although the absolute amount of the index may be uncertain by reason of the constant difference which arises from the divergence between the sixth and tenth magnitudes previously referred to. The question of this difference will be discussed presently.

In the meantime attention is directed to the upper portion of Fig. 1 which illustrates the distribution of the color indices for the region of the Pole. In order that the results might be representative, as many stars as possible, 107 in all, were included. The distances of the points from the axis are proportional to the indices of the individual stars whose photographic magnitudes are indicated in the margin. In calculating the indices Mount Wilson photographic magnitudes were used throughout. For stars brighter

than ten the visual magnitudes were taken from *H.C.*, No. 170. For all the fainter stars Mount Wilson photovisual magnitudes were used.

A possible variation in the average color index is obscured by the sporadic appearance of stars of high intrinsic color; but the gradual and regular increase in the minimum value indicated by the curve bounding the lower edge of the field of points is clearly indicated. From the sixth to the seventeenth magnitudes the change is from -0.1 to $+0.6$ mag. In reality the variation probably is greater, for the brighter objects form a selected group. They include only polar sequence stars, and, with the exception of a few red stars, are almost wholly of the A type. No early B-type stars appear; but in considering the change in the minimum index we must take them into account, for their non-appearance is doubtless due to a restriction in the choice of objects to be used as standards. On the other hand, the faint stars are so numerous that they must be fairly representative, and it is improbable that any increase in the field would modify their minimum index. Since the color index of bright B0 stars is about -0.4 mag., the real change in the minimum index between the sixth and seventeenth magnitudes is probably about one magnitude. Beyond the fifteenth magnitude there appear no stars with indices less than $+0.5$ mag. Inferentially their spectra would be at least F2. Had the Harvard photographic magnitudes been used, the bounding curve would have coincided with the axis for the brighter objects, since the Harvard visual and photographic scales coincide for A-type stars; below the tenth magnitude the two photographic scales are parallel, and the characteristic variation would have appeared, although all the color indices would have been smaller by 0.37 mag.

This very considerable modification of the result raises again the question of the divergence of the Harvard and Mount Wilson photographic scales for the brighter stars. As a possible source of explanation my attention has been called to the following sentence in *H.C.*, No. 170, relating to the method used in deriving the Harvard photographic magnitudes:

> An absolute scale of magnitudes was derived separately from each of about 80 plates taken by the above methods, the starting point in every case

being the mean photometric magnitude of such stars in the Polar Sequence given in Table I, as were measured on that plate.

This statement seems to imply that the zero point of the photographic scale has been based upon photometric, that is, visual magnitudes. As long as the visual standards used are near the sixth magnitude, this means only the adoption of the international zero point. Nor for fainter stars would there be any difficulty in using visual standards were it certain that the visual and photographic scales for any given type of spectrum coincided. This, however, appears not to be the case. Adams,[1] for example, finds striking evidence that stars of the same spectral type may show a very different distribution of intensity in the continuous spectrum background. The color indices in such cases must be different, and the scales which express the corresponding visual and photographic magnitudes cannot coincide. Again, the curve for the minimum color index indicates that the scales do not coincide even for the whitest stars.

This being the case, it follows that visual standards other than those near the sixth magnitude will not give reliable results for the zero point of the photographic scale. If now, as the above quotation seems to indicate, visual standards were used whenever possible, the result would be a necessary coincidence of the visual and photographic scales, at least for the white stars. For an individual plate the divergence assumed to exist between the true scales might appear; but when the results for plates covering different intervals were combined the overlapping portions would cause the divergence to disappear from the mean and produce an apparent coincidence. The photographic zero points for the Mount Wilson plates, on the other hand, were found by a process equivalent to a direct comparison with stars near the sixth magnitude.[2] If the interpretation of the quotation from *H.C.*, No. 170 is correct, the two photographic scales could not agree unless the true photographic and visual scales for the white stars

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 78; *Astrophysical Journal*, **39**, 89, 1914.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 70; *Astrophysical Journal*, **38**, 241, 1913.

coincide. The latter alternative, as already pointed out, seems to be excluded.

This explanation also accounts for the disappearance of the divergence between the Harvard and Mount Wilson photographic scales near the tenth magnitude. Beyond thirteen there is but one visual magnitude for a white star given in *H.C.*, No. 170, so that for the fainter stars photographic standards presumably were used for the determination of the zero point. Moreover, any divergence between the true visual and photographic scales would begin to show at a point some two magnitudes or so above this limit owing to the absence of a smoothing effect in the lower part of the interval covered by the plates on which the faintest visual standards were used.[1]

One further point requires mention. Reference has been made to a divergence between the two photographic scales beyond magnitude 15.5. The Mount Wilson results in this region are still uncertain, as they are based upon stars at the limit of visibility. Had the Harvard values been used, the upward trend of the bounding curve would have been even greater than that shown. The curve itself, however, is probably not greatly in error, for if the Mount Wilson photographic magnitudes are here systematically too bright, it is likely that a similar error affects the visual magnitudes. Plates of longer exposure will be required to settle the point.

[1] During the discussion following the reading of the paper the following statement by Miss Leavitt was presented by Professor Pickering:

"The magnitudes of stars in the Polar Sequence were originally reduced in accordance with the statement in *H.C.*, No. 170, as quoted by Professor Seares, the zero point for each plate being made to coincide with the photometric magnitudes of stars measured on that plate. In order to avoid any error involved in employing stars other than those near the sixth magnitude, a new reduction was made, which, however, gave magnitudes identical with those in *H.C.*, No. 170. For each of the 20 groups, first differences between the magnitudes of successive stars were taken. Adding together the means of these differences gave a scale of magnitudes which was the mean of the 20 individual scales, and independent of the photometric scale. These magnitudes were then reduced to a zero point depending on the photometric magnitudes of stars of Class A between the magnitudes 5.5 and 6.5. The resulting magnitudes, as has been stated, coincided with the magnitudes in *H.C.*, No. 170."

This apparently leaves the question of the divergence between the Harvard and Mount Wilson photographic scales for the brighter stars still unexplained.

If the systematic change of color with magnitude be regarded as established, it becomes at once a matter of interest to determine whether the change is the same for all parts of the sky. For a preliminary examination several regions have been photographed and partial results for one, the field of the variable *S Cygni*, are also given in Fig. 1. The 200 color indices shown are based upon photographic and visual magnitudes obtained by comparisons with the Pole. The mean of three plates was used for each scale, and the accordance between the separate comparisons is good throughout. Diaphragm plates on *S Cygni* for the determination of the magnitudes of the fainter stars were also made, but these have not yet been reduced.

It will be observed that the curve of minimum index is here the same as for the polar region, and it is of interest to note how sharply the limit is defined for the lower end of the scale where the stars are numerous. The sudden increase in the minimum index beyond magnitude 15.5 is probably apparent, and means only that stars with smaller indices are too faint visually to appear on the plates measured.

It is of interest to compare these results with Hertzsprung's measures of effective wave-length for *N.G.C.* 1647. As already stated, his minimum at magnitude 14.5 is λ 4320, corresponding for bright stars to a spectrum of F0 and a color index of +0.4 mag. This is the same as the limit given by the color index curves. Should it prove that the color change for *N.G.C.* 1647 is the same as that for the *S Cygni* and polar regions, the agreement would afford a valuable control of the photographic and visual scales.

The final interpretation of these results cannot now be given, but certain general conclusions may be indicated. Large color index may mean an advanced spectral type, or it may mean a peculiar distribution of intensity in the continuous spectrum similar to that found by Adams. Were we to exclude the latter possibility we should be confronted with a curious result. The faint stars, none of which show any negative, or even any small positive indices, may be faint either because of small luminosity or because of great distance. That stars of small luminosity should show advanced spectra is not surprising. In fact, for those cases in which the

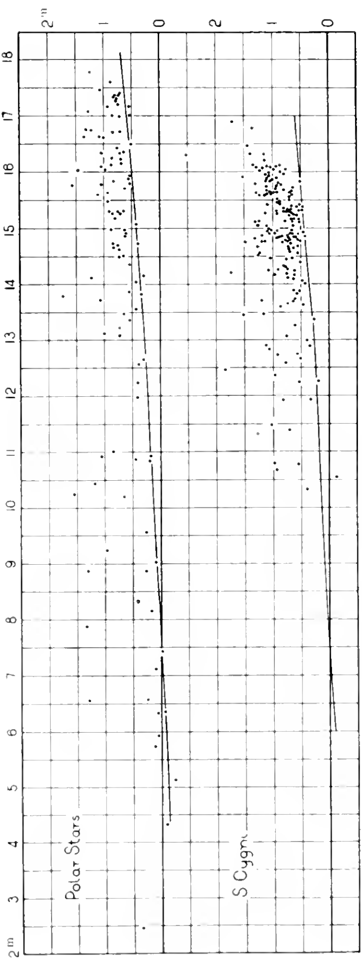


FIG. 1.—Variation of color index with photographic magnitude

absolute magnitudes are known, we find a rapid increase in spectral type with increasing absolute magnitude. But that stars which are faint because of great distance should show no early-type spectra would scarcely be expected, especially when the uniform change in minimum color index is considered. It would imply a gradually increasing suppression of the early types with increasing distance, and any such spectral distribution must be regarded as very improbable. Such is the conclusion resulting from the assumption that the observed increase in color index represents an actual change in spectral type.

The other alternative, namely, that the phenomenon is related to a change in the intensity distribution of the continuous spectrum, may be dependent upon either one or both of two factors—absolute luminosity and space absorption. Kapteyn has shown that, on the basis of the most reliable determinations of star density, the average luminosity must decrease with increasing apparent magnitude. Differences in luminosity may, however, mean differences in the general atmospheric absorption, even for stars of the same spectral type; and it is conceivable that the observed phenomenon might thus be accounted for. The presence of an absorbing medium in space, on the other hand, would also account for the gradual increase in the minimum color index. The results here presented do not permit a separation of the two effects. This phase of the question is, however, soon to be discussed exhaustively by Professor Kapteyn.

Mount Wilson Solar Observatory
December 19, 1913

THE SPECTRA OF MAGNESIUM, CALCIUM, AND SODIUM VAPORS

By JAMES BARNES

The study of the changes produced in the spectra of the elements by different forms of electrical discharges and by varying the pressure and chemical nature of the gas surrounding the electrodes has been the subject of a number of important investigations. The results so obtained have been found very useful in the interpretation of the physical conditions existing in sun-spots as well as in other fields of astrophysics. To these results the author wishes to add a few further observations.

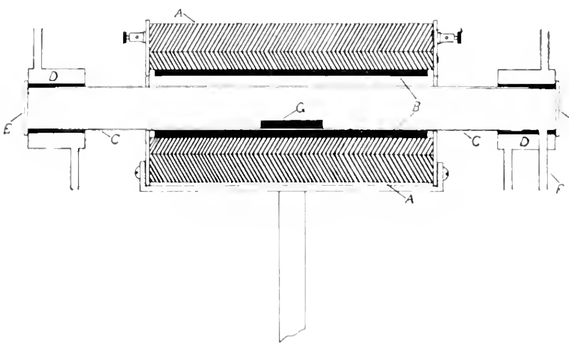


FIG. 1

The arrangement of the apparatus used is shown in Fig. 1. *A* is an electric furnace which was made by winding nichrome wire (No 17, B. and S.) about an alundum tube, *B*. This tube is 10 in. long and 2 in. in diameter and around it is placed a thick pipe-covering of 85 per cent magnesia which in turn is coated with asbestos cement. A long silica tube, *C*, 25 in. long and 1 in. in diameter was run through the alundum tube. In later experiments a glazed porcelain tube of about the same dimensions was used.

The ends of these tubes were closed by iron caps, *D*, and quartz plates, *E*. A stream of water running through the caps keeps them sufficiently cold so that the wax, by which they are sealed to the tube and to the quartz plates, does not melt. The tube *F* is connected with a Geryk exhaust pump.

A piece, *G*, of the metal under investigation is placed in the center of the tube, the caps adjusted, the air removed, and the furnace started. By regulating the current, the temperature of the furnace is held at such a point that the metal is slowly vaporized. The vapor is rendered luminous by the discharge from a large induction coil, parts of the caps, *D*, acting as internal electrodes.

The light coming from one end of the tube was analyzed by a Rowland concave grating of 6 ft. radius, and from the other end by a Hilger prism spectrometer of the constant deviation type. The intensity of the discharge was so strong for the substances used that good photographs of their spectra could be obtained in a few seconds with the prism instrument and in about ten minutes with the grating.

OBSERVATIONS

Magnesium.—I shall cite only the cases where the relative intensities of the lines in the spectrum of magnesium produced by this form of excitation are considerably different from that of the arc in air and *in vacuo* and from that of the spark. Reproductions from the negatives are shown on Plate VI. The strong spark line, λ 4481, which has been the subject for a large amount of investigation and discussion, is very weak in the furnace discharge, in fact on many plates it cannot be found. The strongest lines are λ 2852 and λ 4571. The remarkable intensity of λ 4571 is noteworthy. This radiation is relatively weak in the arcs in air and *in vacuo*.[1] Concerning the spectrum of magnesium, Adams[2] remarks, "The only magnesium line which shows any marked change is λ 4571.275, which is increased from 5 in the sun to 7 in the spot." Brooks[3] regards this radiation as characteristic of magnesium nitride. I tried the experiment of thoroughly washing

[1] *Astrophysical Journal*, **21**, 74, 1905.

[2] *Ibid.*, **30**, 105, 1909.

[3] *Ibid.*, **29**, 184, 1909.

out the tube with dry hydrogen and using magnesium which had been prepared by melting a piece of the metal and condensing it *in vacuo*. The spectrum was not changed, the radiation λ 4571 being just as strong as found above.

The three bands with heads at λ 5622, λ 5211, and λ 4845 are strongly depicted on the spectrum plates. These bands have been attributed to a magnesium and hydrogen compound and their wave-lengths were found by Fowler[1] to correspond with many lines in sun-spot spectra. Recently he found[2] and measured the wave-lengths of a number of additional triplets in the ultra-violet region of the spectrum of the magnesium arc *in vacuo*. These lines also occur in the furnace spectra and were easily found on plates taken with a Hilger quartz spectroscope.

Calcium.—The intensities of the H and K lines and λ 4227 as produced in the furnace discharge do not differ very much from their values as produced in the arcs in air or *in vacuo*. The only line which is strongly affected is λ 6573 as can be seen on Plate VI and is one of the strongest lines in the spectrum. This line has been found by Hale and Adams[3] to be "one of the most strongly affected lines in the entire spot spectrum, showing a rise of intensity from 1 in the sun to 10 in the spot." The bands with heads at λ 6382 and λ 6389 which are also an important feature of sun-spot spectra also occur in the furnace discharge but not with as great an intensity as was found in the arc *in vacuo*.

Sodium.—The spectrum of sodium vapor in the furnace shows the D lines with their intensities very much increased relatively to the other lines. The wings on the D lines are very wide and clear. Adams (*loc. cit.*) found this to be also the case in sun-spot spectra.

In the light of these observations, the author wishes to conclude that the physical conditions density, temperature, and excitation, as used above give spectra which are the nearest he has yet produced to those observed in sun-spots.

Bryn Mawr College
November 1913

[1] *Phil. Trans. Roy. Soc.*, **209** A, 447, 1909.

[2] *Proc. Roy Soc.*, **89** A, 137, 1913.

[3] *Astrophysical Journal*, **30**, 92, 1909.

THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME XXXIX JUNE 1914 NUMBER 5

VISUAL OBSERVATIONS OF HALLEY'S COMET IN 1910

By E. E. BARNARD

Considering its great brightness, and the extraordinary phenomena presented by other comets of recent years which at most only attained the faintest naked-eye visibility, Halley's comet at its return in 1910, though a brilliant and interesting object to the naked eye—especially in the month of May—was, nevertheless, a disappointment when considered from a photographic standpoint. It is safe to say that it did not give us any new information concerning these strange bodies. Photographically, its light was relatively slow in its action on the sensitive plate, and there were few or none of the remarkable phenomena shown by Brooks's comet of 1893, which was faintly visible to the naked eye for about one day, and by Morehouse's comet of 1908, which just attained naked-eye visibility for a couple of days. Had it not been for the previous comets, however, the numerous photographs obtained of it would have put Halley's comet in the first rank among the records of these bodies. While it lacked much in interest as seen with the eye of the sensitive plate, to the human eye it left a lasting impression which, added to its long life-history of more than two thousand years, made it, at its return of 1910, perhaps the most interesting comet of history. The apparent length of its tail when nearest the

earth (120° or more) was probably the greatest on record,[1] though the actual length was much exceeded by many previous comets. As seen from this observatory it was visible to the naked eye from April 29 to June 11, which was not an excessive duration of visibility. With the 40-inch telescope its visual appearance extended from September 15, 1909, to May 23, 1911, which, though a long period, has been exceeded by several comets that never attained naked-eye visibility.

HALLEY'S COMET FROM A POPULAR POINT OF VIEW

In this place it may be well to say a word or two on the popular side of this return of Halley's comet.

It is unfortunate that the newspapers and the general public were so greatly disappointed in the comet—unfortunate from the fact that the general impression left by such reports would be exceedingly misleading when comparing the present return with descriptions of its appearance in earlier times. It was unfortunate, also, from the further fact that even astronomers sometimes have a sentimental side. It would have been a gratification to know that everyone who saw this wonderful object saw it with the same spirit of elation and wonder—one would almost say veneration—with which the average astronomer regarded it. This was, at least, the feeling of the present writer when he looked at this beautiful and mysterious object stretching its wonderful stream of light across the sky.

The great cities that have grown up since 1835, and the smoke and electric lights of today completely robbed the comet of its glory when seen by dwellers in and near the centers of population. The newspapers had excited the public pulse to a high pitch by glowing and sensational accounts of what the comet would do and what it would look like, and had thus raised expectation beyond all reason. When these expectations failed, purely because of local

[1] According to Ellery, comet I 1865, for several days in January, had a tail 150 degrees long (*Monthly Notices*, **25**, 220). I find, however, that this great length is simply a printer's error of 150 degrees for 15 degrees. See a note by Ellery in *Astronomische Nachrichten*, **64**, 219 (same date as the one in *Monthly Notices*), where he gives the length as 15 or 16 degrees. This smaller length is verified by other southern observers.

conditions, it was not possible for them to pile enough contumely upon the comet and upon the heads of those who had made no prediction whatever as to what the comet might really look like. Were such records as these the only ones to depend upon for comparison at future returns, it would indeed be unfortunate. In reality, to those who under favorable conditions saw the comet at its best at the return of 1910, and who would have been justified in making any prediction, it far exceeded the most sanguine expectations in the remarkable display it presented to us.

There was one fact which was brought forth by the comet with startling vividness. It showed that the superstitious terror formerly attending the appearance of a great comet is by no means dead in the human breast. Cases of this kind developed all over this country and abroad—from the stopping-up of keyholes and cracks in doors and windows in Chicago (according to the daily papers) to keep out the deadly comet gases, to the manufacture and sale, among the negroes of the South, of "comet pills," which were supposed to ward off the evil effects of the comet.

The "comet gas" scare seemed to be directly due to the incautious and unwarranted statements of one or two men of science who had painted in rather vivid language the direful effects of breathing the deadly cyanogen gas, which had been shown to exist in the tails of some comets.

Such being the case in our present enlightened day, it is easy to understand how terrifying the comet must have been in former times, even if its display then was no more striking than in 1910. In the calm of the spring night, at a time when one is easily impressed with a mystery that is not present in the day, the comet, with its weird streamer of light reaching far into space, was well fitted either to impress or to terrify the observer, just as his mental temperament might suggest. In the Dark Ages, when the mission of these dread bodies was unknown, and when everything in nature seemed to possess a spirit for good or evil—and mostly for evil—there is little wonder that the unheralded advent of a great comet should inspire, at the least, an uneasiness in the minds of those who saw it. The writer was strongly imbued with this thought on several nights while observing Halley's comet when in its most

impressive stage, and it would not have taken much imagination to have endowed it with a guiding spirit. With the enlightenment of today, however, one could see nothing in it that would disquiet or terrify, but rather that which raised a sense of extreme pleasure and wonder at the magnificent mystery it presented.

POSSIBLE ENCOUNTER OF THE EARTH WITH THE TAIL OF THE COMET

In connection with this account of Halley's comet and its near approach to the earth, it may be appropriate to add some remarks on the probable encounter of the earth with a portion of the tail at, or closely following, the time the comet transited the sun. Indeed, it seems more than probable that the earth actually did encounter one of the branches of the tail—the southern branch—on, or about, May 18 or 19, and more probably on the later date. There is also a suspicion that the influence of this encounter (if such there was) on our atmosphere was apparent for months afterward.

The double tail seen here on the nights of May 17 and 18, the lower, and probably larger branch of which widened toward the southeast horizon, involved the ecliptic, as will be seen by the diagram on p. 389, and without doubt extended beyond the earth. There are strong chances that the earth passed through this part of the tail about May 19. That the tail was long enough to reach to the earth is shown by the fact that as late as May 25 its length (54°) was over 30 million miles, or twice the distance of the comet at its nearest approach to us on May 18.

With the exception of a sketch by Miss Mary Proctor in New York City and a newspaper account by Professor D. P. Todd of Amherst (whose observation seemed to refer to May 16), I have seen no reference from northern observers to the second, fainter and broader tail shown in my drawings of May 17 and 18 south of the bright beam and separated from it by a distinct dark space perhaps 10 degrees wide. In Plate X I have tried to show as accurately as possible the appearance of the tails and their exact location among the stars on these two dates. The head of the comet was, of course, invisible below the horizon. This feature (the broad, faint, southern tail) seems to have been generally overlooked by observers in the Northern Hemisphere. It is, how-

ever, well shown in drawings made in South Africa by Innes and others at the Transvaal Observatory (now the Union Observatory). See *Circulars 3* and *11* of that observatory. It is also shown in a drawing made by Dr. Frank C. Cook, United States Navy, at Bahia Blanca, Argentine Republic, on May 19, 1910, at 5 A.M. In the South African sketches the south tail is generally shown fainter and very much broader, which agrees with my drawings. In my drawings the north edge of the south branch is well determined, but the south edge of it is evidently lost in the zodiacal light, which fills out the space to the southeast horizon.

Professor C. D. Perrine, at Cordoba, Argentine Republic, calls attention to and describes this second and broader tail (*Astronomical Journal*, **26**, 145).

One would have expected considerable parallax in portions of the tail on May 17 and 18. A comparison of the South African drawings with mine, however, does not show any parallax, at least none greater than the uncertainty of the drawings themselves

During the first part of the night of May 18, as will be seen by the notes, the sky was normal. It is probable that the slight mistiness mentioned on that date was in no way connected with the presence of the comet. The slight aurora, also, was nothing out of the ordinary, and certainly had nothing to do with the comet. In the latter part of the night, when the moon had set, the sky seemed to be free from any decided mistiness to the north of the comet's tail. At the same time the southern and fainter branch seemed to spread its effect over the southeast horizon, but there was nothing especially suggestive in its appearance.

The forenoon of May 19, however, developed peculiarities that were very suggestive (*Astronomische Nachrichten*, **185**, 229, 1910). Briefly, these consisted of a peculiar iridescence and unnatural appearance of the clouds near the sun and of a bar of prismatic light on the clouds in the south. This, combined with the general effect of the sky and clouds—for the entire sky had a most unnatural and wild look—would have attracted marked attention at any other time than when one was looking for something out of the ordinary. The sky had been watched carefully during the forenoon of this date but nothing unusual had appeared until close to noon, when the

conditions became abnormal, as stated above. Of course this unusual phenomenon, if seen only at one place, might be considered a coincidence, but something similar was reported on that date at other widely distant places. (See *Transvaal Observatory Circular*, No. 3, p. 19.)

The most suggestive phenomenon, however, was apparent later on, in June and for at least a year afterward. It was first noticed here on the night of June 7, 1910, and consisted of slowly moving strips and masses of self-luminous haze which were not confined to any one part of the sky. I have given an account of these singular features in the *Proceedings of the American Philosophical Society* for May-June 1911. It is true that these peculiarities might in some way have been of auroral origin, but this I do not think is probable, for they do not seem to resemble in any way, either in position or in appearance, any auroral phenomena with which I am familiar. Apparently nothing of the kind has again been visible here since September of 1911. At the same time it is also true that a similar absence of essentially all auroral effects has been very marked here, during the same period. This luminous haze had not been noticed by me in past years previous to 1910—especially in those years in which I was almost constantly out at night comet-seeking.

I would be more disposed to believe that this phenomenon of luminous haze had some connection with the near approach of Halley's comet to the earth were it not for the fact that apparently a similar phenomenon was recorded by Mr. Backhouse at Sunderland, England, through many years (see *Publications of West Hendon House Observatory*, No. 2, p. 109, 1902). Mr. Backhouse's descriptions show that the phenomena seen by him were perhaps of a similar nature to those seen here in the fall of 1910. It is probable, therefore, that this luminous haze was in no way connected with the close approach to us of the tail of Halley's comet. Nevertheless, a record should be made here of the phenomenon for the benefit of posterity. These observations by Mr. Backhouse were not known to me when my paper was prepared for the Philosophical Society.

THE COMET WITH THE LARGE TELESCOPE

Observations of the physical appearance might have been very interesting if it had been possible to follow the comet closely with

the large telescope. It was necessary, however, for it to have attained a considerable altitude before it could be seen with that instrument. As it was, the comet could be observed with the large telescope only after dawn, when the nucleus and its brighter appendages alone could be seen. The smallness of the field (5′5) and the great power of the telescope would have militated much against its successful use. From the clouded condition of the sky very few observations could be made during the morning visibility in April and May of 1910. A few rather unsatisfactory views were had, mainly when observing the comet for position later, in the last of May and in June. The most interesting observations, however, were obtained on the mornings of May 4 and 5, when the comet was watched in the coming daylight as it faded from view.

On May 3 (astronomical date), after the exposures with the Bruce telescope, the comet was observed with the 40-inch. Its aspect in the large instrument was rather singular. At first there were two wings to the nucleus, the southern of which was the brighter. The northern one, indeed, was so much fainter that it gave the nucleus and its appendages a very unsymmetrical appearance. Daylight soon blotted out the northern wing, leaving the nucleus with the southern one alone visible. It then very greatly resembled the naked-eye appearance of a great comet, with nucleus and tail. The accompanying sketches show the nucleus and wings as seen in the 40-inch telescope just before dawn obliterated the northern wing, and at $16^h 20^m$ when only the southern wing and nucleus were visible (upper two sketches of Plate VII).

On May 4 at $16^h 15^m$, with the 40-inch telescope the nucleus and its appendages were more symmetrical. While on May 3 the matter was nearly all on the southern side of the nucleus, it was evenly distributed on May 4 (see lower sketch of Plate VII).

VELOCITY OF THE PARTICLES OF THE TAIL

Of the physical phenomena presented by the comet, the most interesting was shown on June 6, 1910. On that date a long receding mass appeared in the tail. This seemed to be a disconnected streamer. From photographs made here with the Bruce telescope, at Honolulu by Mr. Ellerman, and at Beirut, Syria, by Mr. Joy, the writer obtained the results shown in Table I for the

motion of the object and hence, also, for the motion of the particles forming the tail (*A.N.*, **186**, 11, 1910). At this time the recession of the comet's head from the sun was 16.6 miles (26.7 km) per second.

TABLE I

Station	Interval	Hourly Motion	Recession per Second			
			From Comet		From Sun	
			Miles	Km	Miles	Km
Y.O.–Honolulu ..	4^h25	3′60	23.1	37.2	39.7	63.9
Y.O.–Beirut ..	15.15	5.17	33.1	53.3	49.7	80.0
Honolulu–Beirut ..	10.90	5.78	37.3	59.7	53.9	86.4

These results show a decided acceleration in the motion of the mass, which in the last two photographs amounted to an increase of 14 miles, or 22 km, per second. It should have been stated, however, that some uncertainty exists in the results owing to the possible change in the form of the mass. In all cases the end nearest the comet's head was measured, but this end itself may have shortened by dissipation of its material, and thus produced an apparent motion larger than the real one.

I have combined these three photographs (in the negative form) in Plate VIII. They are herewith presented so that the reader may judge for himself of the probability of any change in the actual form of the end of the mass.

Plate IX is a reproduction of the photograph of May 4 Attention has been called in the notes to the fact that heavy smoke from the power-house was drifting over the comet throughout the exposure of this plate, and that in effect it must have cut down the actual exposure-time by one-half From this cause the full width of the tail is perhaps not shown.

It was unfortunate that May 4 and 5, the only two good mornings on which the whole comet was visible, were both spoiled by the smoke from the power-house that was driven south, directly over the comet, by a heavy north wind.

The tail of the comet on May 29 (Plate XI) shows considerable structure, which is more or less lost in the reproduction.

Thanks are due to Mr. Leon Barritt, editor of the *Evening Sky Map*, for the loan of the half-tone block made from my photograph of June 6 (Plate XII).

One rather striking feature of the tail during the last stages of its visibility was that the star 87 *Leonis* remained in it or close to it for a long time. The tail seemed to be slipping eastward by or over the star. This, of course, was due to the motion of the comet. and the changing position of the sun.

THE RETURN OF THE COMET AND ITS EARLY APPEARANCE IN THE LARGE TELESCOPE

During the fall and winter of 1908–1909, the writer made a careful search for Halley's comet, both photographically with the Bruce telescope, and visually with the 40-inch. At that time it was too faint for either instrument. As if in acknowledgment of the discovery of photography since its last return in 1835 and the wonderful progress made in the application of that science to astronomy, the comet was destined to be seen first by the sensitive photographic plate. It was actually discovered, photographically, by Dr. Max Wolf, with the 30-inch reflecting telescope at Heidelberg, Germany, on September 11, 1909. The first visual observations of the comet were made by Professor S. W. Burnham with the 40-inch telescope of the Yerkes Observatory, on September 15, 1909.

At my first observations with the large telescope, beginning September 17, 1909, the comet was a small and rather faint speck of light, very much like a faint stellar nebula, of which there are so many in the sky. It was by no means at the limit of the great telescope, and under favorable conditions could have been seen much earlier with that instrument. The increase in brightness was not very rapid and as late as the last observations in February 1910, before the comet passed behind the sun, it gave very little promise of the splendid display it was destined to make later, in the month of May. Its reappearance from behind the sun in the morning skies of April and May could not have been under more unfortunate circumstances for observation at the Yerkes Observatory. That part of the year is always unpropitious here, and it seemed as if everything combined, on this particular occasion, to

hide from us the growth of the comet and its approach to the earth. Forest fires in the northern part of the state produced a densely smoky sky which, even when the clouds were merciful to us and would have let us see the comet, cut off with a thick yellow veil all but a glimpse of the bright head. The sky, on every morning, was watched until strong daylight for a chance to photograph or observe the comet. Similarly every care was taken in the evenings to secure results as long as the comet was visible.

The transition from the morning to the evening skies by the passage of the comet between us and the sun on May 18 was coincident with a change in the weather conditions, and we were thus enabled to watch it in its recession from the earth and sun. After a long cloudy period the sky suddenly cleared at midnight, on May 17, and gave us a splendid opportunity that night and the night of May 18, at the most critical time, to observe the phenomenon of its nearest approach to us.

NAKED-EYE AND TELESCOPIC OBSERVATIONS

The following notes descriptive of the comet's appearance to the naked eye and with the telescope are given nearly in full in the hope that they may be of value at its future returns. Of course photography took care of the general features of the comet, and thus preserved an accurate record of its appearance to the sensitive plate. At the same time it is a noteworthy fact that the photograph usually gives but little information as to the naked-eye appearance of a comet. A careful description, therefore, of its appearance to the eye alone must have a special value, independent of that of the photographs, and supplemental to them. From a historical standpoint, for comparison with its appearance in times past, it must have a value beyond that of the photograph.

In the descriptions which follow I have, in some cases, gone rather extensively into the details of the naked-eye appearance of the comet. I feel that this is justifiable for the following reason. In looking up the published information concerning its appearance in 1835 to form some opinion as to how the comet would look at the present return, I was surprised at the meagerness of the records, and I determined to prepare as faithful an account as possible of its

appearance to the naked eye for the benefit of observers at future returns. I therefore made as accurate a record as I could of its appearance to the eye alone. These results were obtained while guiding on the comet in photographing it, and at other times when a few moments could be spared to examine it. The descriptions, after the comet came into the evening skies, were written down, from my dictation at the time, by my niece, Miss Mary Calvert, and therefore have the accuracy of the inspiration of the moment.

A pair of large, old-fashioned field-glasses were available in these observations, and were used to supplement the naked-eye views when necessary. These glasses were specially suited for the purpose, and far better, because of their large lenses, than the more modern field-glasses, which for such work are deficient in light, and generally are too powerful.

The Bruce photographic telescope is supplied with a 5-inch visual guiding telescope, with a field of about 20′. When photographing the comet, notes were kept concerning its appearance in this instrument.

THE NAKED-EYE AND TELESCOPIC NUCLEUS

One striking fact that was noticeable when the comet was bright in the evening sky, especially noticeable on or about May 26, was that it had a nucleus within a nucleus. To the naked eye the nucleus was stellar and as bright as δ *Leonis*, of magnitude 2.7. With field-glasses the nucleus was small, but of sensible diameter, and of a beautiful bluish-white color; it was surrounded by a much fainter hazy nebulosity, which ran out to form the tail—the view being rather an intensification of that with the naked eye. The naked-eye and field-glass "nucleus" was not the true nucleus. In the 5-inch guiding telescope a small planetary nucleus of the magnitude 8 or 9 was seen in a denser nebulosity. It was very well defined and very yellow. About this date, therefore, naked-eye and telescopic observations of the nucleus would refer to two entirely different things of exactly opposite colors. That which formed the nucleus to the naked eye was simply the small denser nebulosity about the real nucleus (see *Astronomische Nachrichten*, **185**, 234).

Following is a careful summary of the notes. The records belonging to the earlier part of the observations (containing also micrometer positions), when the comet was visible only in the telescope, and those similarly made in its later stages, have already been printed in the *Astronomical Journal*, **26**, 43, 62, 76, 1909–1910, and **27**, 147, 1912. The present notes all refer to the year 1910.

All the times recorded in this paper are Central Standard Time, or 6^h0^m slow of Greenwich Mean Time.

April 11, 15^h35^m. Examined the sky but could see no traces of the tail. There was a broad strip of haze in the east, but the horizon seemed clear for about 2° altitude. At 16^h30^m the comet was well seen in the 5-inch guiding telescope, but it did not look any brighter than when last seen in March. There appeared to be a dim hazy nucleus with some nebulosity. The brightest part of the comet was at least two magnitudes less than *B.D.*$+7°5121$ of magnitude 6.3. There was no trace of the tail; the sky was too bright and hazy to show it. The comet was visible in the guiding telescope until 16^h51^m, when it was lost. It is probable that it would have been faintly visible to the naked eye if the sky had been clear and dark.

April 13, 16^h5^m. It was quite easily seen in the 5-inch telescope as a brightish, ill-defined, nebulous star, with no trace of tail, and was lost in dawn at 16^h58^m. The sky at that time was more or less hazy. The comet was certainly brighter on this date than on April 11. Each previous morning, before the brightest dawn, the sky had been examined for any trace of the tail, but none could be seen.

April 16, from 15^h45^m to 16^h55^m. The comet was bright in the 5-inch telescope. When at a considerable altitude the nucleus was starlike, almost white, and of the sixth magnitude. It was not quite as bright as the star *B.D.*$+7°5121$ (magnitude 6.3) with which it was compared for brightness, though it was more conspicuous than the star. The tail could be traced for 15′ or 20′, but the comet was not visible to the naked eye. Judging, however, from its brightness in the 5-inch, it must have been close to naked-eye visibility. Clouds prevented any successful photographs.

April 19. When first seen at about 15^h20^m the comet was in a clear space close to the horizon. It was beautiful in the 5-inch, with a bright nucleus and a fine parabolic outline to the head, from

which the tail streamed out of the field of view. It was not visible with the naked eye, but the sky was too poor for one to have seen it.

April 20. The comet rose in dense haze, and was first visible in the 5-inch telescope at 15^h45^m, but was very dim. At 16^h20^m the nucleus was of the same brightness as the star *B.D.*$+7°5101$ (magnitude 7.0), but did not seem to be so intense in its light—it was more planetary and not quite starlike. The comet could not be seen with the naked eye at any time, the sky being too poor.

April 29. The comet was hidden by clouds until 15^h45^m, when it came out on a very bright sky, and could be seen with the naked eye for the first time. The nucleus was bright and was of magnitude 2 or 2.5. The tail was visible for a couple of degrees, but with field-glasses it could be traced for 4° or 5°. The comet remained visible to the naked eye until 16^h7^m, when it was lost, but it was visible in the 5-inch until 16^h30^m. To the naked eye it did not appear so bright as Daniel's comet in September of 1907 when in a similar position with respect to daylight.

May 2, 15^h40^m. The comet was seen for about one minute in a thin streak of clearer sky. The tail stretched out of the field of the 5-inch guiding telescope, but thick haze prevented its being seen with the naked eye.

May 3. The comet was beautiful to the naked eye, with a long tail streaming upward toward the right. The tail, however, was not bright. Before moonrise it could be traced for 17° or 18°. The head and nucleus were of about the second magnitude, and were estimated to be one magnitude brighter than γ *Pegasi*. Even after the moon rose the tail could be traced for nearly 15°. The following notes were made before the comet rose, the sky being examined carefully.

14^h17^m	No trace of tail.
14 22	No trace of tail.
14 29	No trace of tail.
14 34	No trace of tail. Sky good.

The comet was first seen at 14^h40^m. The smoke from the power-house was passing over it during most of the exposure, and must have cut the light down seriously.

May 4. The comet was beautiful. The tail stretched about one-half the distance to θ *Pegasi*, a length of 15°, and seemed a little shorter than on May 3. It became very gradually fainter toward the end, where it seemed to fade out as if that were really its end, and not so much as if it simply became too faint to be seen. The head, however, seemed brighter than on May 3, and was fully of the second magnitude. At about 15^h it was one magnitude brighter than γ *Pegasi*. At 16^h7^m the comet was still faintly visible to the naked eye, but one minute later it had disappeared. The smoke from the power-house was passing in front of the comet and partly hiding it during the observations, so that the exposures must have been cut down in effect at least one-half.

May 5. Dense, thick sky. No trace of the comet could be seen with the naked eye. It was very faintly visible in the 5-inch telescope.

May 6. The sky was very thick. The comet was fairly well seen with the naked eye when it rose, but hazy clouds at once covered it. At first the tail could be traced, even in the hazy sky, for a distance of 17° or 18°. The whole comet must have been brighter than at previous observations. It could still be seen faintly between the clouds with the naked eye at 14^h53^m.

May 8. The sky was very thick and was constantly being covered with heavy clouds. The comet was seen with the naked eye several times between the clouds. After 15^h10^m it seemed pretty bright with a long tail. The views were fragmentary through the breaks in the clouds.

May 9. The sky was very thick. At 15^h5^m the comet was seen very faintly with the naked eye. It must have been very bright to be seen at all under the conditions. The tail could be traced for 15°. The head was at least of the second magnitude. At 15^h48^m it was still faintly visible with the naked eye.

May 13. No trace of the comet because of dense haze and smoke all around the horizon. If the tail had been very long it would perhaps have been seen above the smoke bank.

May 14. The sky was very thick and bad. At 14^h40^m the tail could be traced slightly beyond θ *Pegasi*, a distance of about 53°, and passed about 2° or 3° to the right of and below that star. It must

have been 3° or 4° broad near the south side of the square of *Pegasus*. It was fairly noticeable when looked at with averted vision, but could not be traced anywhere near the head, which was invisible in the haze.

At 15^h40^m the comet was faint in the 5-inch telescope, while *Venus*, at the same altitude, was fairly well seen with the naked eye, but was very dull and red. At 16^h0^m the nucleus, which was yellow and starlike, with some coma, was quite noticeable in the 5-inch telescope, but neither the head nor any of the tail near it could be seen with the naked eye at any time, because of the smoky haze. Where the tail could be seen it was very straight and broad.

From the foregoing observations the head must have been many times less bright than *Venus*. The observations also show that the tail must have been about 50° in length.

May 17. After a stormy period the sky cleared brilliantly at midnight. As observations at this time are of the utmost importance in connection with the nearest approach of the comet to the earth, the notes will be given nearly in full.

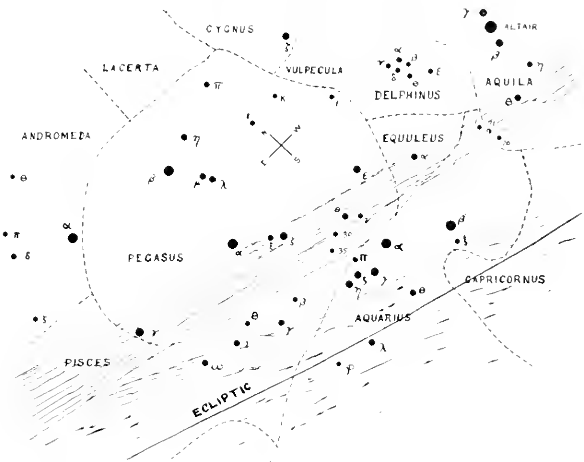
13^h0^m. A narrow twilight (which later proved to be the tail) seemed to extend along the eastern horizon. This was more marked at 13^h5^m. "There is a diffused dawn effect near the east horizon about 4° or 5° high." At 13^h10^m this seemed either to have risen rather rapidly or to have become more pronounced. The sky was very clear, but the moon was still above the horizon. At 13^h25^m this "dawn" effect was as high as ϵ *Pegasi*. At 13^h30^m distinct traces of the tail were certainly visible a little south of the square of *Pegasus*, and reaching nearly to *Altair*. At 13^h35^m the axis of the tail would pass through θ *Pegasi*. It was perhaps 5° or 6° broad near θ and apparently rose to a point $10^\circ \pm$ southeast of *Altair*. At 13^h45^m θ *Pegasi* was in the axis of the tail. At 13^h55^m ζ *Pegasi* was on the north edge of the tail. At 14^h15^m θ *Pegasi* was close inside the south edge and slightly in the tail, while γ *Pegasi* was in the tail and perhaps one-half degree north of its middle or axis and θ *Aquilae* exactly on its north edge. At 14^h20^m the tail between ζ and γ *Pegasi* was perhaps brighter than any portion of the Milky Way. It seemed somewhat brighter in the middle and faded slightly toward the edges. It joined the Milky Way and could be

traced beyond θ *Aquilae*. At this time it appeared straight, but at about 13^h40^m it was thought to be slightly convex to the north. The width of the tail was a little greater than the distance from β to η *Pegasi* (5°). At 14^h25^m the tail, beyond ϵ *Pegasi*, was about one-fourth as bright or less than that part between ζ and γ *Pegasi*. The star 71 *Aquilae* (*B.D.* $-1^\circ4016$) was in the middle or axis of the tail. At 14^h47^m α *Equulei* was just free of the north edge. At 15^h10^m the tail was faint from dawn and could be seen only by averted vision. At 15^h12^m it was still feebly visible near ζ and γ *Pegasi*, and could be traced as far as ϵ *Pegasi*. The sky was very clear. At this time γ *Pegasi* seemed to be still a little north of the axis. Miss Calvert watched it a little longer while I went to the 40-inch. She says that at 15^h18^m she could no longer see the tail, though she had seen it one or two minutes earlier.

The tail was only a little brighter toward the axis—it was very flat and did not diffuse much at its edges. Indeed it seemed to be nearly uniform in light with respect to its width, but it tapered very much toward the end, near which it would not be over three-fourths as wide as at a point near ζ *Pegasi*. This of course was an effect of perspective. Streamers or irregularities were carefully looked for but none was seen. The edges of the tail were smooth and uniform. At about 13^h15^m or 13^h30^m I could see the sky dark, below and above the tail, and there appeared to be a brightening along the southeast horizon, as if another portion of the tail were present. At 14^h45^m α *Equulei* was just free of the north edge of the tail. The head of the comet could not be seen when it rose, either with the 5-inch or the 40-inch telescope, because of the thick sky near the horizon. The observations show that the tail was at least 107° long on this date.

May 18. Beautifully clear all day, with a few flecks of clouds in the afternoon. A beautiful night with a three-fourths full moon. Every preparation had been made to photograph any phenomena that might develop during the night. There was a slight mistiness in the air. This was noticed only when, on hiding the moon, a feeble illumination was seen near it. At 8^h37^m and later, certain phenomena developed which are believed to have been auroral. The notes on these have been collected and are given later. At this

time the sky appeared unusually good. There was still a considerable twilight effect in the low northwest. At 8^h56^m some faint luminosity was visible under *Cassiopeia*. The sky had a feeble misty look everywhere, which was not due to ordinary haze, for apparently the sky was very clear. Up to 10^h35^m nothing out of the ordinary was noticed in the appearance of the sky, except the faint mistiness which had been visible since dark. At 10^h40^m the eastern horizon was bright with a diffused luminosity while the



Key-map for drawings of the tail of Halley's Comet, on May 17 and 18

western horizon was free from anything of this kind. At 12^h0^m the illumination of the eastern horizon seemed to be a little brighter. At 12^h42^m the eastern horizon for perhaps one-fourth the way up was very bright. (This later proved to be the comet's tail.) At 14^h10^m the tail was certainly visible just east of ϵ *Pegasi*. Sky still moonlit. At 14^h20^m the tail was surely visible in the east. It was quite noticeable at this time, even in the moonlight. It seemed to be a little north of its position of the previous morning (A.M. of May 18). At 14^h23^m γ *Pegasi* was just inside the south edge of the

tail, while *α Pegasi* was just inside it on the north edge. It lay between *θ* and *ϵ Pegasi*, nearer to *θ*. It was strongly visible (moon nearly down). At 14^h27^m the north edge of the tail diffused very gradually and reached halfway from *γ Pegasi* to *α Andromedae*. Both *θ Aquilae* and *ζ Pegasi* were in the axis of the tail. At 14^h42^m *γ Pegasi* was 3° inside of the south edge of the tail. The north edge diffused three-fourths of the way to *α Andromedae*, and was 10° wide near that star. At 14^h52^m the tail could be traced to the horizon, widening out toward the east horizon. At 15^h4^m the tail could still be seen (though it was dim) and the dark region in it. At 15^h10^m the tail and the dark space were still feebly seen, but they were badly dimmed by dawn.

The brightest portion of the tail near *α* and *γ Pegasi* was as bright as the Milky Way, but did not seem to be more than half as bright as on the previous morning. The tail could be traced to the Milky Way beyond *θ Aquilae*, where it became faint and somewhat tapered. It could not be traced across the Milky Way. Several times the impression was given—I was almost sure of it—that the brightest part of the tail fluctuated in brightness as if its light were unsteady. The illumination below the dark space in the tail, though feeble, I think was real. It apparently extended to the southeast horizon as if it passed below the horizon, and there seems no question that it was a separate part of the tail, and that the dark space was a rift that separated the tail into two parts. There was no evidence of any streamer north of the bright tail. The south edge was rather definite, though softly blended, but the north side was very diffused. At times it seemed to diffuse beyond *α Andromedae*. The light of the tail was very similar to that which forms the *Gegenschein*, or like light reflected from dust particles; that is, it did not have a nebulous appearance. The bright northern part of the tail could be said to be roughly cone shaped, with its base along the horizon, and tapering out and becoming faint toward *θ Aquilae*. As dawn approached, say a little after 15^h, the whole sky seemed to assume a feeble glow that did not appear to be entirely due to dawn. The observations, located on a celestial globe, make the length of the tail at least 120°.

On both nights (May 17 and 18) there was no decided light north

of the tail, that is, all the sky above the tail was apparently pure and free from unusual illumination, with the exception of the slight mistiness mentioned in the preceding paragraph. The illumination below the brighter part of the tail was decided. It was soft and seemed to reach to and beyond the southeast horizon.

Although the slight aurora which developed on May 18 certainly had nothing to do with the proximity of the comet, it seems best to give it here as a part of the record for that night for comparison with observations that were doubtless made elsewhere and for other reasons. I have therefore collected all the phenomena that certainly seemed to belong to the aurora in the notes that follow.

At 8^h37^m there seemed to be some horizontal streaks of diffused light in the north above *Cassiopeia*. They had disappeared five minutes later. At this time considerable twilight effect still remained in the northwest. At 8^h56^m some feeble luminosity was visible under *Cassiopeia*. At 9^h2^m apparently a slight aurora was visible to the right of and below *Cassiopeia*. At 9^h14^m a faint luminous band was visible halfway from the horizon to the stars of *Cassiopeia*. This seemed to be an auroral effect. At 9^h20^m an active aurora with streamers flashed up very suddenly. By 9^h28^m it had become a uniform glow extending almost as high as *Cassiopeia*. There were feeble attempts at activity at 9^h40^m consisting of a great number of short streamers. At 10^h4^m the altitude of the bright part of the arch (which was fairly strong) was exactly halfway from the horizon to α *Cassiopeiae*. At 10^h12^m streamers were ascending to the left of the summit of the arch and moving to the left. At 10^h16^m the arch was rather strong but indefinite. At 10^h18^m the aurora was again active (but not bright), with diffused streamers. At 10^h28^m there was no definite arch, but a diffused general illumination was present reaching nearly as high as *Cassiopeia*. At 10^h32^m still diffused, with no definite arch, and one streamer moving west. The brightest part of the illumination extended halfway to *Cassiopeia*. By 10^h40^m the aurora had almost faded out—apparently dead. At 11^h10^m a very slight auroral glow. At 11^h24^m there seemed to be no aurora. At 11^h32^m, no aurora. At 13^h52^m the aurora started up again with a very low feeble arch. At 14^h14^m the altitude of the arch was $3°$ or $4°$.

There are no other notes on the aurora, and I assume that it finally disappeared about this time.

Notes were also made of the appearance of any meteors, but they are not given here because but very few were seen on May 18 and none was noted on May 17. They seemed to have no connection with the comet.

May 19. Cloudy at night. No observations of the comet possible.

May 20. At $7^h 50^m$ to $8^h 30^m$ with the naked eye the head was one-half degree in diameter. The head and nucleus were of about the second magnitude, and resembled a yellow nebulous star. There seemed to be a faint diffused tail. The sky was muggy, and the comet was in clouds most of the time.

In the 5-inch the nucleus at first was very stellar and very yellow, with some of the hazy yellow light about it, but no tail was seen with certainty.

The sky was examined repeatedly as late as 11^h, but no trace of the comet's tail was anywhere visible. The moon was nearly full.

At 14^h the sky was hazy in the west. At about $14^h 30^m$ a hazy luminous streak $4°-5°$ broad extended from θ *Aquilae* to the east —fainter toward θ *Aquilae*—through α *Pegasi*. This resembled the comet's tail, but was doubtless a strip of haze. I looked at it several times, taking it for a strip of haze, but it did not seem to move. There were masses of moving haze overhead toward the north. To all appearances it looked like the comet's tail of the mornings of May 18 and 19. I cannot be certain that this was not haze, but it was a singular coincidence of position, appearance, etc., if it was. It was visible for fully 15 or 20 minutes. Then the sky got worse with the haze and moonlight and it disappeared. At the same time there seemed to be a similar strip in the low south which stretched from the Milk Dipper in *Sagittarius* to *Antares*. This was $3°$ to $4°$ wide. I think these must have been merely strips of haze, and had nothing to do with the comet, but they are given here as a matter of record. It may be well to note, however, in this connection that the tail (both branches) was still visible in the morning sky in South Africa on this date, at $15^h 30^m$ G.M.T., or

only five hours earlier than the supposed observation recorded above (see *Circulars 3* and *11*, Transvaal Observatory). Both tails were also seen by Perrine at Cordoba on the 20th at the same moment as my observation (see *Astronomical Journal*, 26, 145).

May 21 and 22. Cloudy.

May 23, 8^h40^m. Sky cloudy, but the comet shone for a few minutes through a break in the clouds. It was very bright—perhaps brighter than the first magnitude. To the naked eye the nucleus and coma appeared like a nebulous star. There was some faint tail. During the exposures the sky was white with a full moon and haze, patches of which frequently covered the comet. The total eclipse of the moon on that night unfortunately came too late to aid in the observations of the comet.

May 24. At 7^h55^m the comet was quite bright to the naked eye, with traces of the tail. It was bluish white and a striking object. The head was large and hazy and about $15'$ or $20'$ in diameter. The nucleus resembled a first-magnitude star in haze. The tail was $25°$ long. For $5°$ or $6°$ it was noticeable and then became rapidly fainter. At 8^h35^m it was quite noticeable for $10°$ or $15°$. At $10°$ from the head the tail was about $2°$ wide. The light of the comet was still bluish white. At 8^h50^m the tail could be traced with the naked eye for $29°$. At 9^h10^m the sky was very bright with moonlight and did not seem to be very clear, but the tail was still noticeable to the naked eye. Ten minutes later the tail was very feeble for want of contrast and at 10^h0^m it was scarcely visible to the naked eye, but the head was still very bright like a hazy star. At 10^h30^m the comet was still visible, very low and dim.

In the 5-inch telescope the nucleus was sharply defined, not a point, but more like a small bright planet with coma. At 8^h45^m it was nearly white. It also appeared white to the naked eye. In the last part of the exposure the nucleus was about five times greater in diameter than when the exposure began, and more ill defined—it seemed to swell in size.

[The last part of the foregoing paragraph is in accord with the observations of Professor A. E. Douglas at Tucson, Arizona, who later, on this same date, saw the nucleus double. See *Harvard Observatory Bulletin*, No. 412.]

May 25. At 8^h30^m the tail could be traced for 21°. The head was decidedly less intense than ϵ *Hydrae* south of it. The sky was quite good. At 8^h50^m the tail could be traced for eight-tenths the distance between the head and *Jupiter*, or a length of 43°. At 8^h55^m the comet was seen on a fairly dark sky, only a little twilight effect remaining. It was very beautiful, though the head did not seem relatively so bright as on other nights. The tail, for about 20°, was pretty bright, and increased very much in width. At 9^h0^m it seemed to extend in a very diffused manner nearly to the same right ascension as that of *Jupiter*, a distance of 54°. Prolonged, its axis would pass about 8° south of *Jupiter*. The tail was very diffused at its end and seemed to extend northward nearly to *Jupiter*. At 9^h10^m the central brightness of the head was almost bluish white in the field-glasses. At 9^h30^m the sky had begun to whiten with moonlight, but the comet was still in good relief, the moon being behind clouds in the east. At 9^h35^m the tail could be faintly traced several degrees beyond 87 *Leonis* = *B.D.* − 2°3360 (magnitude 5.0), which at this time was in the axis of the tail, or for about 43°. At 9^h55^m the tail was very dim on the bright moonlit sky, but was still faintly visible for 10° or more.

At 8^h30^m with the 5-inch telescope the nucleus was very small, like a ninth- or tenth-magnitude star. The coma was very large and fairly bright. Before the wires were illuminated the nucleus did not appear double, nor were there any other nebulosities in the field of view. It was very dim and hazy. At 8^h50^m the nucleus was of the same brightness as *B.D.* + 7°2055 (magnitude 8.4). At 9^h15^m the nucleus was a little brighter and hazy. At 9^h35^m it was fairly stellar and a little brighter—brighter than any of the stars in the field of view: (*B.D.* + 7°2048 [magnitude 9.0], + 7°2052 [magnitude 9.2], + 7°2055 [magnitude 8.4]). At 9^h50^m the coma was very dense and extended perhaps 5′ all around the nucleus, which was very small and dim.

May 26, 7^h55^m. The comet was visible to the naked eye as a faint hazy star. At 8^h5^m it was quite noticeable, with perhaps faint traces of tail. At 8^h22^m the tail was showing faintly to the naked eye. The comet seemed less bright than on the previous night. At 8^h30^m, with field-glasses, there seemed to be a central

nucleus, fairly well defined, large, and bluish white, surrounded by much fainter hazy nebulosity which extended from it to form the tail. It had the same appearance to the naked eye. This, however, was not the true nucleus, which was very small and seen only in the telescope. The tail was visible for 10° or 15°. At 8^h40^m the tail was very noticeable for about 15°, but it faded very rapidly toward the end. To the eye the nucleus was bright—of the second magnitude. At 8^h45^m the sky was very good and the tail was very noticeable. It could readily be traced to 87 *Leonis* With field-glasses the nucleus was an intense bluish white. The whole head seemed to be of a bluish-white color. At 8^h50^m the tail was conspicuous halfway to 87 *Leonis*, after which it became diffused and faint. It seemed a little brighter in the middle near the head. No streamers were seen. The sky was fairly dark and the comet a conspicuous and strikingly beautiful object. But the nucleus was very much inferior to *Regulus*. At 8^h55^m the tail was conspicuous as far as 87 *Leonis*, and, though rather faint near that star, it could be traced feebly 10° beyond it. To the naked eye the nucleus was *very* much brighter than the rest of the head. At 9^h0^m the tail could be very feebly traced beyond *Jupiter*. The axis would pass 6° or 7° south of the planet. At 9^h10^m the nucleus was about as bright as δ *Leonis* (magnitude 2.6). The comet was a very striking object to the naked eye, with the tail, which seemed to be straight, reaching as far as 87 *Leonis*, where it became faint. At 9^h15^m, by hiding *Jupiter*, the tail could be feebly traced to α *Virginis*, or a length of about 65°. The nucleus was perhaps one-half a magnitude less bright than γ *Leonis* of magnitude 2.6. At 9^h20^m the sky was still good. The tail for 15° from the head was everywhere brighter than *Praesepe*. Within 5° or 10° of the head it was 4 or 5 times as bright as *Praesepe*. At 9^h45^m the comet was seen on a fine dark sky and was very conspicuous. In the field-glasses the tail widened out very much. The nucleus was large and bluish white and was surrounded for a short distance by a hazy glow of the same color. There seemed to be no structure in the tail. At 9^h50^m the moon was whitening the eastern sky, but the tail was still noticeable as far as 87 *Leonis*, where it became faint. It gradually widened out, with the south side perhaps a little the

brighter. To the naked eye the nucleus was rather dull. In the field-glasses it was still bluish white and hazy, like a star shining through a bluish-white mist. There did not seem to be any evidence of streamers, either with the naked eye or with field-glasses. At $10^h 20^m$ the sky was very bright with moonlight, but the comet's tail was still noticeable (though not very strong) for some $10°$, and could be traced faintly as far as 87 *Leonis*. At $10^h 30^m$ the tail could still be traced feebly for $10°$ or more from the head. At $10^h 50^m$ the comet was very near the horizon and disappearing in some tree tops, and nothing could be seen of the tail with the naked eye.

At $7^h 55^m$, with the 5-inch telescope the nucleus was small and planetary, and with the coma was very yellow. The nucleus was larger and perhaps slightly brighter than *B.D.*$+6°2129$ (magnitude 8.0), which was in the field. At $8^h 40^m$, though less intense, it was brighter, and more yellow than the star. It seemed to be very much brighter than on May 25. The nucleus was estimated to be decidedly brighter than the star at $9^h 5^m$, and at $9^h 10^m$ it was stated that it must have brightened since the exposure began. It was ill defined and perhaps $5'' \pm$ in diameter. At $9^h 35^m$ the nucleus was very much brighter. It seemed to have increased greatly in brightness but was very ill defined. At $10^h 5^m$ it was decidedly more yellow and perhaps a little brighter than the same star, though its light was not so intense. It was very hazy and much larger than the star. At $10^h 50^m$ the comet and nucleus were both very faint.

May 27, $8^h 0^m$. To the naked eye the comet resembled a small dim cloud, in which the nucleus was small and faint. The sky was smoky, and had been so almost all the late afternoon. At $8^h 15^m$ the comet was dull to the naked eye, like a dull nebula some $10'$ or $15'$ in diameter. One could not be sure of seeing any tail at this time. At $8^h 25^m$ the tail could be feebly seen for $4°$ or $5°$. At $8^h 35^m$ it was only feebly visible for perhaps $5°$ or $6°$, but it could be seen fairly distinctly. The sky was very poor with some twilight illumination. At $8^h 38^m$ the tail could be seen rather dimly for about $10°$—sky still luminous. In the field-glasses the condensation or nucleus was of a bluish-white color. The rest of the head and

tail were whitish. At 8^h42^m the tail could be traced faintly to 87 *Leonis* (about $33°$), and near the head for $6°$ or $8°$ it was quite noticeable. The comet did not seem as bright as on May 26, but the sky was poor and whitish. The nucleus was about midway in brightness between γ (magnitude 2.6) and ζ *Leonis* (magnitude 5.1), or about magnitude 3.8. At 9^h0^m the tail seemed to be brighter on the south side and could be traced quite distinctly to 87 *Leonis* (which star was apparently in the axis, or perhaps a little south of it), after which it became faint. For one-half that distance it was conspicuous. The sky was fairly dark but it was not pure. With the field-glasses the tail near the head was feebly brighter in the middle. At 9^h10^m the condition of the sky, though not pure, was fair. Possibly the tail was slightly curved, with the convex side south. It was quite noticeable as far as 87 *Leonis*. At 9^h15^m there did not seem to be any structure in the tail as seen with the field-glasses. To the naked eye the comet was a conspicuous object. The tail near the head was very much brighter than *Praesepe*, but it faded off rapidly near 87 *Leonis*. By hiding *Jupiter* it could be traced to a point halfway between *Jupiter* and α *Virginis*, or for a distance of $53°$ or $54°$. It seemed certainly to be curved when the whole tail was considered, with the convex side toward the south. Near *Jupiter* it was perhaps $3°$ in width and faint. At 9^h30^m, with the field-glasses, what appeared to be the nucleus was of sensible diameter and hazy and was very strongly conspicuous. At 9^h35^m there seemed to be a diffusion from that part of the tail near *Jupiter*, extending to the north as high as the planet (α 12^h19^m, $\delta -0°31'$). The star 87 *Leonis* was perhaps a little south of the middle of the tail. The nucleus was in pretty strong contrast to the tail near the head. At 9^h40^m the tail, from 87 *Leonis* to the end, became exceedingly faint and diffused. At 9^h45^m the comet was still a conspicuous object, with the tail extending to 87 *Leonis*. The nucleus resembled a dull hazy star of the third magnitude. The sky was fair, though not specially pure. At 10^h20^m the comet's head was getting down into the haze near the horizon, but was still strongly conspicuous. By 10^h30^m the head was becoming dim to the eye. The tail was still noticeable and could be traced readily to 87 *Leonis*. At 10^h40^m the head

was very dim. The tail also was very dim, but could still be traced to 87 *Leonis*. The sky up high and overhead was very clear, but near the horizon there was a good deal of smoky haze.

At 8^h5^m the nucleus, in the 5-inch telescope, was faint and ill defined, with some haze about it. It was very much fainter (perhaps two magnitudes) than the star *B.D.*$+5°2171$ of magnitude 8.1, which was in the field with it. At 8^h45^m the nucleus was very dim, and was very feebly contrasted with the nebulosity. At 8^h50^m what was so conspicuous as a nucleus to the naked eye could not have been the true nucleus, for in the 5-inch the nucleus was very small and faint and was apparently only a condensation in the coma. It must, therefore, have been the brighter part of the coma which formed a nucleus to the naked eye. At 9^h10^m it seemed to have grown dimmer. It was rather difficult to guide on and was very small and ill defined. At 9^h35^m the nucleus and coma appeared very much like the nucleus and close nebulosity of the Great Nebula of *Andromeda* when seen in an ordinary telescope, and showed about the same amount of contrast, the bright part of the coma being about $1'-2'$ in diameter and very diffused. At 9^h50^m the true nucleus was very small and dim, and was several times fainter than the star *B.D.*$+5°2171$. It was surrounded by a dense nebulosity $1'$ or more in diameter. This nebulosity must have been what appeared to the naked eye as the nucleus. At 10^h5^m the nucleus was difficult to guide on. The glow about it was very strong and there was no contrast. It was simply a central condensation of the coma. At 10^h15^m the nucleus was just discernible, being all but lost in the coma. At 10^h35^m it was no longer visible to guide on. I do not think its faintness was due entirely to the condition of the sky.

May 29. At 10^h5^m the tail was conspicuous as far as 87 *Leonis*, a distance of 27°, its axis passing about one-half degree north of that star. It could be feebly traced beyond the line between *Jupiter* and *Spica*, or about 52°, but only feebly. It was noticeably curved convex to the south. The southern side, from the head to 87 *Leonis*, was a little the brighter and more definite. The nucleus was about as bright as η or θ *Leonis*. The tail seemed to diffuse to the north to *Jupiter*, and perhaps beyond. The southern

edge would about bisect the line between *Spica* and *Jupiter*. The comet was decidedly less bright than on May 27.

In the 40-inch the nucleus was not yellow, but was pale in color. The measured diameter north and south from one setting was 2″6.

May 30. At 8^h5^m the comet could be feebly seen with the naked eye like a faint nebula. At 8^h20^m it was not as bright nor as noticeable as the star π *Leonis*. There was no tail visible at this time. The sky was very clear. At 8^h30^m with the field-glasses the tail could be traced for a couple of degrees. The head was quite bright and seemed to be a nebulous mass without any special nucleus. There was perhaps a faint suggestion of a tail for a couple of degrees, but very faint. The head was about as bright as π *Leonis* (magnitude 4.9), perhaps a little brighter, but more noticeable than that star. At 8^h40^m the tail could be traced as far as 87 *Leonis*, a distance of 25°, but was faint toward its end. The sky was still bright with twilight. At 8^h50^m the comet was quite conspicuous. The tail was noticeable as far as 87 *Leonis* and was seen faintly 7° or 8° beyond that star (which seemed to be nearly in the axis of the tail). For about 15° it was conspicuous. The head was of about the third magnitude, and with the field-glasses resembled a bright hazy nebulosity. At 9^h15^m the tail seemed decidedly curved between the head and 87 *Leonis*, with the convex side to the south. At about 5° from the head it was of about the same brightness as *Praesepe*. Nearer the head it was brighter. With the field-glasses the condensation in the head looked like a large diffused nucleus, bluish white in color, surrounded by a fainter nebulosity which extended back to form the tail. The central brightness was very strong as compared with the rest of the head. At 9^h25^m the comet, to the naked eye, was very dull as compared with its appearance a few nights earlier, but it was still conspicuous. The tail, where it passed below *Jupiter*, had the same appearance of diffusing and spreading out toward that planet previously noted, but *Jupiter* was too bright to make this certain. The sky was very transparent, especially in high altitudes. At 9^h40^m the comet was a striking object. The tail was conspicuous as far as 87 *Leonis*, after which it became faint, but by hiding

Jupiter it could be feebly traced as far as the line between *Jupiter* and *Spica*, a length of 47°. With the field-glasses no structures or irregularities could be seen in the tail, which diffused very softly toward the edges and was not especially brighter in the middle. No streamers were visible either with the naked eye or with the field-glasses. At this time it was still a conspicuous object. At 10^h30^m, the comet, though low, was still conspicuous. The sky seemed to be very good in its direction. The head was quite bright, and the tail could be readily traced to 87 *Leonis*. At 10^h45^m the head was quite bright, like a second- or third-magnitude star, but the tail was lost in clouds.

During the exposures on the comet there seemed to be a denser part some 10° back from the head, as if the tail sagged a little south at that point.

At 8^h5^m, though the head was distinct to the naked eye, it was very small and faint in the 5-inch guiding telescope. At 8^h35^m the nucleus was very small and starlike and shone in the middle of a dense nebulosity about 0\'5 in diameter. It was one magnitude brighter than the star *B.D.*+3°2273 of magnitude 9.2. At 9^h10^m the nucleus was very small and starlike in a very dense nebulosity which diffused gradually for 1′±. It was very much brighter than *B.D.*+3°2273—about $1\frac{1}{2}$ magnitudes brighter. The sky was very clear. At 9^h45^m the nucleus was very small and stellar with some haze close about it. It was not very much brighter than *B.D.*+3°2273. At 10^h17^m the nucleus was almost lost in the strong condensation about it.

May 31. The comet was first seen with the naked eye at 8^h17^m. The sky was covered more or less with hazy clouds, but at about 10^h45^m, when seen for a few minutes below the clouds, it was conspicuous.

In the 40-inch telescope at 8^h42^m the measured diameter of the nucleus, north and south, was 4\"9.

June 1. At 8^h10^m the comet was faintly visible to the naked eye. At 8^h40^m only faint traces of the tail could be seen. The sky at this time was covered with hazy clouds from the northwest. At 9^h0^m, in spite of the condition of the sky, the tail could be traced to 87 *Leonis*, a distance of 22°. The comet was covered with hazy

clouds nearly all the time. There were long strips of these clouds moving southwardly, which were, most of the time, only a few degrees wide, and if they had been displaced 4° or 5° the comet would have been seen on a good sky throughout the observations. Once in a while it came out for a few minutes only to be covered again. The rest of the sky was good. After 10^h it got on to a better sky and there was very little interference from clouds, but the sky was not good in the direction of the comet. The tail could be traced for several degrees beyond 87 *Leonis*, or perhaps for about 25°, and was noticeable as far as that star. The head, which seemed brighter on this date, was about midway in brightness between γ and η *Leonis*, or $3^m.1$. At 11^h0^m, though the comet was very low and dim, the tail, in moments of freedom from clouds, could be seen up to 87 *Leonis* fairly well, and could be traced some degrees farther. Its axis passed slightly north of that star.

At 8^h10^m in the 5-inch the nucleus was very faint and small —just visible—in a strong condensation. At 8^h58^m it could no longer be seen to guide on.

June 5, 9^h7^m. To the naked eye the head was about one-half magnitude brighter than the star 15 *Sextantis* = *B.D.* +0°2615 (magnitude 4.1) and more conspicuous than that star. The tail, which seemed to be straight, could quite readily be traced to 87 *Leonis* and perhaps a few degrees beyond, but it was dim. It was visible in a diffused manner to the star χ *Virginis*, or about 33°. To the naked eye a faint nucleus was doubtfully visible. The head was about as bright as 87 *Leonis*, and not much more noticeable than that star.

The comet was first seen in the finder of the 40-inch at 8^h9^m. With the 40-inch telescope itself at 8^h20^m, the nucleus was very small, 2″ or 3″ in diameter, and surrounded by a dense nebulosity. At 9^h0^m it was a very small point in dense hazy light which was placed in a very strong nebulosity which faded away rapidly, and was perhaps 3′ or 4′ in diameter. The minute nucleus was about three magnitudes less than the comparison star (estimated magnitude 9±; see *Astronomical Journal*, **27**, 149, 1912), but the general brightness of the head would be about $2\frac{1}{2}$ magnitudes less than the star.

In the 5-inch telescope no nucleus was visible. There was only a strong condensation which was rather hard to guide on.

June 6. At about 8^h27^m the comet became visible to the naked eye as a faint hazy spot. At 8^h47^m the tail was not yet visible. The head was not quite as noticeable as the star ρ *Leonis* (magnitude 3.8). At 8^h50^m the tail could be traced for a distance of $5°$. At 8^h57^m it could be seen faintly to 87 *Leonis*, a distance of $18°$. At 9^h10^m it was noticeable as far as 87 *Leonis*, which was on its upper (north) edge, and could be seen feebly several degrees beyond. Sky good. At 9^h57^m the tail could be traced to χ *Virginis*, or for $32°$. Though very faint, it was noticeable as far as 87 *Leonis*. The comet had faded sadly, however, since June 1, and though a noticeable object, was only the ghost of its former self.

At 8^h25^m it was quite conspicuous in the 5-inch telescope, with a bright starlike nucleus, which was about one-half magnitude less bright than the star *B.D.*$+0°2641$ (magnitude 8.0). At 8^h35^m the nucleus was beautifully starlike, and imbedded in a very strong condensation that faded rapidly and was itself nebulous. When best seen at 9^h42^m the nucleus was about one magnitude less than *B.D.*$+0°2641$, or about the ninth magnitude.

June 7. At 9^h40^m the tail was faint, but could be traced to 87 *Leonis* (which star was in the north edge of the tail), a distance of $17°$. The entire comet was fainter than on June 6. The head was of about the same brightness as the star ρ *Leonis*. It was relatively fainter with respect to the tail than at previous observations. The sky was poor and the Milky Way dull. There were no clouds, however.

The comet was first visible in the finder of the 40-inch at 8^h8^m. In the 40-inch telescope itself the nucleus, which was in a very strong condensation, was very ill defined and blurred.

June 9. At 10^h35^m the sky was murky and broken with clouds. The comet was only fairly visible to the naked eye. At best the tail could be very faintly traced to 87 *Leonis*, or for $15°$. In the latter part of the observations the sky was good everywhere else but in the region of the comet, which was covered with misty clouds.

June 10. Crescent moon. At 8^h47^m the comet was not visible with the naked eye, but later it could be seen faintly, with possibly a trace of tail. It was very faint in the 5-inch telescope.

June 11. The sky was clear at dark but a crescent moon was shining. From about 9^h15^m the comet could be seen faintly at intervals for perhaps half an hour. The head alone was visible as a dim hazy star, and was only just seen with certainty. Clouds kept covering the place so that the exposures were badly interrupted.

A faint small nucleus could be seen in the 5-inch telescope.

This was the last date on which the comet was seen with the naked eye.

June 12. In the 40-inch the nucleus was very ill defined—not stellar. It was placed in a small dense nebulosity $5''$ in diameter, which diffused into the general nebulosity of the head. The nucleus was of about the same brightness as the comparison star (estimated magnitude $10\pm$; see *Astronomical Journal*, **27**, 149, 1912), or a little less bright, but was less definite. The sky was very white with moonlight.

June 14. With the 40-inch telescope the nucleus was almost stellar and about one-half magnitude less than the comparison star (estimated magnitude 9.5–10; see *Astronomical Journal*, **27**, 149, 1912). It was in the center of a strong condensation about $1'$ in diameter. The sky was thick and bright with strong moonlight.

June 15. Only the faint nucleus and central condensation were visible in the 5-inch telescope. The comet was very faint and dim throughout the exposures.

June 24. At 9^h10^m in the 5-inch the comet was somewhat strongly condensed with no nucleus, and was perhaps of the eighth magnitude. It was not certainly seen with the naked eye. At 9^h42^m it became too faint in dense haze to guide on with the 5-inch.

June 25. At 8^h40^m it was of about the eighth magnitude and rather small and dim in the 5-inch telescope. By 9^h52^m it could no longer be followed. The sky was very good. The comet could not be seen with the naked eye, but it was quite noticeable in the field-glasses, with which perhaps faint traces of the tail could be seen.

June 27. At 9^h0^m the comet, though seen in the 5-inch telescope, was very feeble and too faint to attempt an exposure.

GREATEST VISIBLE LENGTH OF THE TAIL

For the convenience of those interested in the matter, Table II contains the greatest lengths of the tail as seen with the naked eye during these observations.

TABLE II

Date		Length of Tail	
May	3	17°–18°	
	4	15	
	6	17 –18	On hazy sky
	9	15	On very bad sky
	14	53	
	17	107	
	18	120 or more	
	24	29	
	25	54	
	26	65	
	27	53	
	29	52	
	30	47	On poor sky
June	1	25	
	5	33	
	6	32	
	7	17	
	9	15	

A list of the photographs obtained with the various lenses of the Bruce photographic telescope is given in the catalogue of the report of the Comet Committee of the Astronomical and Astrophysical Society of America.

YERKES OBSERVATORY
WILLIAMS BAY, WIS.
March 7, 1914

ELEMENTS OF THE ECLIPSING VARIABLE STARS *Z DRACONIS* AND *RT PERSEI*

By HENRY NORRIS RUSSELL AND HARLOW SHAPLEY

The very accurate light-curves determined by Dugan for *Z Draconis* and *RT Persei* give an opportunity for carrying the solutions for the orbital elements to a much higher degree of precision than is usually practicable. The following discussion will give the application of the method of least-squares to the derivation of definitive elements from photometric work, and will treat of an annular eclipse when the star's disk is supposed to be darkened at the limb.[1] The observations afford conclusive evidence that in both cases the stars are really darkened at the limb, and in the case of *RT Persei* make possible the determination of the eccentricity and the longitude of periastron from the photometric data alone.

I. The star *Z Draconis* (*B.D.*$+73^\circ 533$, $11^h 39^m 8$, $+72^\circ 49'$) was found to be an *Algol* variable of large range by Mme. Ceraski in 1903.[2] The present discussion is based upon the photometric observations by Dugan, published in No. 2 of the *Contributions from the Princeton University Observatory*. These number 1149, each representing the mean of 16 settings, and cover the entire period, somewhat more thickly during the times of eclipse than elsewhere. The period, $1^d 8^h 34^m 40^s.95$, and the epoch of principal minimum are taken from Dugan's discussion (*op. cit.*, p. 16), and normals have been formed from the data given in his table of observations arranged in order of phase (pp. 19–39).

The resulting normals for that part of the light-curve which lies outside the principal minimum are given in Table 1. The first column gives the number of observations combined into a normal—18 during the secondary minimum, and usually 36 outside eclipse—the second the mean phase, counted from principal

[1] For a summary of the formulae and notation, and references to the original discussions and tables, see *Astrophysical Journal*, **36**, 404, 1912. All references in the text to tables with Roman numbers are to these papers.

[2] *Astronomische Nachrichten*, **161**, 159, 1903.

minimum, and the third the mean observed light-intensity, derived from the values given by Dugan. The unit of intensity corresponds to a stellar magnitude fainter by 1.477 than his comparison star, *B.D.*+72°545—that is, according to the data of p. 4 of his memoir, to the magnitude 10.46 on the Harvard scale. The fourth column gives the residuals from the definitive solution described below, and the remaining columns the coefficients of the equations of condition from which this solution was derived.

TABLE 1

No. Obs.	Phase	Observed Intensity	O−C	Coefficients of			
				δb	δc	δd	e
36	$+3^h26^m$	0.073	−0.008	−0.79	−0.63	...	...
36	4 48	0.082	−0.006	−0.60	−0.36		
42	5 38	1.002	+0.010	−0.46	−0.21		
36	6 20	0.981	−0.014	−0.34	−0.12		
36	7 01	1.001	+0.003	−0.20	−0.04		
36	7 43	0.992	−0.010	−0.10	−0.01		
36	8 35	1.010	+0.004	+0.09	−0.01		
42	9 25	1.011	+0.002	+0.25	−0.06		
36	10 08	1.018	+0.007	+0.38	−0.15		
36	11 08	1.009	−0.003	+0.55	−0.30		
42	+12 50	1.006	−0.008	+0.70	−0.62		
18	+14 04	1.007	−0.005	+0.02	−0.85	−0.04	+0.04
18	15 06	1.008	+0.022	+0.08	−0.07	−0.40	+0.12
18	15 38	0.941	−0.013	+1.00	−1.00	−0.85	+0.12
18	16 30	0.945	+0.002	+1.00	−1.00	−0.99	−0.02
18	17 11	0.959	0.000	+0.09	−0.08	−0.68	−0.13
18	17 48	0.990	+0.016	+0.07	−0.03	−0.30	−0.12
18	18 27	0.989	−0.014	+0.01	−0.83	−0.16	−0.07
24	+19 05	1.002	−0.011	+0.85	−0.72	−0.03	−0.02
36	−12 00	1.017	+0.003	+0.68	−0.46		
36	−10 30	1.021	+0.010	+0.44	−0.19		
36	−8 51	0.998	−0.011	+0.14	−0.02		
36	−7 13	0.991	−0.008	−0.18	−0.03		
36	−5 45	0.992	0.000	−0.44	−0.19		
36	−4 36	1.000	+0.023	−0.63	−0.40		
27	−3 27	0.982	+0.001	−0.79	−0.63		

The characteristics of this portion of the light-curve have been clearly interpreted by Dugan (pp. 41-42). There is a shallow but unmistakable secondary minimum, which follows the primary at an interval slightly greater than half the period. Outside eclipse the intensity rises toward secondary minimum, showing that the companion is brighter on the side which receives the radiation of the principal star than on the other; and the light-curve is

slightly convex upward, indicating a small ellipticity of the stars. Values of the constants involved in these relations are given by Dugan; but these have not been derived by a least-squares solution, and the number and accuracy of the observations justify such treatment.

If θ is the orbital longitude of the bright star, measured from the principal conjunction, and l the observed intensity, we should have:

$$l=a-b\cos\theta-c\cos^2\theta-nd\ ,$$

where $2b$ represents the difference in light-emission of the two sides of the companion, $1-c$ is the ratio of the minor to the major axes of the prolate spheroidal stars, d is the depth of secondary minimum, and n the loss of light at the corresponding phase during principal minimum, in terms of the loss at mid-eclipse. (This assumes that the intensity-curves of the two minima are of similar form, and differ only in depth—which will be fully justified by the elements given later.)

From a preliminary solution, the approximate values were found $a=1.004$, $b=0.021$, $c=0.010$, $d=0.072$, the middle of the secondary minimum being at phase 13^h30^m. In the equations of condition summarized in Table 1, δa, δb, δc, δd represent corrections to be added to the foregoing values, and $200\ e$ is the correction in minutes to the assumed time of secondary conjunction. The coefficients of this last quantity were read from a plotted curve. All the normals outside eclipse were given weight 1, and those during the secondary minimum weight $\frac{1}{2}$. The resulting normal equations are:

$$\begin{aligned}
+22.00\delta a+2.60\delta b-8.08\delta c-1.72\delta d-0.04\ e&=-0.0065\\
+\ 2.60\delta a+8.08\delta b-3.00\delta c-1.70\delta d-0.04\ e&=-0.0025\\
-\ 8.08\delta a-3.00\delta b+5.30\delta c+1.68\delta d+0.04\ e&=-0.0006\\
-\ 1.72\delta a-1.70\delta b+1.68\delta c+1.22\delta d+0.00\ e&=-0.0012\\
-\ 0.04\delta a-0.04\delta b+0.04\delta c+0.00\delta d+0.034e&=+0.0001\ ,
\end{aligned}$$

whence we find:

	Least-Squares	Dugan
$\delta a=-0.0008$,	$a=1.003\pm0.0023$	1.006
$\delta b=-0.0008$,	$b=0.020\pm0.0020$	0.016
$\delta c=-0.0014$,	$c=0.009\pm0.006$	0.016
$\delta d=-0.0013$,	$d=0.071\pm0.009$	0.059
$e=+0.002\pm0.037$		
Secondary conjunction	$13^h30^m\pm7^m$	13^h30^m

The corrections are all insensible, which is not surprising, as the preliminary solution was made on least-squares principles. The quantities O−C given in Table 1 are therefore at the same time the absolute terms of the equations of condition and the final residuals. The probable error of the unit of weight, corresponding on the average to the mean of 37 observations, is ±0.0067 in terms of the whole amount of light measured, which corresponds to $\pm 0^{m}.0073$ in stellar magnitude. This gives for the probable error of the mean of six observations the value $\pm 0^{m}.018$. For the same quantity Dugan finds, from the residuals from a free-hand curve, the value $\pm 0^{m}.016$. The values of the other constants found by Dugan, which are given above, are also in satisfactory agreement with those here found.

The secondary conjunction comes 13 minutes later than the moment halfway between principal conjunctions. As Dugan has shown (p. 42), this is evidence of a small orbital eccentricity, such that $e \cos \omega = +0.01$, where ω represents, as usual, the longitude of periastron measured from the ascending node of the orbit of the brighter star. The secondary minimum is so shallow that there can be no hope of determining $e \sin \omega$ from the light-curve, and it is legitimate to assume a circular orbit in the calculations which follow.

We have first to rectify the observed light-curve, that is, to remove the effects of ellipticity and the radiation effect, which is done by adding to each observed intensity l the quantity 0.020 $(1+\cos\theta)+0.009\, l \cos^2\theta$, and then dividing by 1.023, to reduce the intensity outside eclipse to unity. The rectified intensity at the middle of the secondary minimum is 0.930, which makes the depth of this minimum $0^{m}.079$. The observed and rectified intensities during the principal minimum are given in Table 2. The normals give the mean of 12 observations, except within $1^{h}20^{m}$ of the middle of eclipse, when they depend on 6 observations. The first column gives the mean phase for each of these normals, the second the mean observed intensity, and the third the rectified intensity. It will be observed that the difference between the two amounts to a considerable fraction of the observed intensity near the time of greatest eclipse. The reason for this is that, in recti-

fying the curve, we assume that the fainter side of the companion (which sends us most of the light received at this time) has been increased in brightness until it equals the other side. The fourth and fifth columns of the table give the residuals resulting from the final solutions made on the two hypotheses of uniformly bright star-disks and of disks darkened to zero intensity at the limb.

TABLE 2

Phase	Obs'd Int	Rectified Int.	Uniform O−C	Darkened O−C	Phase	Obs'd Int.	Rectified Int	Uniform O−C	Darkened O−C
-2^h53^m3 ...	0.968	0.989	−0.011	−0.011	$+0^h03^m0$.	0.097	0.135	+0.001	+0.002
−2 41.1....	0.967	0.988	−0.012	−0.009	+0 16 0.	0.127	0.164	−0.004	−0.004
−2 28 4..	0.942	0.964	−0.027	−0.016	+0 24 4 .	0.178	0.214	+0.004	+0.003
−2 16.9 ...	0.920	0.943	−0.008	+0.002	+0 32 2	0.228	0.264	+0.003	+0.003
−2 03 8....	0.878	0.902	+0.002	+0.007	+0 41 7..	0.273	0.308	−0.022	−0.020
−1 47 8...	0.787	0.813	+0.004	+0.002	+0 48 1.	0.340	0.373	−0.006	−0.004
−1 32.3..	0.688	0.717	+0.007	+0.005	+0 55 1	0.403	0.436	+0.006	+0.004
−1 21.0.	0.608	0.637	+0.010	+0.007	+0 59 9	0.439	0.471	+0.004	+0.003
−1 12.1..	0.527	0.558	−0.004	−0.005	+1 04 4 .	0.491	0.523	+0.020	+0.018
−1 04 7....	0.466	0.499	−0.008	−0.010	+1 09 0..	0.515	0.546	+0.006	+0.006
−0 58 2.	0.424	0.457	+0.003	0.000	+1 13 2 .	0.538	0.569	−0.001	−0.003
−0 52.2...	0.390	0.423	+0.013	+0.014	+1 17 3	0.567	0.597	+0.005	+0.004
−0 47 0...	0.342	0.376	+0.007	+0.009	+1 26 0 .	0.650	0.679	+0.016	+0.013
−0 42.5..	0.305	0.339	+0.002	+0.005	+1 36 0	0.707	0.735	+0.001	0.000
−0 37 5..	0.271	0.306	+0.007	+0.008	+1 45 2 .	0.757	0.784	−0.008	−0.010
−0 32.7...	0.234	0.271	−0.004	−0.005	+1 53 8	0.809	0.835	−0.010	−0.012
−0 28 4 ..	0.192	0.228	−0.007	−0.007	+2 01 6...	0.848	0.873	−0.013	−0.009
−0 22.3 .	0.157	0.194	−0.005	−0.005	+2 12 8 .	0.903	0.927	−0.012	−0.003
−0 15.5 .	0.126	0.163	−0.002	−0.002	+2 23.5..	0.937	0.959	−0.019	−0.007
−0 08 6...	0.108	0.146	+0.004	+0.003	+2 37.7..	0.973	0.994	−0.006	0.000
					+2 57 4..	0.976	0.996	−0.004	−0.004

We have now to determine the elements of the system from these data, with the aid of our previous determination of the depth of secondary minimum. The ellipticity constant z, which appears in our equations, may be derived from the quantity c already determined. If the star-disks are uniformly bright, $z=2c=0.018$; if they are completely darkened toward the limb, $z=\frac{5}{4}c=0.011$.

We next transform the observed phases into longitudes in the orbit, by means of the equation

$$\theta=\frac{2\pi t}{P}=0\ 003215t$$

(in which the phase t is to be expressed in minutes of time), and plot the rectified intensities against $\sin\theta$. The light-curve appears

to be practically symmetrical, the differences between the ascending and descending branches having the aspect of residual errors of observation. Drawing a free-hand symmetrical curve to represent the observations, we find for the rectified intensity at minimum $\lambda=0.133$. Reading from the curve the values of $\sin\theta$ corresponding to a loss of light of $n(1-\lambda)$, we find:

n	0.00	0.25	0.50	0.75
$\sin^2\theta(n)$	0.290	0.1063	0.0528	0.0203

The first of these values, corresponding to the beginning of eclipse, is decidedly uncertain, but the others can be read with accuracy from the curve. From the equation

$$\chi(k,\ a_0,\ n)=\frac{\sin^2\theta(n)(1-z\cos^2\theta(\frac{1}{2}))}{\sin^2\theta(\frac{1}{2})(1-z\cos^2\theta(n))}$$

we find, as the observed values of these functions:

$$\chi(k,\ a_0,\ \tfrac{3}{4})=0.384\ ,\qquad \chi(k,\ a_0,\ \tfrac{1}{4})=2.010\ ,\qquad \chi(k,\ a_0,\ 0)=5.47: \quad (1)$$

We will first determine the elements of the system on the hypothesis that the star-disks appear uniformly bright. To find approximate values, we have the equation:

$$a_0=1-\lambda_1+\frac{1-\lambda_2}{k^2}\ , \qquad (2)$$

in which $1-\lambda_1$ denotes the depth of that minimum during which the larger star eclipses the smaller. To find whether this is the principal or secondary minimum, we must compute a_0 for various values of k, on both assumptions, take $\chi(k,\ a_0,\ \frac{1}{4})$ from Table III, and compare with the observed value. Thus we find:

TABLE 3

Large Star in Front at Principal Minimum			Large Star in Front at Secondary Minimum		
k	a_0	$\chi(k,\ a_0,\ \frac{1}{4})$	k	a_0	$\chi(k,\ a_0,\ \frac{1}{4})$
1.00	0.937	2.290	1.00	0.937	2.290
0.90	0.953	2.127	0.980	0.972	2.324
0.80	0.977	2.002	0.965	1.000	2.362
0.726	1.000	1.900			

To obtain the observed value of $\chi(k,\ a_0,\ \frac{1}{4})$ we must have $k=0.809$, $a_0=0.975$. The larger star is in front at the principal eclipse,

which is very nearly total. We have now for the light of the smaller but brighter star:

$$L_2 = \frac{1-\lambda_1}{\alpha_0}, \tag{3}$$

whence $L_2 = 0.890$, and $L_1 = 0.110$.

We may now compute a light-curve from these constants, using the equations

$$l = 1 - \alpha L_2\,; \qquad \sin^2\theta = \frac{A + B\psi(k,\alpha)}{1 - z'B\psi(k,\alpha)}\,; \qquad A + B\psi(k,\alpha_0) = 0 \tag{4}$$

in which $z' = \frac{z}{1-z}$, and A and B are constants to be determined. The last equation, which expresses the condition that the obscuration at mid-eclipse shall be α_0, gives $A = 1.161B$. Computing l for $\alpha = 0.00$, 0.05, etc., and reading the corresponding values of $\sin\theta$ from the free-hand light-curve, we find that a very good representation of them may be obtained by setting $B = 0.0310$, whence $A = 0.0360$; but on plotting the computed curve on a large scale, and reading off the residuals for the individual observations, it appears that some improvement should be possible.

A differential correction by the method of least-squares was therefore attempted. From equations (3) and (4) we find, neglecting the very small quantity z':

$$l = 1 - \alpha + \frac{\alpha}{k^2\alpha_0}(1-\lambda_2)\,, \qquad \sin^2\theta = B\{\psi(k,\alpha) - \psi(k,\alpha_0)\}$$

which for brevity we may write

$$\sin^2\theta = B(\psi - \psi_0)\,.$$

Differentiating these, and remembering that λ_2 and $\sin\theta$ (that is, the time of observation) do not vary, we find:

$$\left.\begin{aligned} &(\psi-\psi_0)\frac{dB}{B} + \left(\frac{\delta\psi}{\delta k} - \frac{\delta\psi_0}{\delta k}\right)dk - \frac{\delta\psi}{\delta\alpha_0}d\alpha_0 + \frac{\delta\psi}{\delta\alpha}d\alpha = 0 \\ &dl = \left(\frac{1-\lambda_2}{k^2\alpha_0} - 1\right)d\alpha - \frac{2\alpha(1-\lambda_2)}{k^3\alpha_0}dk - \frac{\alpha(1-\lambda_2)}{k^2\alpha_0^2}d\alpha_0 \end{aligned}\right\} \tag{5}$$

whence, eliminating $d\alpha$, we find dl in terms of $\frac{dB}{B}$, dk, and $d\alpha_0$. The numerical values of the derivatives of ψ may be found from the tabular differences of the function. Since it appears that the

provisional value of k is a little too great, and that of α_0 too small, these derivatives were computed for $k=0.800$, $\alpha_0=0.978$.

In forming the equations of condition, the observations given in Table 2 were combined into normal places, by taking means of the phases and the residuals for groups of from two to four of these. In all but three cases, the resulting normals depend upon an equal number of observations upon the ascending and descending branches of the curve. Just what observations went to form each normal can be determined by inspection, if it is remembered that the unit of weight for these equations corresponds to the mean of 24 of the original observations—that is, to four of the quantities given in Table II of Dugan's paper between the phases $\pm 1^h 20^m$, and to two of the tabular entries for larger phases. If these equations should be so written that the absolute terms were expressed in light-intensity, they would be of very different weights, for the observations, whose probable error in stellar magnitude varies but little, give the intensity with much greater precision when the star is faint. To allow for this, each equation is divided through by the corresponding rectified intensity, so that the absolute terms are of the form $\frac{dl}{l}$, and may be converted into residuals in stellar magnitude by changing their signs, and multiplying them by 1.08. The equations so obtained are given weights proportional to the number of observations. The use of the rectified, rather than the observed intensity, in this reduction, is equivalent to giving the observed magnitudes a weight which diminishes as the star becomes fainter, being 1.00 at maximum (mag. 10.5), 0.85 at 11^m5, 0.66 at 12^m5, and 0.51 at minimum (13^m0). This appears to be justified by Dugan's remark, "When there was haze, or dew, or moonlight, observations at faintest light were mere guesswork" (*op. cit.*, p. 41), and by the fact that under these conditions he usually stopped observing.

Table 4 gives these equations, with the residuals remaining after their solution, and also the residuals resulting from the solution to be described later, in which the star-disks are supposed to be completely darkened toward the limb. To make the equations

more nearly homogeneous, the unknowns have been taken as $x=\frac{1}{2}\frac{dB}{B}$, $y=dk$, $z=3d\alpha_0$.

TABLE 4

EQUATIONS OF CONDITION

Weight	Mean Phase		Uniform O−C	Darkened O−C
½	$0^h\ 5^m8$	$-0.15x-1.72y-2.49z=+0.008$	+0.017	+0.017
½	0 15.8	$-0.42x-0.93y-1.05z=-0.030$	−0.017	−0.018
1	0 26.9	$-0.76x-0.23y-1.27z=-0.020$	−0.005	−0.006
1	0 38.6	$-0.92x-0.05y-0.75z=-0.023$	−0.013	−0.008
1	0 50.6	$-0.98x-0.11y-0.48z=+0.007$	+0.012	+0.014
1	1 01.8	$-1.00x-0.20y-0.32z=+0.009$	+0.010	+0.005
1	1 12.9	$-0.99x-0.29y-0.24z=+0.005$	+0.003	0.000
1	1 23.5	$-0.96x-0.36y-0.18z=+0.024$	+0.020	+0.014
1	1 34.2	$-0.92x-0.41y-0.14z=+0.011$	+0.006	+0.003
1½	1 48.9	$-0.84x-0.45y-0.09z=+0.001$	−0.006	−0.009
1	2 02.7	$-0.73x-0.43y-0.06z=+0.001$	−0.006	−0.001
1	2 14.8	$-0.62x-0.40y-0.04z=-0.004$	−0.010	−0.001
1	2 26.0	$-0.44x-0.30y-0.02z=-0.020$	−0.025	−0.012
1	2 39.4	$0.00x\quad 0.00y\quad 0.00z\quad -0.009$	(−0.009)	−0.005

The normal equations resulting from these, and their solution, are:

$$8.39x+3.12y+3.76z=-0.0015 \qquad x=-0.0008 \quad \text{weight } 5.4$$
$$3.12x+3.13y+3.79z=+0.0029 \qquad y=-0.0163 \quad \text{weight } 1.0$$
$$3.76x+3.79y+7.63z=+0.0489 \qquad z=+0.0140 \quad \text{weight } 3.0$$

The weighted sum of the squares of the residuals is diminished by the solution from 0.002681 to 0.001987. The resulting value of the probable error of the unit of weight is ±0.0095, corresponding, in stellar magnitude, to ±0.0104.

Applying the corrections thus derived to the provisional constants, we have the corrected values:

$$B=0.03095\pm0.00026, \text{ whence } 1-\lambda_1=0.868$$
$$k=0.793 \quad \pm0.010 \qquad L_2=0.886$$
$$\alpha_0=0.980 \quad \pm0.002 \qquad L_1=0.114$$

The light-curve computed from these data gives values for the residuals which agree, within the limits of error of the graphical process, with those derived from the equations of condition. Hence the values of the constants just found may be accepted as final.

For the beginning of eclipse we find $\sin^2\theta' = 0.2269$, corresponding to a semi-duration of $2^h 34^m 6$. To find the remaining elements, we have the equations:

$$\left.\begin{array}{l} a_1^2(1-z\cos^2\theta')(1+k)^2 = \cos^2 i \cos^2\theta' + \sin^2\theta' ; \quad b_1^2 = a_1^2(1-z) \\ a_1^2(1-z)\{1+kp(k, a_0)\}^2 = \cos^2 i ; \quad a_2 = ka_1 ; \quad b_2 = kb_1 \end{array}\right\} \quad (6)$$

From Table I we find $p(k, \alpha_0) = -0.934$, and then

$$a_1 = 0.2695 , \qquad a_2 = 0.2137 , \qquad \cos i = 0.0695 .$$

To find the probable errors of these quantities, we must express their differential increments in terms of the quantities x, y, z which appear in our least-squares solution, and also make allowance for the fact that the probable errors of these three quantities are not determined independently of one another. From the form of the normal equations, we have for the sum of the squares of the residuals (if x, y, z denote variations from the values which make the sum of the squares a minimum M):

$$[pvv] = M + 8.39x^2 + 6.24xy + 7.02xz + 3.13y^2 + 7.08yz + 7.63z^2 ,$$

or

$$[pvv] = M + (2.90x + 1.08y + 1.30z)^2 + (0.98y + 2.42z)^2 + (1.005y)^2 .$$

The quantities in parentheses are determined independently by the observations, with weight unity. We will call them p, q, r. If the small quantity z is neglected, a_1, a_2, and i are given by the equations:

$$a_1^2(1+\cot^2 i) = \frac{B}{\phi_1(k)} ; \quad \cot^2 i = \frac{B}{\phi_2(k)} - A .$$

$$a_2 = ka_1 \qquad\qquad A + B\psi(k, a_0) = 0 .$$

These are abundantly accurate enough for our present purpose (giving, in fact,

$$a_1 = 0.268 , \qquad a_2 = 0.212 , \qquad \cos i = 0.071) .$$

Differentiating them, and introducing the numerical values

$$\phi_1(k) = 0.431 , \quad \phi_2(k) = 0.749 , \quad \frac{\delta\phi_1}{\delta k} = -0.01 , \quad \frac{\delta\phi_2}{\delta k} = +0.93$$

(which are readily found from Table IIa), and also

$$\psi(k, a_0) = -1.17 , \quad \frac{\delta\psi}{\delta k} = +0.44 , \quad \frac{\delta\psi}{\delta a_0} = -2.18 ,$$

we find without difficulty:

$$\frac{da_1}{a_1}=1.99x+0.06y+0.02z=0.69p-0.36q-0.33r\,.$$

$$\frac{da_2}{a_2}=1.99x+1.32y+0.02z=0.69p-0.36q+0.93r\,.$$

$$d(\cot^2 i)=0.010x-0.040y-0.024z=0.004p-0.012q-0.032r\,.$$

The weights of these three quantities are the reciprocals of the sum of the squares of the coefficients of p, q, r, in these equations.

The final values of the elements thus derived on the hypothesis of uniformly bright star-disks are given below, in Table 6, and the outstanding residuals for the observations and normal places above, in Tables 2 and 4. These residuals are distinctly systematic, and are larger than might have been expected, giving a probable error of $\pm 0\overset{\mathrm{m}}{.}0104$ for the mean of 24 of the original observations, or of $\pm 0\overset{\mathrm{m}}{.}021$ for the mean of 6, while from the observations outside principal minimum we found $\pm 0\overset{\mathrm{m}}{.}018$ for this latter quantity. The discrepancies between observation and calculation, and especially the large negative residuals near the beginning and end of eclipse—which indicate that the actual duration is longer than the computed—are such as would be caused by darkening of the star-disks toward the limb.

We therefore proceed to a second solution, in which we assume that the apparent brightness of the disks varies as the cosine of the angle of emission of the light, and falls off to zero at the limb. We must now use the formulae and tables prepared for this case of completely darkened stars.

For the relation between k and α_0 we now have:

$$Q(k,\alpha_0)=\frac{1-\lambda_2}{\alpha_0-(1-\lambda_1)}\,.$$

If we suppose the larger star to be in front at principal minimum (so that $1-\lambda_1$ is greater than $1-\lambda_2$), we compute Q for various values of α_0, find the corresponding values of k from Table V,[1] and then take the χ-functions from Table IIIx, with arguments k, α_0. If, however, we assume that the smaller star is in front, we must

[1] It aids in this process to note that $Q(k,\alpha_0)-k^2$ changes but slowly with k and α_0.

interchange the numerical values of λ_1 and λ_2, compute Q and k as before, and α_0'' by the equation

$$\alpha_0''=\alpha_0\frac{Q(k,\alpha_0)}{Q(k,1)}$$

and take the χ-functions from Table IIIy, with arguments k, α_0''.

In this way we find (assuming $1-\lambda_1=0.867$, $1-\lambda_2=0.070$):

TABLE 5

LARGE STAR IN FRONT

α_0	$Q(k,\alpha_0)$	k	α_0''	$\chi(k,\alpha_0,\frac{3}{4})$	$\chi(k,\alpha_0,\frac{1}{4})$	$\chi(k,\alpha_0,0)$
1.000	0.569	0.689	....	0.502	1.680	3.53
0.980	0.619	0.740	..	0.459	1.773	3.93
0.950	0.844	0.884	....	0.412	1.905	4.62
0.940	0.959	0.961	..	0.389	1.965	4.89
0.937	1.000	1.000	..	0.373	2.012	5.09

SMALL STAR IN FRONT

α_0	$Q(k,\alpha_0)$	k	α_0''	$\chi(k,\alpha_0,\frac{3}{4})$	$\chi(k,\alpha_0,\frac{1}{4})$	$\chi(k,\alpha_0,0)$
0.950	0.985	0.982	0.942	0.371	2.024	5.17
0.980	0.953	0.950	0.968	0.365	2.048	5.30
1.000*	0.934	0.925	1.000	0.360	2.080	5.49
$1+x$†	0.934	0.915	$1+x$	0.357	2.090	5.51
Observed values of χ-functions			...	0.384	2.010	5.47:

* Grazing annular eclipse. † Central annular eclipse.

It appears from this table that the smaller star must undergo eclipse at the principal minimum. The observed value of $\chi(k,\alpha_0,\frac{3}{4})$ is exactly represented if $\alpha_0=0.939$, $k=0.972$, and that of $\chi(k,\alpha_0,\frac{1}{4})$ if $\alpha_0=0.937$, $k=0.998$. That of $\chi(k,\alpha_0,0)$ is too uncertain to be of value in this connection.

We therefore assume $k=0.985$, $\alpha_0=0.938$—whence $L_2=0.925$, $L_1=0.075$—and compute a light-curve from these elements, using formula (4), but taking the ψ-functions from Table IIx, and using the value $z=0.011$, already found to be appropriate in this case. We thus find $B=0.0306$, $A=0.0330$, and obtain a curve which is already very good but appears capable of some improvement. In this case the adjustment was made by empirical changes of the constants, as it did not seem worth while to undertake the labor of another least-squares solution. A very satisfactory representation

was obtained by diminishing k by 0.005, B by 0.0002, and λ_1 by 0.002, leaving α_0 unchanged. These changes reduce the depth of the secondary minimum by $0^m.002$, which is practically negligible.

The residuals outstanding from the light-curve finally adopted are given in the last columns of Tables 2 and 4. In the latter, the weighted sum of the squares of the residuals is 0.001126, as against 0.001528 for the preliminary darkened solution. Both of these sums include the normal place at phase $2^h39^m.4$, which falls outside the eclipse on the uniform hypothesis. If this observation is included in the uniform case also, the sum of the squares is 0.002068 for the best possible uniform solution—almost twice as great as on the assumption of darkening at the limb. That this star does not present a uniform disk seems, therefore, to be beyond question.

The residuals from the darkened curve still show a systematic run of signs, and are similar in character to, though less in numerical magnitude than, those of the uniform solution. This would indicate that the apparent brightness of the star's disk actually falls off still more rapidly than we have assumed. It would, however, hardly be safe to draw this conclusion for the representation of the observations is already all that could be expected—the probable error of the mean of 24 observations, according to the darkened solution, being $\pm 0^m.0073$, and that of 6 observations $\pm 0^m.015$, while the latter quantity for the observations outside principal minimum was $\pm 0^m.018$.

Proceeding to find the elements corresponding to the darkened curve, we have again a set of equations of the form (6), in which now the function $p(k, \alpha_0)$ must be taken from Table I*x*. We find, from the computation of the light-curve, $\sin^2 \theta' = 0.2642$—corresponding to a semi-duration of eclipse of $2^h48^m.0$—and then $a_1 = 0.262$, $a_2 = 0.257$, $\cos i = 0.054$.

The final elements derived for the system of *Z Draconis*, on the two hypotheses of uniform disks and disks completely darkened toward the limb, are summarized in Table 6, along with various related quantities. The probable errors of the darkened elements are not given, since these were not obtained by a formal least-squares solution; but they are presumably somewhat less than those of the uniform elements. The elements found by Dugan

(*op. cit.*, p. 43) are also given, for comparison. These differ so little from the final elements derived on the hypothesis of uniform star-disks that the only return for the labor of the least-squares solution has been the certainty that the darkened curve fits the observations better than the other can possibly do.

TABLE 6

ELEMENTS OF THE SYSTEM OF *Z Draconis*

		Uniform			Darkened
		Dugan	Least-Squares		
Maximum radius of larger star	a_1	0.270	0.2695	±0.0020	0.262
Minimum " " " "	b_1	0.266	0.2671		0.261
Maximum " " smaller "	a_2	0.217	0.2137	±0.0025	0.257
Minimum " " " "	b_2	0.214	0.2118		0.256
Ratio of the radii of the stars	k	0.805	0.703	±0.010	0.980
Ratio of the axes of the spheroidal stars	$1+\frac{1}{2}z$	1.016	1.000	±0.006	1.0055
Least apparent distance of centers	$\cos i$	0.074	0.0695	±0.0024	0.054
Inclination of orbit plane	i	85°44′	86°01′	±7′	86°55′
Eccentricity of orbit	$e \cos \omega$	+0.010	+0.010	±0.005	+0.010
Maximum fraction of light of the smaller star obscured during eclipse	α_0	. . .	0.980	±0.002	0.938
Difference of light of the sides of the larger star	$2b$	0.032	0.040	±0.006	0.040
Light of the smaller star	L_2		0.886		0.927
Light of the larger star					
Brighter side	L_1		0.114		0.073
Fainter side	L_1-2b		0.074		0.033
Ratio of surface-brightness					
Of the bright sides of the two stars	$\frac{J_2}{J_1}$	15.8	12.3		13.1
Of the sides of the fainter star		1.58	1.54		2.2
Stellar magnitude					
Of the brighter star			10.57		10.50
Of the fainter star					
Bright side			12.80		13.26
Faint side			13.27		14.12
Density of the brighter star	ρ_2	0.36	0.39		0.22
Density of the fainter star (assuming equal masses)	ρ_1	0.19	0.19		0.21

The unit of light is the combined light of the brighter sides of the two stars, and corresponds to a stellar magnitude of 10.24. The unit of length is the mean radius of the orbit, which must

remain unknown, as the star is far too faint for spectrographic study. The unit of density is the sun's density. It is probable, by analogy with investigable cases, that the brighter component is more massive than the other, and therefore that it is denser, and the fainter component less dense, than the tabular values.

The general characteristics of *Z Draconis* are fairly typical of eclipsing variables. There is nothing unusual about the system, except the great accuracy of the observed light-curve, and the consequent precision of the elements. There can be little doubt that the brighter component, at least, is considerably darkened at the limb, and the eclipse theory, on this hypothesis, gives a very satisfactory account of the observed variations in brightness. The principal difference between the uniform and darkened elements is that the smaller and brighter component comes out considerably larger and less dense on the latter assumption (as is almost always the case). Even so, it is decidedly denser than the majority of eclipsing variables.

In response to an inquiry from the writer, Miss Cannon very kindly examined the spectrum of *Z Draconis* on several plates, but found it too faint to classify. From analogy with other eclipsing variables of similar density, it might be expected to be of class A or class F.

While the data are not sufficient to justify an estimate of the distance of this system, they are enough to make it probable that it is very remote. Assuming the sun's stellar magnitude to be -26.8, it is easy to show that a star of magnitude m, mass M, density ρ, and surface brightness J (with reference to the sun as a standard) must have the parallax $\pi''=(0.630)^{m+0.2}\rho^{\frac{1}{3}}M^{-\frac{1}{3}}J^{-\frac{1}{2}}$.[1] Applying this to the bright component of *Z Draconis*—using the data of the darkened solution—we find,

$$\pi''=0.''0044M^{-\frac{1}{3}}J^{-\frac{1}{2}}.$$

The mass and surface brightness are unknown; but in most eclipsing variables, when they can be estimated, they are found to be greater than in the case of the sun. It seems likely, therefore, that the distance of this system is of the order of magnitude of 1,000 light-years.

[1] Cf. Schlesinger, *Publications of the Allegheny Observatory*, **2**, 61, 1910.

II. The series of photometric measures of *RT Persei* by Dugan, published in No. 1 of the *Contributions from the Princeton University Observatory*, comprises nearly fifteen thousand comparisons, and fixes the light-curve with a precision that is not surpassed by observations on any other star. The unusual accuracy of the comparisons and the existence of a well defined secondary minimum have permitted the most complete application of the theory of eclipsing binaries. For this star more information has been derived from the light-curve than has ever before been obtained from a single curve—in fact all the quantities have been derived that are possible from photometric observations alone. In addition to the nine independent unknown quantities found for *Z Draconis*, the line-of-sight component of the eccentricity and a definite periastron effect are determined from the curve of *RT Persei*, and in this case also the existence of darkening at the limb is demonstrated. The method of discussion closely follows that for *Z Draconis*, and only those points which are more or less unique will be mentioned in detail in this presentation. It may, in fact, facilitate matters if the points of special interest in the discussion are simply enumerated in the order in which they arose.

1. The asymmetry of the light-curve at principal minimum becomes practically negligible after a shift of the epoch of mid-eclipse of -1.0 minute. The sum of the residuals from the computed curves is reduced 20 per cent by such an adjustment. With this slight change the formula for minima derived by Dugan is $2417861^{d}.63026+0^{d}.8494222\,E$. The matter of the asymmetry outside the minima will be referred to later.

2. The normal places of six observations each, published in Table IV (*op. cit.*), were combined into supernormals of 30 observations each for maximum light, of 24 observations each for secondary minimum, and of 12 observations each for the primary. From the maximum-light groups 13 equations of condition were formed and values were derived for ellipticity, differential reflection, and mean maximum light. All observed intensities were then rectified by means of the relation:

$$\text{Rectified intensity}=\frac{\text{Observed intensity}+b(1+\cos\theta)+d\cos^2\theta}{L_0},$$

where from the least-squares solution $b=0.011\pm0.0046$ and $c=0.017\pm0.012$; the divisor, $L_0=1.015$, reduces the rectified intensity outside minima to unity. The maximum-light groups are given in Table 7. The phases are computed from the nearest primary minimum, and the first column of residuals refers to the mean maximum light of 1.000.

TABLE 7

Adjusted Phase	Observed Intensity	Rectified Intensity	$O-C_1$	$O-C_2$
			L	L
$+2^{h}11^{m}$	0.983	0.998	−0.002	−0.008
2 30	0.995	1.006	+0.006	−0.001
3 24	0.993	0.998	−0.002	−0.010
4 27	1.017	1.017	+0.017	+0.007
5 32	1.015	1.010	+0.010	0.000
6 58	1.017	1.012	+0.012	+0.004
+7 54	1.012	1.010	+0.010	+0.004
−8 1	0.995	0.992	−0.008	−0.002
6 47	0.989	0.983	−0.017	−0.009
5 24	0.995	0.990	−0.010	0.000
4 17	1.000	1.000	0.000	+0.010
3 22	0.978	0.984	−0.016	−0.008
−2 28	0.991	1.004	+0.004	+0.010

3. As noted by Dugan, the intensity is distinctly higher between primary and the following secondary than in the half of the curve preceding the primary. Maximum intensity in the latter case, determined from 180 observations, is 0.992 ± 0.002. During the first half of the maximum light 210 observations give the mean value 1.007 ± 0.002. The sum of the squares of the residuals in the foregoing table is reduced by applying the appropriate sine curve correction[1] from 1402 to 595. The amplitude of the sine curve is 0.020. From the residuals in the last column above the probable error of the mean maximum light is ±0.0015 after this adjustment; it was ±0.004 from the least-squares solution.

Since the middle point of secondary eclipse comes nine minutes earlier than the point midway between primary minima, the periastron passage occurs between primary eclipse and the following secondary, and therefore the asymmetry just discussed is evidently

[1] *Astrophysical Journal*, **36**, 69, 1912.

to be attributed to the periastron effect. The difference of two-hundredths of a magnitude should probably be considered the result of secondary heating and reflection arising from the eccentricity of the orbit, or the result of a secondary tidal action. It has been shown by Dugan (*op. cit.*, p. 40) that simple reflection alone is not sufficient to account for the mean reflection effect considered in the last section. In like manner, considering the small eccentricity found later, it is immediately evident that the increase in the differential reflection at periastron is insufficient to account for the observed difference, and recourse must be had again to the reasonable supposition that there exists a cumulative heating effect.

4. The solution for the orbit was made completely by the graphical method, experience having shown that the adjustment by least squares is an unnecessary refinement. The circular elements for uniform disks obtained by Dugan served as a first approximation to the interpretation of the principal eclipse. The main difference between the new circular uniform elements and those already published arises from the use of the new ellipticity constant. The two solutions are compared below:

	Uniform Circular Elements Derived from Principal Minimum						
	k	a	b	i	L_b	L_f	$\frac{J_b}{J_f}$
Dugan	1.00	0.274	0.268	85°38′	0.801	0.130	6.2
Shapley	1.00	0.272	0.267	85°44′	0.830	0.161	5.2

These sets of elements represent the primary accurately, but they are not adequate for the secondary eclipse which is noticeably of longer duration.

5. From the displacement of secondary minimum we derive:

$$e\cos\omega=\frac{\pi\left(t_2-t_1-\frac{P}{2}\right)}{P(1+\operatorname{cosec}^2 i)}=-0.012\,.$$

The line-of-sight component of the eccentricity is obtained from the equation:

$$e\sin\omega=\frac{(p_s-p_p)k}{2+(p_s+p_p)k}\,,$$

where the quantities p_s and p_p are the functions $p(k, \alpha_0)$ taken from Table I for $k=1.00$ and for the values of the maximum eclipse, α_0, at secondary and primary minima, respectively. The value of the fraction of the light eclipsed at the primary, α_{op}, used in the circular solution above, is 0.823. The fraction at the secondary eclipse would be the same for a circular orbit; but it is in general different when the orbit is not circular, and in this particular case must be smaller since the relative widths of the two minima indicate that the stars are nearer apastron at secondary eclipse.

For an eccentric orbit the depths of the two minima and the maximum percentages of light eclipsed are connected by the relation (when $k=1$):

$$\alpha_{os}=\frac{\alpha_{op}(1-\lambda_s)}{\alpha_{op}-(1-\lambda_p)} .$$

A few approximations showed that for $1-\lambda_s=0.13$, $1-\lambda_p=0.69$, and consequently $\alpha_{os}=0.807$, the light-curve computed for the secondary minimum was entirely satisfactory. Its duration is exactly four hours, and is 20.6 minutes longer than the primary. Adopting these values of α_{op} and α_{os} we derive immediately $e \sin \omega = +0.043$, $e=0.045$, and $\omega=164^\circ$.

6. The light-curve for the secondary minimum was computed by transforming the circular elements of principal minimum into another set of circular elements for secondary minimum by means of the equations (30) and (32), *Astrophysical Journal*, **36**, 57, and working the problem in reverse order from elements to light-curve. By means of equation (30) the final elliptic elements were readily derived and are given in a subsequent table. The light of the two stars and the relative surface intensities are obtained from the relations:

$$L_b=\frac{1-\lambda_p}{\alpha_{op}}=1-L_f=\frac{L_f J_b}{J_f}=0.839 .$$

7. The solution for darkened elements of *RT Persei* developed a novelty in the apparent existence of a non-central annular eclipse at principal minimum. This condition is to be expected upon an examination of the uniform elements. Experience has shown that

the general effect of the introduction of the assumption of complete darkening at the limb is to increase the inclination of the orbit and

TABLE 8

Adjusted Phase	Observed Intensity	Rectified Intensity	Uniform O−C	Darkened O−C
			L	*L*
$-2^h\ 2^m6$	0.980	0.996	−0.004	−0.004
1 47.7	0.963	0.980	−0.015	−0.010
1 35.5	0.928	0.949	−0.013	−0.012
1 23.7	0.891	0.911	+0.007	+0.003
1 11.5	0.795	0.816	−0.013	−0.017
1 0.5	0.727	0.749	0.000	−0.004
0 51.9	0.664	0.686	+0.006	0.000
0 44.2	0.593	0.615	0.000	−0.002
0 33.9	0.508	0.530	+0.003	+0.008
0 24.5	0.421	0.444	−0.008	+0.003
0 18.4	0.377	0.399	−0.003	+0.009
0 10.3	0.330	0.353	+0.007	+0.013
−0 2.5	0.291	0.314	0.000	−0.003
+0 4.2	0.294	0.316	−0.002	−0.005
0 12.1	0.328	0.351	−0.005	+0.003
0 19.7	0.378	0.400	−0.013	+0.001
0 26.9	0.432	0.454	−0.018	−0.007
0 34.4	0.505	0.527	−0.003	+0.001
0 41.6	0.566	0.588	−0.004	−0.005
0 48.5	0.626	0.648	0.000	−0.007
0 55.6	0.694	0.716	+0.005	0.000
1 3.1	0.758	0.780	+0.010	+0.005
1 12.0	0.824	0.846	+0.013	+0.007
1 22.2	0.871	0.891	−0.004	−0.009
1 32.0	0.950	0.970	+0.022	+0.022
1 42.2	0.960	0.979	−0.005	−0.003
1 51.3	0.968	0.985	−0.015	−0.010
+1 59.3	0.995	1.011	+0.011	+0.011
		SECONDARY MINIMUM		
−2 12.1	1.017	1.013	+0.013	+0.013
1 41.8	0.985	0.983	−0.007	−0.005
1 7.5	0.946	0.946	−0.008	−0.009
0 44.8	0.923	0.924	+0.001	+0.002
−0 17.7	0.888	0.890	+0.008	+0.007
+0 10.9	0.877	0.879	+0.003	0.000
0 34.3	0.895	0.896	−0.010	−0.009
0 57.6	0.938	0.939	−0.003	−0.002
1 26.4	0.970	0.970	−0.007	−0.007
+1 56.2	0.980	0.985	−0.014	−0.010

invariably to increase the size of the brighter star, usually without appreciable change in the size of the darker companion.[1] In the

[1] *Astrophysical Journal*, **36**, 281, 1912.

uniform solution the stars are found to be sensibly equal, with large partial eclipses at both minima. An increase of 20 per cent in the radius of the brighter star (as was found for *Z Draconis*) with a small increase in the inclination of the orbit would project the darker companion entirely upon the disk of the larger star at the middle phase of primary eclipse. This general result is so definite that we can safely assume at once that the secondary eclipse is total and the loss of light at that phase is $1-\lambda_s=1-L_b=L_f=0.121$, the light of the smaller star. Also $\alpha_{os}=1$, but since the orbit is eccentric (we adopt here the values of e and ω derived from the uniform solution) the notation of equation (3), *Astrophysical Journal*, **36**, 390, should be altered for this occasion to allow for the difference in the percentages of eclipse at the two minima, i.e.:

$$Q(k, \alpha_{op})=\frac{1-\lambda_p}{\alpha_{os}-(1-\lambda_s)} \,.$$

The rectification of the intensity-curve for uniform disks suffices for the darkened case, but $z_d=\frac{5}{4}z_u=\frac{5}{4}c=0.021$. The darkened curve for the same depths near minima is so much less pointed than the uniform that it is now better to take the ranges of variation smaller; but it is to be observed that the annular eclipse has no constant minimum phase and resembles a partial eclipse very closely. For $1-\lambda_p=0.682$ and $1-\lambda_s=0.121$, $Q(k,\alpha_{op})=0.777$. It is obvious that α_{op} must lie between unity, which corresponds to a grazing annular eclipse, and $1+x$, corresponding to central transit. A value nearer the latter limit is to be expected, since the inclination approximates 90° and the secondary eclipse near apastron is total. Assuming the values $1+0.4x$, $1+0.6x$, $1+0.8x$, $1+x$, the corresponding k was taken from Table V and the light-curve for the principal minimum computed. The best representation was found for $\alpha_{op}=1+0.8x$, $k=0.80$. The light-curve for the secondary minimum and the elliptical elements were computed in the same manner as for the uniform orbit. For $k=0.80$, Table Iv gives $\alpha_{op}=1.038$, which is the greatest loss of light at primary eclipse in terms of the loss of light at internal tangency.

8. The evidence of the deviations from the computed curves is decidedly in favor of darkening toward the limb. The sums of the squares of the residuals are as follows:

	Uniform	Darkened
Principal minimum	2507	1938
Secondary minimum	710	562

The small, but apparently systematic deviations at principal minimum of the observed curve from the computed uniform curve occur at the same phases and are in the same direction as the deviations of the computed darkened curve from the uniform. Moreover, a mere inspection shows that no adjustment of the uniform curve can possibly conform so closely to the observations as the darkened solution.

TABLE 9

ELEMENTS OF THE SYSTEM OF *RT Persei*

		Uniformed	Darkened
Maximum radius of brighter star	a_b	0.283	0.333
Minimum " " " "	b_b	0.278	0.329
Maximum " " fainter "	a_f	0.283	0.266
Minimum " " " "	b_f	0.278	0.263
Ratio of radii of the stars	k	1.000	0.800
Ratio of the axes of the spheroidal stars	$1+\frac{1}{2}z$	1.017	1.010
Least apparent distance of centers	$\cos i$	0.0805	0.0286
Inclination of orbit plane	i	85°23′	88°22′
Eccentricity of orbit	e	0.045	0.045
Longitude of periastron	ω	164°	164°
Maximum loss of light at primary minimum*	α_{0p}	0.823	1.038
Maximum loss of light at secondary minimum*	α_{0s}	0.807	1
Difference of light of the sides of the fainter star	$2b$	0.022	0.022
Light of the brighter star	L_b	0.839	0.870
Light of the fainter star			
Brighter side	L_f	0.161	0.121
Fainter side	L_f-2b	0.139	0.099
Ratio of surface brightness			
Of the bright sides of the two stars	$\frac{J_b}{J_f}$	5.2	4.6
Of the sides of the fainter star		1.16	1.22
Mean maximum intensity			
Near periastron		1.007	1.007
Near apastron		0.992	0.992
Density of the brighter star	ρ_b	0.43	0.26
Density of the fainter star (assuming equal masses)	ρ_f	0.43	0.50

* In terms of the loss of light which would occur at the moment of internal tangency.

The observational groups are given in Table 8 (p. 424), the phases being referred to the middle points of the minima.

9. In Table 9 the final uniform and darkened elliptic elements and various related quantities are given. The units are the same as those employed for *Z Draconis*. The stellar magnitude of *RT Persei* is 10.63 at maximum; the spectral type is F? The computed parallax is $0.0041 M^{-\frac{1}{3}} J^{-\frac{1}{2}}$, indicating that the distance is of the same order of magnitude as that of *Z Draconis*.

PRINCETON UNIVERSITY OBSERVATORY
December 15, 1913

THE ILLUMINATION-CURRENT RELATIONSHIP IN POTASSIUM PHOTO-ELECTRIC CELLS

By HERBERT E. IVES

SYNOPSIS

1. HISTORICAL

The photo-electric current is a function of voltage, of electrode distance, of the kind of gas between electrodes, of pressure, and of illumination.

The *voltage-current* relation and the *electrode-distance-current* relation, under constant illumination, were the subject of study by Stoletow[1] and others some twenty years ago.

The effects of pressure variations and of different gases, again under constant illumination, were studied by Stoletow, Varley, and other investigators. All of these studies revealed a complicated relationship between the current and the variables in question, for which qualitative explanations have been offered in terms of saturation, ionization by collision, etc.

[1] *Journal de physique*, ii, **9**, 468, 1890.

One of the most important relationships to be determined, both from its theoretical bearing and from its practical application, is that between photo-electric current and intensity of illumination.

Elster and Geitel[1] concluded from some early experiments that the current is directly proportional to the illumination. Lenard[2] reached the same conclusion. In his results, however, the fact on which greatest emphasis is laid is that the final voltage acquired by the sensitive surface is a constant, independent of the intensity of illumination, while the current or rate of acquisition of voltage is certainly not a constant. Actually his figures show a current increasing more rapidly than the illumination. Lenard, accepting the linear illumination-current relationship as proved, has recently used alkali metal cells for measuring the decay of light in phosphorescence.

Griffith,[3] working with a zinc plate, illuminated by a spark, made careful correction for air absorption and measured the current by a balancing method in which the electrometer served merely as a detector. His results plotted take the form of a curve concave toward the current axis (similar to Figs. 8–10 and Lenard's values).

Richtmyer,[4] using a sodium cell, found a strictly linear relation between illumination and current, over an enormous range. He suggested various laboratory applications of the cell for photometric work.

Elster and Geitel,[5] in a paper appearing after a large part of the work here described was completed, found the photo-electric current, in cells having a gas atmosphere of a fraction of a millimeter, directly proportional to the illumination over an even greater range than Richtmyer investigated. They have developed a special form of cell, having a surface of colloidal metal or hydride, with an atmosphere of inert gas at a pressure determined by them as giving great sensibility. These cells are now obtainable on the market.

[1] *Annalen der Physik*, **48**, 625, 1893.

[2] *Ibid.*, **8**, 149, 1902.

[3] *Philosophical Magazine*, **14**, 297, 1907.

[4] *Physical Review*, **29**, 71, 404, 1909.

[5] *Physikalische Zeitschrift*, **14**, 741, 1913.

2. OBJECT OF PRESENT STUDY

It appeared proved from the work of Elster and Geitel and of Richtmyer (which appeared subsequently to that of Griffith) that the photo-electric current is truly proportional to intensity of illumination. The extreme sensibility of the alkali metals is well established. Elster and Geitel, and later Pohl and Pringsheim[1] have shown the sensitiveness of the metals sodium, potassium, rubidium, and caesium to extend well down into and through the visible spectrum, the maximum (of the "selective" effect) lying at progressively greater wave-lengths in the order in which the metals are above given.

It appeared to the writer to be desirable at this time to study thoroughly the alkali metal cell as a possible substitute for the eye in photometry, particularly in colored-light photometry. If it should be possible to produce cells of uniform wave-length sensibility, to develop a colored absorbing screen which should make the resultant spectral-sensibility curve that of an average eye,[2] then it should be possible to tie down to a purely physical instrument the characteristics of that wonderful, but most troublesome, physiological one—the human eye. The work was therefore undertaken as the logical continuance of the writer's study of heterochromatic photometry.

In order for a physical photometer to be available for anything except as a detector in a null method with lights of the same color, or for the measurement of lights of the same color where the intensity-response relationship has been determined, the relationship between the intensity of illumination and the resultant current or reaction must be a simple one, the same for all colors. The most desirable relationship, as well as the simplest, is the linear one, which has been credited to the photo-electric cell. Granted that this simple relationship exists, the investigation as planned was to have been chiefly directed to the questions of the method of construction, performance, and reproducibility of the cells, their behavior under various photometric tests, the choice of alkali metal, and the specification of the proper color screen.

[1] *Berichte der Deutscher Phys. Ges.*, **12**, 215, 349, 1910, and subsequent papers.

[2] Ives, "Photometry of Lights of Different Colors," *Phil. Mag.*, **24**, 149, 352, 744, 846, 854, 1912.

As will appear, the work took a different direction. The mode of construction of the cells and the methods of using them as heretofore described were not found satisfactory. Finally when the first difficulties were overcome the illumination-current relation was not found to be linear in photo-electric cells as heretofore constructed.

3. PRELIMINARY APPARATUS AND RESULTS

Five methods of measuring photo-electric current are to be found in the literature: (1) by the rate of drift of an electrometer needle; (2) by the ballistic method, or the charge acquired in a definite exposure time by an electrometer connected to the cell; (3) by measuring the potential across the terminals of a high resistance in series with the cell; (4) by balancing the photo-electric current with a current variable in a known manner, using either an electrometer or sensitive galvanometer as a detector; (5) by the deflection of a sensitive galvanometer.

As a large part of the problem was anticipated to be the study of the wave-length sensibility-curves, the comparatively insensitive galvanometer method was not considered. A Dolazalek electrometer (made by the Cambridge Scientific Instrument Co.) was accordingly employed throughout the work. In order in starting to have the benefit of all previous work, a cell was purchased on the market. The metal was potassium, the cell had a quartz window (which was not necessary in this work) and was made by Müller-Uri. It is shown in the diagram, Fig. 6, *a*.

The arrangement of cell, electrometer, and light-source was copied closely from that described by Richtmyer[1] and is shown in Fig. 1. The cell and electrometer were placed on a metal shelf on a brick pier in the laboratory basement; a galvanized-iron cover fitted over them, pierced with openings for a set of keys[2] for connecting and disconnecting cell, electrometer, and known e.m.f. for calibration purposes. Glass windows permitted the exciting light and that illuminating the electrometer mirror to enter. In a separate iron box, connected to the first by metal tubes, was a six-volt

[1] *Loc. cit.*

[2] See McClung, *Conduction of Electricity through Gases and Radioactivity.*

storage cell, discharging through a 2500-ohm resistance, from which various voltages might be taken by a sliding contact. This latter was connected to the potassium and served to neutralize the contact e.m.f. which in the dark causes a strong current. More will be said about this later.

The needle was charged to a potential of 100 volts by contact with a set of dry batteries. The needle was suspended by a quartz fiber of 9 μ diameter (maker's figure), having a period of swing of about 18 seconds, and a sensibility when charged as above of about 33 cm per volt, as read by a telescope placed with the scale at 1.80 m distance.

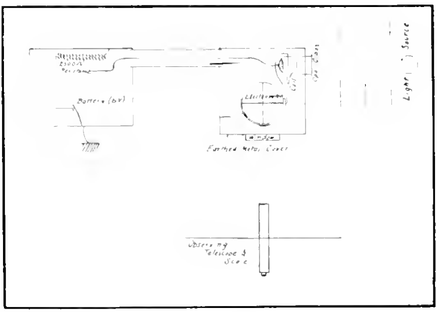


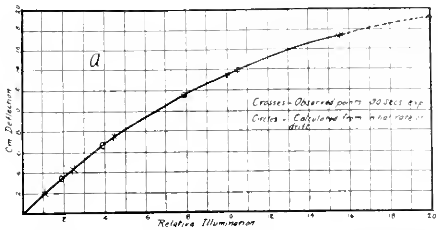
FIG. 1.—Arrangement of apparatus for "rate of drift" and "ballistic" methods of reading photo-electric current.

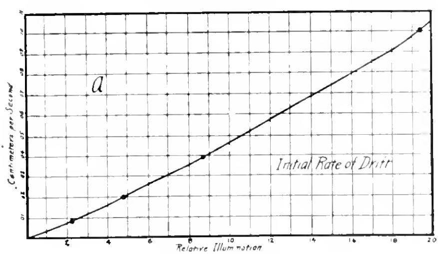
A standard carbon incandescent lamp of 10 candle-power mounted on a long track served as the light-source. Its light fell upon a piece of flashed opal glass covering the opening to the cell.

With the apparatus so arranged, following the procedure as outlined by Richtmyer, the first experiments made were on the response to various illuminations.

The rate-of-drift method was first used. Contrary to the findings of others, the needle did not "move at a uniform rate," but continuously and rapidly decreased in speed. All tests of insulation and other ordinarily suspected causes of trouble were negative.

Suspecting that a critical condition might exist, caused by the period of the needle having an unusual relationship to the rate of drift, attention was next turned to the second method of measurement above—the ballistic one. In this the cell is exposed for a fixed convenient length of time, the needle allowed to come to rest, and the deflection read.





FIGS. 2 AND 3.—Illumination-current relationships obtained from cell *a* with preliminary apparatus.

Again an unexpected result was obtained, namely, that with each different exposure-time a different illumination-current relationship was found. For short exposure the plotted curve was convex toward the illumination axis. For long exposure it was concave. Fig. 2 shows the curve obtained for 30 seconds exposure. Fig. 3 shows the curve obtained for zero exposure—in other words, the initial rate of drift, as extrapolated from the rate of drifting over successive centimeter divisions on the scale. This latter,

convex to the illumination axis, is of the character obtained by Griffith.

By proper choice, then, of time of exposure, it appeared possible to obtain any curve desired, among others a straight line. But this apparent dependence on time of exposure called for explanation. It was consequently decided to make a trial of the third or steady deflection method, in which the effect of both needle period and choice of exposure-time are eliminated. This led to the construction of the apparatus as finally used, which will now be described, not as chronologically developed, but under appropriate headings.

4. FINAL FORM OF APPARATUS

The electrometer.—After much trouble with defective insulation in damp weather and difficulty in making adjustments, each

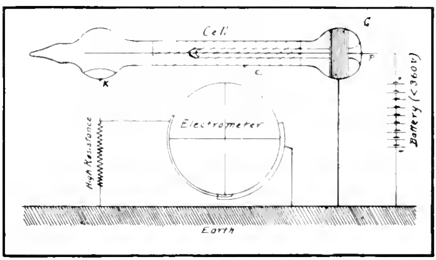


FIG. 4.—Diagram of connections for "steady deflection" method of measuring photo-electric current.

necessitating the removal of the sheet-iron cover, it was decided to inclose the electrometer completely in a dry air-tight box and arrange to operate its adjusting screws from without. Fig. 4 shows diagrammatically the arrangement for the steady deflection method, while Fig. 4*a* shows in section the electrometer and its accessories as practically arranged. The lower portion of the inclosing box is of heavily shellacked wood, lined with tin foil. Around the top is a narrow trough containing mercury. Into this trough sets the sheet-iron top, which in turn has an opening through which the

needle support of the electrometer projects into a cut-off glass bottle, also seated in a mercury trough. By this latter means the electrometer mirror may be turned to bring any part of the scale into view. The leveling screws each rest on a small brass pillar, which moves as a piston in a sleeve mounted in the supporting shelf, stopcock grease making the piston air-tight. Each piston is moved up and down by a tapering rod, which is threaded into a

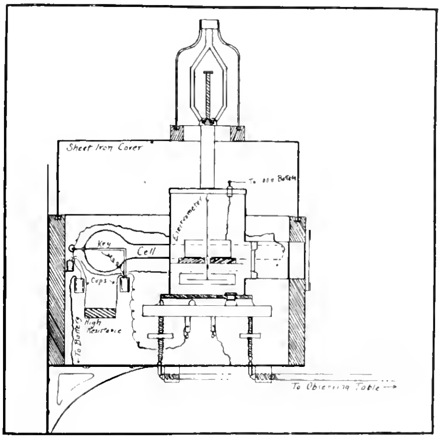


FIG. 4*a*.—Diagrammatic section of final arrangement of apparatus

fixed nut. Long handles carry this adjustment over to the observation table, two meters away. Adjustment is made by charging and discharging the needle with all quadrants earthed and altering the level until no change of needle zero occurs. When adjustment is apparently complete it may be found that the needle does not hang symmetrically with respect to the quadrants. It is then raised or lowered slightly until on adjustment it is symmetrical. The raising or lowering need only be done on first setting up. Slight changes in level are found necessary from time to time as shown by the disturbance of the zero on charging. The adjustment

of the electrometer is a delicate matter and without some means of working from the position of observation is almost prohibitively tedious. When adjustment was completed the deflections were found strictly proportional to voltage over the whole range of the scale.

The needle was charged with the aid of a small ionization chamber containing a small sample of polonium, as described by Erikson.[1] The arrangement of cell, screen, etc., is sufficiently well shown in the diagram. A single key, working through a glass-ground joint, served to earth the electrometer quadrants attached to the cell. Three mercury cups on sealing-wax stands were connected respectively with the source of high potential, the electrometer, and the earthed walls of the box. A metal tube leading down from the box carried the wires to needle and cell from the high-voltage batteries. Wires soldered to the electrometer shelf, cover, and tubes, and to gas pipes insured complete earth connection.

Two trays of calcium chloride rested on top of the wooden part of the box and, in addition, a small tray of phosphorous pentoxide assisted in keeping the inclosure dry. During the later and more important part of the work the basement was steam-heated, whereby everything was made thoroughly dry, even without the drying material. After the cell and electrometer are closed up, they need not be touched for days or weeks, the best possible conditions being thus insured.

Source of high potential.—A battery of small dry cells provided the high voltage necessary to work satisfactorily with the steady-deflection method. These were cells of the kind used in electric flash-lamps, coming in cartons of five each. To insure perfect contact they were taken out of their cases and connected by soldered wires. They were then arranged in rows in four shellacked wooden trays, sixty to the tray. A switchboard on top of the tray-holder was arranged so that one, two, three, or four groups could be put in series, giving, when fresh, 90, 180, 270, or 360 volts. By direct wire connection from the cells intermediate voltages were available when desired. Each tray was connected by fine fuse wires, and a fixed resistance of 2000 ohms was kept in series with the photo-

[1] *Physical Review*, **36**, 253, 1913.

electric cells for their protection. A key, worked by a cord, over a pulley, effected the charging and discharging of the electrometer needle from 65 of the dry cells (100 volts). The entire battery system was inclosed in a galvanized-iron box.

High resistance.—Much time was spent in the search for a satisfactory high resistance. Alcohol in a capillary tube,[1] xylol and alcohol,[2] mannite solution with non-polarizable electrodes,[3] various forms of carbon resistances,[4] besides selenium, Welsbach mantle oxides, etc., were investigated. Alcohol and xylol-alcohol were found to become polarized and were, therefore, abandoned. Mannite solution in a long thermometer capillary proved free from polarization, but had to be made of very low concentration to secure high enough resistance in the length of the tube which it was feasible to use (perhaps owing to the conductivity of the water used as solvent). Its chief defect, however, was the difficulty of preventing evaporation and leakage in spite of the plentiful use of paraffin over all cocks and joints. The most satisfactory resistance was a modification of a carbon one described by Stewart.[5] A piece of dull-surfaced hard rubber had two machine screws tapped into it. Each machine screw carried a nut which could be screwed tightly down against the rubber. Lamp black was mixed in a commercial lacquer and painted around the electrodes, forming an adherent conducting coat. The two spots of lamp black were then joined by a fine lead-pencil line. In this way the chief difficulty with carbon resistance—erratic contact—was overcome. The resistance used in obtaining the curves here shown had a value of 150 megohms. With the electrometer sensibility used, one centimeter deflection thus corresponded to 0.2×10^{-9} amperes.

Source of light.—Electric incandescent lamps were used uniformly, of various candle-powers, their light usually falling upon a piece of flashed opal glass 5 cm from the cell surface. Occasionally when an insensitive cell was experimented with, the opal glass was

[1] Nichols and Merritt, *Physical Review*, **34**, 475, 1912.

[2] Campbell, *Philosophical Magazine*, **22**, 301, 1911; **24**, 668, 1912.

[3] Pohl and Pringsheim, *Berichte d. Deutsch. Phys. Ges.*, **6**, 174, 1913.

[4] Aust, *Physical Review*, **32**, 256, among others, 1911.

[5] *Physical Review*, **26**, 302, 1908.

removed, allowing the light to fall directly on the sensitive surface. The highest illumination used (Figs. 8–15) was about 500 meter-candles on the opal glass, and the glass had an effective absorption of at least 90 per cent as used.

The electric lamps were controlled from storage batteries in the usual way and were mounted upon regular photometer carriages and tracks carrying screens to exclude all stray light. Exposure of the cell to the light was made by a shutter moved from the observing table.

5. CONSTRUCTION AND MANNER OF CONNECTING THE PHOTO-ELECTRIC CELLS

The cell first used was, as above stated, a purchased one, and is shown in section in Fig. 6, *a*. In order to make clear why this cell was not entirely satisfactory and why radical changes in construction were introduced, the manner of connecting up the apparatus will now be given. Fig. 4 shows diagrammatically the essential parts. The alkali metal electrode K is connected to one pair of quadrants of the electrometer and to earth through the high resistance. The other electrode P is connected to the positive terminal of the batteries.

Insulating and guard rings.—Two spurious currents, present in the dark, are found in the photo-electric cell as heretofore constructed. The first of these is opposite in direction to the current produced by light, and is ascribable to the contact difference of potential between the potassium and the other (platinum) electrode P. The second is what has been called a "dark current," in the same direction as the light current. These two currents may be so large as to be very troublesome, especially as they are likely to be variable in amount. No satisfactory results can be secured unless they are reduced to negligible values.

A series of experiments was carried out to learn the cause and method of obviating these currents. A recent paper by Elster and Geitel, appearing since the final form of cell here adopted was under trial, contains all the essential points about these effects, so that it suffices here to say that the "dark current" is the result of conduction over the surface of the glass, due either to the glass

itself, to occluded water vapor, or to a thin film of carbon, deposited from the hydrocarbon with which the alkali metal is usually covered before use. It, in common with the contact e.m.f. current, can be greatly reduced by using glass of good insulating quality, or by separating the two electrodes as far as practicable. A still more complete protection is afforded by the use of internal and external guard rings put on by chemical silvering and connected to earth. The cell shown in Fig. 4 represents the best form. At *G* are the internal and external guard rings, *C* represents an insert of cobalt glass tubing, which has a very high resistance compared with the ordinary clear soda-lime glass. The whole cell is mounted on a

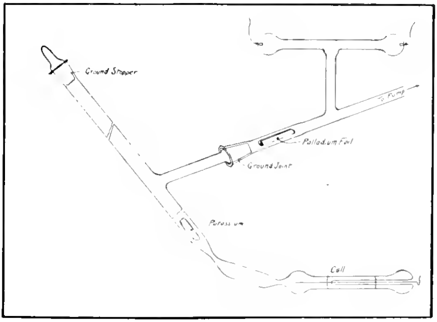


Fig. 5.—Arrangement for filling and exhausting cells

glass plate by a sealing-wax support at *G*, as shown in the sketches of Fig. 6. The insulating and guarding of the electrometer from *P* and from earth connection other than *R* and *G* is, therefore, excellent, as was shown by the fact that a cell similar to *C*, but made of ordinary clear white glass, gave with a certain high resistance and voltage a current due to contact e.m.f. represented by 7 cm deflection, whereas with the cobalt glass inserted, as in *C*, this was reduced to less than a millimeter. The guard ring and cobalt glass together entirely eliminate the "dark current."

Details of filling.—The process of making and filling the cells was not found to be entirely easy, despite the published descriptions

of the process. The pouring of liquid potassium through constricted tubing is very different from pouring mercury, although the two liquid metals look so much alike. Potassium has a very great surface tension, combined with a pronounced tendency to stick to the glass. As a consequence it is likely to pile up in front of a constriction or, on going through, to leave a thread of metal which can be removed only by heating to the distillation point, a

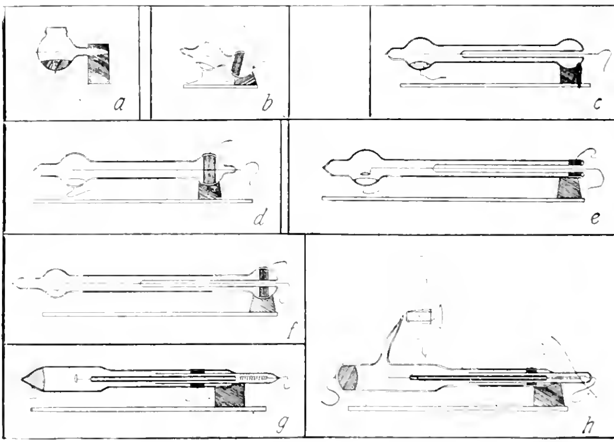


FIG. 6.—Types of cells used in the investigation

process dangerous to the glass. Another difficulty is that of maintaining the surface clean, for, in spite of the most elaborate cleaning, first with hot chromic acid and later with nitric acid, caustic potash, and distilled water as a preliminary to the silvering operation, the glass surface gives up impurities which collect and float on the molten metal. As a result of a great deal of experiment it was found that a large body of potassium could be obtained clean if it was practically shot to place through the filtering constrictions and then allowed to cool at once. Any flowing about caused the collection of a film of scum. In Fig. 5 is shown the essential part

of the apparatus for filling cells. The long tube has at one end a ground stopper through which is introduced a piece of potassium which has been scraped clean under benzol or has previously been distilled into a short glass tube. The Gaede mercury pump is then operated, the cell being inclosed in an electric oven raised to 200° C. Exhaustion is continued for about an hour to allow all the gases to escape from the cell. Then a small flame is played under the metal and glass tube. The potassium presently melts, breaks through its coating of oxide, and, because of the steep slope of the tube, rushes through the two constrictions and into *C*. If necessary, the residue in the lower constriction is driven away by heating. After cooling and then renewed pumping the cell is sealed off. When desired, a small amount of hydrogen is introduced by heating palladium foil in the side tube. Pressures are read by a Gaede short McLeod gauge.

Far easier and more satisfactory is the method of filling by distillation. In this case the unfilled cell is silvered on the lower half of the bulb containing the electrode. The filling tube is inclined at a much less steep angle. A much smaller piece of potassium is needed. A flame is played carefully on the potassium until it vaporizes and the condensing film is driven over and down on to the silver. In order to obtain deposits of uniform character, the expedient was adopted of seating the bulb in a cup of mercury cooled by a jacket of carbon dioxide snow. By slow distillation a perfectly matt fine-grained layer is obtained. If done very slowly on to a highly chilled surface, colored colloidal films may be made.

Description of various cells used.—By the time the serious work on the illumination-current relation was begun, quite a stock of experimental cells was on hand, representing various stages of design. A number of the later ones were pressed into service, and as their individual characteristics probably affect the results obtained with them, a brief description is here given of all the cells for which performance-curves are shown below. Pressures where mentioned are those of the pump system when the cell was sealed off. The gases given off by the molten constriction probably made the actual pressures greater. The cells are shown diagrammatically in Fig. 6.

a) Cell purchased from Müller-Uri. Solid mass of potassium; electrode distance 10 mm; originally possessed no guard rings, but later given an external ring of silvering to be connected to earth; pressure unknown.

b) Short double bulb cell, furnished with inside and outside guard rings; potassium distilled on silver; shortest distance electrode to potassium 10 mm; sealing-off pressure about 0.03 mm.

c) Cell with cobalt glass insert, but no guard ring; potassium distilled; shortest electrode distance 12 mm; pressure not recorded.

d) Cobalt glass insert; guard rings; distilled metal; electrode distance 14 mm; pressure unrecorded.

e) Cobalt insert; guard rings; distilled metal; ring electrode close to metal, distance 8 mm; pressure 0.002 mm.

f) Cobalt glass insert; guard rings; distilled metal, transformed to hydride by glow discharge; electrode distance 15 mm; pressure 0.3 mm.

g) Cobalt glass insert; guard rings; solid mass metal; electrode distance made variable; pressure 0.005 mm.

h) Cobalt insert; guard rings; solid mass of potassium; electrode distance variable; side tube with stop-cock for attachment to pump to permit variation of pressure.

Tubes *g* and *h* will be described more fully under section 9.

Tubes *j* and *k* were kindly loaned the writer by Mr. Saul Dushman, of the General Electric Co. Research Laboratory, near the conclusion of the work. *j* is a cell purchased abroad recently; it has a colored hydride surface, is furnished with exterior tin foil guard rings, and is presumably one of the recent argon-filled type. *k* is a cell prepared by Mr. Dushman, similar in appearance to *a*, but several times larger and without the quartz window. It was exhausted to the best vacuum attainable with a Toepler pump.

6. THE USE OF THE QUADRANT ELECTROMETER FOR THE MEASUREMENT OF PHOTO-ELECTRIC CURRENT

It is unfortunate that students of the photo-electric effect have usually been interested either exclusively in determining currents or exclusively in determining the potential acquired by the sensitive surface. Had they been studying both, there would probably

be fewer unqualified users of the rate of drift method of measuring current, working on the assumption that the electrometer needle moves at a uniform rate indefinitely. As will be shown below, this is true only to a degree of approximation conditioned by the sensibility of the instruments, the effective voltage, and the character of the current. This would seem to be obvious, but the writer has not seen a satisfactory discussion of the matter in any text or article.

The most important fact to keep in mind when using the electrometer to measure current is that the instrument forms part of the electrical system and as such may exert on the phenomenon under study an effect far from negligible. It is imperative that the possible disturbances due to the instrument be thoroughly understood before conclusions are drawn from its indications.

The most general case to consider is the steady deflection method, for it goes over into the rate of drift when the leak becomes zero.

Consider, first, the measurement of a current which is obeying Ohm's law—current proportional to difference of potential.

Let V = the voltage applied to the whole system.
v = potential of the electrometer at any time, t.
R = the resistance, such as a photo-electric cell, through which passes the current to be measured.
r = resistance of the high-resistance leak.

Then, the quantities being represented in such units that the constants of proportionality are all unity, we have for the rate of charging of the electrometer:

$$\frac{dv}{dt}=\frac{V-v}{R}-\frac{v}{r}. \tag{1}$$

Integrating,

$$\frac{V}{R}-v\left(\frac{1}{R}+\frac{1}{r}\right)=ke^{-\left(\frac{1}{R}+\frac{1}{r}\right)t};$$

when

$$t=0, \quad v=0,$$

whence

$$k=\frac{V}{R}$$

and

$$v=\frac{V}{R}\left(\frac{rR}{R+r}\right)\left(1-e^{-\left(\frac{1}{R}+\frac{1}{r}\right)t}\right). \qquad (2)$$

When t becomes ∞; i.e., steady deflection,

$$v=V\left(\frac{r}{R+r}\right). \qquad (3)$$

Writing this $v=\left(\frac{V}{R+r}\right)r$, it is seen that the deflection v is proportional to the current $\frac{V}{R+r}$ through the whole system, and it follows that if r is made small with respect to R, v will represent the current $\frac{V}{R}$ to any desired degree of approximation. The factor of uncertainty is then the effect of the high-resistance leak. If it is large, then any change in R (such as exposure to light with a photo-electric cell) is not represented by a proportional change in v.

A measure of the relative size of r and R is afforded by the size of the deflection. Let equation (3) be rewritten:

$$\frac{v}{V}=\frac{r}{R+r}.$$

Now $\frac{r}{R+r}$ differs from $\frac{r}{R}$ by an amount immediately calculable for various values of $\frac{r}{R}$. Let us choose arbitrarily the permissible deviation as determined by the experimental accuracy attainable and call this $\left(\frac{r}{R}\right)$. Find that value of $\frac{r}{R+r}$ for which

$$\frac{r}{R+r}=\frac{r}{R}-\left(\frac{r}{R}\right).$$

This value is at once the value of $\frac{v}{V}$; in other words, the ratio of the electrometer voltage to the total applied voltage may be used as the criterion of the influence of the high-resistance leak. Taking, for instance, 1 per cent as the permissible error, we have

$$\frac{r}{R+r}=0.99\left(\frac{r}{R}\right)$$

from which

$$R = 99r$$

$$\frac{r}{R+r} = 0.01 = \frac{v}{V}.$$

Consequently, if the current obeys Ohm's law, the largest safe deflection is that corresponding to 1 per cent of the applied voltage.

If, now, the current, instead of obeying Ohm's law, is a saturated one, it is easy to see that the criterion for safety is that the applied voltage shall be sufficiently above the saturation voltage, so that the voltage difference between the electrometer and the source of high potential remains a saturation voltage for all voltages acquired by the electrometer. The difference possible between working with an ohmic current and a saturation current is illustrated in the hypothetical case where 100 volts are applied. If the current obeys Ohm's law one volt deflection is the permissible limit. If the saturation voltage is below 100, say 80, then the electrometer may charge up to 20 volts without the current being misread, i.e., the high-resistance leak may be of 20 times larger value and the sensibility consequently 20 times higher.

Where the current is not clearly of either of the foregoing characteristic types, it is perhaps best to make an experimental determination of the effect of the high resistance. In the present work two resistances of relative strength about 1 to 5 were kept on hand, and in doubtful cases the ratio of large and small illumination-currents was taken with both resistances. Thus in one case where an error of this sort would have vitally affected conclusions, two illuminations gave deflections in the ratio of 3.127 with the higher resistance, and 3.129 with the lower, showing the value of r to be negligibly small.

Turning, now, to the rate of drift and ballistic methods, we obtain the limiting conditions by making $r = \infty$ in equations (1) and (2):

$$\frac{dv}{dt} = \frac{V - v}{R}. \tag{4}$$

$$v = V\left(1 - e^{-\frac{t}{R}}\right). \tag{5}$$

It is evident from equation (4) that the rate of drift of the electrometer needle will be proportional to $\frac{V}{R}$ only if v is negligibly small compared to V. The limiting deflection may be obtained by solving (5) for v for different values of $\frac{t}{R}$, and determining when $\frac{V-v}{R}$ differs from $\frac{V}{R}$ by $\Delta\left(\frac{V}{R}\right)$ in a similar manner to the case just considered. Expressing this in terms of $\frac{v}{V}$ gives the limiting deflection for a given assumed allowable error. For the ballistic method the procedure is exactly similar, the value of $\frac{v}{V}$ being determined for which $\left(1-e^{-\frac{t}{R}}\right)$ differs from $\frac{t}{R}$ by the permissible amount. Allowing 1 per cent as above, this calculation gives $\frac{v}{V}=0.02$ as the limiting deflection.

In case of saturation, it is again obvious that the voltage applied to the system should be large enough so that the difference between the electrometer voltage and the applied voltage is still a saturation voltage.

It is evident from this discussion that the common manner of use of the electrometer for current measurement is valid only if the relation of sensibility to applied voltage is such that the largest charge acquired by the electrometer leaves practically unaffected the effective voltage over the cell or other device. Neglect of this precaution leads to erroneous results, as is pointed out in the next section.

One point worth noting in passing is that with a straight ohmic resistance, it follows from equation (5) that relative current measurements can be made, notwithstanding the non-linear type of the curve connecting voltage and resistance change (current). This is done by determining the *relative times necessary to attain the same deflection;* which follows because v is the same function of t and of $\frac{1}{R}$. Let this be clearly distinguished from determining the *deflection attained in a given time*, which is represented by equa-

tion (5). Both these measures of current are found used indiscriminately by investigators of the photo-electric effect.

7. APPLICATION OF ELECTROMETER DISCUSSION TO PRELIMINARY RESULTS

The application of the preceding discussion makes possible an explanation of the results obtained with the preliminary apparatus.

In Fig. 2 the largest deflection is 19 cm. The sensibility was about $23 \frac{\text{cm}}{\text{volt}}$. Now the potential acquired by a potassium surface under the light of the visible spectrum is not much more than one

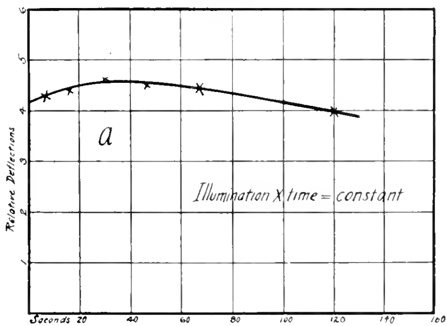


FIG. 7

volt. Substituting the value of 1 for V in equation (5), it is evident that $\frac{19}{23}$ volts is very far from the 0.02 volts determined on as the limiting condition. The curve shown is, therefore, exactly of the type one should expect from an approximately linear illumination-current relationship.

Further points of interest also follow. On the supposition of the cell behaving as an ohmic resistance, it follows that if the current $\frac{1}{R}$ is proportional to the illumination, then the same deflection should be obtained for the same values of the product intensity × time. This assumption has been made in using the ballistic method, in order to keep the deflection on the scale. Fig. 7 shows

the values of the deflections obtained for constant intensity$\times$time. The deflections are not constant, which would lead to the suspicion that the photo-electric current is not proportional to the intensity. The deviation might, perhaps, be due to a slight leak, for by equation (2) it is $\left(\frac{1}{R}+\frac{1}{r}\right)$ and not $\frac{1}{R}$ that is interchangeable with t. By the use of the steady-deflection method any slight leak merely adds to the purposely introduced leak and hence this alternative explanation may be tested out by recourse to that method.

Now, as to the curve shown in Fig. 3, given by the initial rate of drift. This and all the other results on this cell are consistently explainable on the ground that (1) the true illumination-current relationship is of the character shown in Fig. 3; (2) the effective voltage in the cell is altogether too low to warrant the use of the ballistic method with the electrometer-sensibility employed.

Assuming the current proportional to the voltage, as it probably is to a first approximation, we may substitute in equation (5) above, choosing suitable constants, the current values as given by the initial rate of drift (Fig. 3). The equation $\frac{d}{2}=(1-e^{-25I})$, where d=deflection in centimeters and I=current values from Fig. 3, yields the points marked in Fig. 2 by circles. It is evident from the perfect coincidence that there is nothing thus far to contradict the belief that the curve convex to the illumination axis (as found by Griffith) represents the true illumination-current relationship in this cell.

Further support for this is found in the next section.

8. COMPARATIVE RESULTS ON SEVERAL CELLS

With the apparatus as described under section 4, a large number of miscellaneous illumination-current curves were taken for all the cells above described. These were made with different applied voltages. At first the aim was to use one voltage throughout, and one was selected which would not cause a dark discharge in any of the cells. Afterward it became evident that the phenomenon under study was a function of voltage, so some extra curves were made at different voltages. The results are exhibited in Figs. 8–15.

Before discussing them as a group it is perhaps best to complete the discussion of cell *a*.

At various times during the experiments on high resistances, curves similar to Fig. 3 were obtained from this cell. As long, however, as polarization and other troubles were not entirely eliminated these results could not be accepted as conclusive. Fig. 8 shows the illumination-current curve obtained under conditions believed to be subject to no reservation or correction. It is, like

FIG. 8 FIG. 9 FIG. 10 FIG. 12

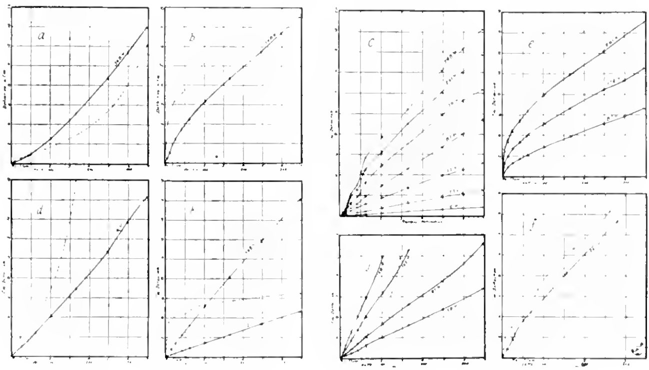


FIG. 11 FIG. 15 FIG. 14 FIG. 13

FIGS. 8, 9, 10, 12, 11, 15, 14, 13.—Characteristics of eight photo-electric cells

Full lines: Illumination-current relationship
Broken lines: Voltage-current relationship
Illumination unit, roughly 10 meter-candles, 1 cm deflection = 0.2 $\times 10^{-9}$ amperes

Fig. 3, the curve given by Griffith, and Lenard's values when plotted, convex toward the illumination axis.

Had no other cell been at hand the conclusion could well have been drawn that this is the true relationship between photo-electric current and illumination. The opportunity afforded by the possession of several cells, however, made it possible to demonstrate that the phenomenon is much more complicated. Each of the several cells, as is evident from Figs. 8–15, shows a different

illumination-current relationship. Curves both concave and convex, of most extreme type, as well as some with double curvature, appear. It appears, too, that the relationship is, to a varying degree, in different cells, a function of the applied voltage.

In no case is this relationship linear. If only a comparatively short range of illumination be used, few points taken, and a certain voltage range not overstepped (see, for instance, *c*, 7.3 volts), curves may be obtained which appear to be straight lines. When compared with other curves of the same family, that is, obtained with higher and lower voltages, the points which deviate by amounts apparently within the errors of measurement are actually found to be true indications of curvature. Furthermore, with the cells *j* and *k*, in which the nearest apparent approach to linearity was found on first measurement, the observations were repeated many times, running up and down the curves, proving the curvatures to be real and of significance. The four curves of cell *j* are obviously developments of each other. Cells such as *j* and k might easily be hastily assumed to show the heretofore believed linear relationship. Only the refinement of measurement called for by the requirements of photometric application would make clear with these particular cells that this is not so. Fortunately, the extreme differences among all the cells leave no doubt of the exceptional and accidental character of the linear relationship when found.

9. SUGGESTED REASONS FOR THESE RESULTS

A study of the illumination-current relationship exhibited in Figs. 8–15 shows that in any one cell the relationship is a function of the applied voltage, although to a varying degree in different cells. Thus the variation in character of curve in cell *c*, in going from 2 to 348 volts, is much greater than in cell *k*. It is evident that different voltages correspond in different cells to different characteristic curves. Thus cells *b*, *f*, and *c* show the same type of curves at voltages 348, 261, and about 100. Nor does it appear possible in any one cell to obtain all types of curves by mere voltage change. For instance, the two extremes exhibited in cell *a* (concave upward) and cell *e* (concave downward) are not attained in cell *c* at the extreme voltages used, other phenomena entering to

interfere at the high voltage and lack of sensibility preventing a study of lower voltage effects.

Differences of pressure, of gas, of electrode distance, of surface, exist among the cells, and it is to these that we must look for explanation of the phenomena. These differences have been recorded in the description of the cells. Additional evidence as to the different electrical conditions holding in the cells is afforded by the voltage-current curves (made subsequently to the illumination-current curves) for a chosen medium illumination. These show differences, ascribable to different pressures and electrode distances, which are as extreme as those between the illumination-curves, which they resemble in some ways.

A study of previous work on the photo-electric effect[1] shows that curves possessing nearly all the characteristics of Figs. 8–15 have been obtained where the variables were voltage, electrode distance, and pressure, illumination being maintained constant. J. J. Thompson[2] develops two equations, the first dealing with the conditions well below the voltage, pressure, and electrode distance at which discharge occurs in the dark and the second with the conditions near the point of discharge. From the first of these equations it follows that for low voltages the current should obey Ohm's law, for higher voltage saturation would set in. From the second of these equations it follows that the current should increase, owing to ionization by collision, according to a power of e determined by electrode distance, etc. These equations were found represented in a general way by replotting the data of cell c, Fig. 10, in terms of voltage and current for fixed illumination, Fig. 16. It will be seen that curves showing various stages of the two equations are represented. It must be clearly understood, however, that the difference between any two curves in this figure is not produced by change of pressure, electrode distance, or gas, but by changing that factor which figures as a constant in Thompson's equations—illumination.

As the most inclusive summary of these results it may be said that the reasoning and equations just quoted are applicable to the

[1] See J. J. Thompson, *Conduction of Electricity through Gases.*

[2] *Ibid.*

present case if variation of illumination and variation of voltage are considered as similar in influence. But certain peculiarities of these illumination-curves are not to be overlooked. The voltage-current curves of cell *c* show at the lowest illumination apparent complete saturation; at slightly higher illuminations approach to saturation and subsequent increase ascribed to ionization by collision. But at higher illuminations the increased current again

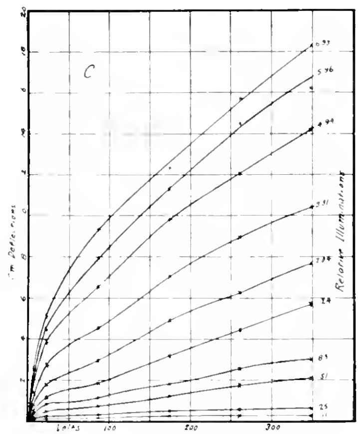


FIG. 16.—Voltage-current relations in cell *c* at different illuminations

approaches saturation, making a saturation-curve with a depression. At still higher illuminations this depression disappears and a simple characteristic curve is obtained, apparently approaching saturation uniformly. This steplike character of the voltage-current curve occurs in a number of cells, as shown by broken lines in Figs. 8–15. These data are of interest in connection with the characteristic curves for the inert gases, recently published by Franck and Hertz,[1] which show a series of steps.

The most striking feature of the illumination-current curve *c* is

[1] *Berichte der Deutsch. Phys. Gesel.*, **20**, 929, 1913.

the series of steps shown at higher voltages. These steps, caused by varying illumination, are similar to the voltage-current curve steps just referred to, and are additional evidence that illumination variation must in these cells be considered as of similar effect to voltage variation. The data of cell *j* (argon-filled) indicate similar stepped curves, due to illumination variation. A peculiarity of curve *c* is indicated by the broken-line continuations of the first approach to saturation. In each case the point of illumination 0.8 was the second point of rest of the electrometer needle, attained after the needle had nearly come to rest at a lower value. Undercooling-curves are at once suggested.

The next step in the study appeared to be the intentional variation of electrode distance and pressure, the factors not variable in the completed cell.

10. FURTHER EXPERIMENTS TO TEST CONCLUSIONS

Cell with variable electrode distance.—The easiest factor to vary and, as it was thought (noting the results of Stoletow), the most likely to show in the results, was electrode distance. A special cell was, therefore, constructed, shown in Fig. 6 (*g*). The platinum electrode was attached to an iron rod which slid in a glass tube and was connected by a fine coil of copper wire to a platinum wire sealed in the glass and going to the batteries. The cell was first filled in a horizontal position and sealed off at the best vacuum obtainable with the Gaede mercury pump, the iron rod was then held in position by a solenoid, the cell turned to the upright position, and the potassium melted and flowed into its final place. The electrode was afterward easily put in any desired position by inclining and tapping the tube.

Fig. 17 shows two curves obtained with the cell, for electrode distance of 2 and 40 mm. These are both concave to the illumination axis, and while the short-distance curve is somewhat less concave than the other, it is evident that variation of electrode distance alone, at this pressure, does not produce rapid changes in curve type.

Cell with variable pressure.—A cell similar to *g* was next constructed, differing in the possession of a side tube containing a

ground stopcock and a ground cone, fitting the ground sleeves of the pump system *h*. This cell was made a little differently from the last, being filled in an upright position through a lateral constricted tube, the solenoid being in position throughout. Of course the proper procedure would be to have the cell constantly connected to pump and gauge, but this was not possible to arrange in the present case. Trouble was expected from the stopcock, and leakage did, in fact, spoil the potassium surface after two days. In that

Fig. 17

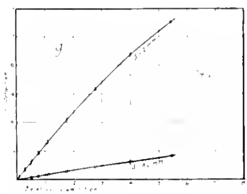
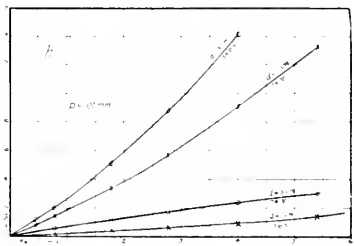


Fig. 18



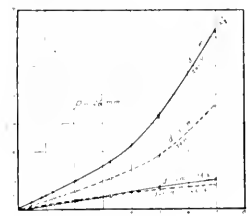


Fig. 19

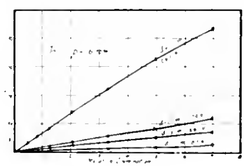


Fig. 20

Figs. 17, 18, 19, 20.—Illumination-current relationships in cells of variable electrode distance and gas pressure.

time, however, it was possible to allow the surface to reach a steady state after its first rapid drop in sensitiveness and to run several curves at several pressures of hydrogen. The change in pressure was made by replacing the cell on the pump, securing a good vacuum, and introducing gas from palladium foil. Measurements were made as soon after as practicable. Results for three pressures 0.01, 0.06, and 0.6 mm are shown in Figs. 18, 19, and 20, while Fig. 17 may be considered as exhibiting part of the same series of pressures, the

value of p being about 0.005 mm. These curves, while by no means offering a complete picture of the variation of the illumination-current relation, show clearly that it is a function of pressure, voltage, and electrode distance. From the results of Varley it is to be expected also that the nature of the gas filling the tube will affect the relationship.

II. DISCUSSION

Two generalizations appear to be justified from study of these curves:

A. There is a qualitative agreement throughout between the effects of (1) increase of electrode distance; (2) increase of pressure; (3) decrease of voltage. For illustration, take Fig. 19, $p=0.06$ mm, electrode distance 1 cm, 261 volts. This gives a curve convex to the illumination axis. Lowering the voltage to 174, distance remaining constant, yields a nearly straight line, i.e., a more concave curve. Leaving the voltage at 261, an increase of electrode distance to 5 cm yields a concave curve. With this greater electrode distance an increase of voltage to 348 brings back the convex type of curve. Again changing from 174 volts, 1 cm distance at 0.01 mm (Fig. 18), to the same voltage and distance for 0.06 mm (Fig. 19) alters the curve from convex to straight, while with pressure remaining constant at 0.01 mm an increase of electrode distance to 5 cm yields a concave curve. There appears to be no exception over the range of voltages, etc., here used, to this relationship, which, of course, is not an unnatural one.

B. The course of the changes from the most intense conditions (high voltage, low pressure, small electrode distance) to the least intense (low voltage, high pressure, large electrode distance) appears to consist of at least three and perhaps more stages.

First stage.—At most intense conditions (Fig. 15) a current which approaches saturation with increase of illumination (curves concave toward illumination axis).

Second stage.—A current increasing with illumination in the same manner that a constant illumination current increases as the dark discharge voltage is approached (ionization by collision) (Figs. 8, 11, 18, 19).

Third stage.—This second current approaches saturation (Figs. 9, 12, 13).

A fourth stage is suggested in the curves of Fig. 10, cell *c*, where at relative illumination 0.8 for the higher voltages, a new upward turn is taken, followed again by apparent slow approach to saturation. In fact it appears probable that the illumination-current relationship may be plotted as a curve whose ordinates increase in value in steps.

The most general statement of the effects produced by light is that already made, that variations of illumination appear to act in a similar manner to variations in voltage, producing currents obeying Ohm's law over short ranges (in this case $c = \frac{\text{Illumination}}{\text{Resistance}}$), currents which approach saturation, which increase in value as though through ionization by collision, etc. The quantity of electrons liberated is apparently as potent a factor as is voltage in altering the character of the discharge.

Other differences between the originally studied cells may have been of effect. For instance, the somewhat different character of surface with consequent different amounts of normal and selective effect. It would be of interest in a more complete and detailed study of this relation to examine the normal and selective effects separately in sodium potassium alloy.

A point of extreme importance which must here be emphasized is that these results apply of necessity only to gas-filled cells. The best vacuum attained in the cells here described was not sufficient to produce saturation. Traces of gas, and probably mercury vapor, were always present. What will happen in a cell exhausted by the new Gaede molecular pump with the assistance of liquid air is a question still open for investigation. It must be remembered, however, that all previous work on this relationship has been done with no better vacua than here used; that the most widely exploited cells, those of Elster and Geitel, are purposely filled with a gaseous atmosphere. In short, this work stands in contradiction to all researches which have hitherto been considered proof of the linear illumination-current relationship.

Looked at from the standpoint of photometric application,

these results are the opposite of encouraging. The most that can be said is that by careful choice of electrode distance, gas, pressure, and voltage, cells may be produced which for a more or less limited range show a linear relationship between illumination and current.

Every cell would, therefore, have to be either tested for this relation while still on the pump, or have its calibration-curve determined under conditions to be rigidly adhered to afterward. Any change in pressure, or possibly (a point not touched on here) any aging of the surface would necessitate a checking of this calibration.

A further question, should colored-light photometry be attempted with cells made with the characteristic thus determined, would be the uniformity between different cells and the permanence in any one cell of the distribution of sensibility through the spectrum. Data on wave-length sensibility obtained during the course of this investigation indicate possibilities of wide differences from cell to cell, but this question is left for further study.

It must not be forgotten, however, that the photo-electric cell possesses enormous sensitiveness and that the conclusions above reached do not affect at all its use as a detector, or for measurement by substitution methods where the lights under comparison are of identical quality. A set of electric lamps of the same type could, for instance, be brought to the same candle-power by so regulating their voltage that they gave the same photo-electric current in the cells, or relative candle-powers could be determined by finding the distance at which different lamps gave the same deflection. A degree of sensitiveness greatly exceeding the eye should be attainable for such work. There is not the need, however, for this kind of photometric adjunct that exists for an "average eye" for standardization of color measurements.

12. SUMMARY AND CONCLUSIONS

1. Photo-electric cells and auxiliary apparatus have been developed by which the character of the photo-electric current may be studied without disturbance by spurious effects.

2. The use of the quadrant electrometer for measuring photo-electric current has been studied and the conditions determined such that its characteristics introduce no distortions in the results.

3. The illumination-current relationship has been found not to be linear, but to be a complicated function of voltage, electrode distance, and pressure, similar to the voltage-current relationship.

4. It is concluded that gas-filled photo-electric cells do not possess the qualities most desirable in a physical photometer.

The writer takes pleasure in acknowledging the assistance and co-operation of Mr. E. Karrer during a portion of this work.

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United Gas Improvement Co.
Philadelphia
January 1914

PHOTOMETRIC TESTS OF SPECTROSCOPIC BINARIES

By JOEL STEBBINS

For some years, I have been interested in the possible small light-changes of spectroscopic binaries, synchronous with the orbital motion. In 1904 I began to test some of these stars with a visual photometer, but did not succeed in finding any new variables, and the problem was laid aside until some means could be found of improving the accuracy of the observations. When the selenium photometer was perfected, I at once planned to renew the tests of short-period binaries. It had only been my hope, from the start, to detect continuous variations of something like 0.05 or 0.10 magnitude, and the possibility of finding eclipsing systems had not appealed to me until eclipses had been actually discovered in β *Aurigae* and δ *Orionis*. The success in these two cases brought out the high probability that there are plenty of eclipsing variables which may easily be detected by observations at the proper times. All we need to do is to compute from the spectroscopic elements the instants when the longitude from the node, u, is equal to 90° or 270°. An eclipse of the second component is just as important as that of the primary, for, neglecting the eccentricity of the orbit, where there is one minimum there must always be two. Even if the fainter body is nonluminous, it will reflect the light of the primary, and theoretically at least this second eclipse will be present.

This program of taking photometric measures at the favorable epochs sounds easy enough, but the difficulties are considerable. We hear some complaint that the observers of *Algol* variables do not determine the light-curves thoroughly, but a little experience will convince anyone that, at an ordinary station, it is not often that a critical point on a light-curve, such as the beginning or end of a minimum, happens to come on a good clear night, with the star in proper observing position. In work on bright stars with the selenium photometer, the times are more than ordinarily restricted because of the exacting requirements. When a variable and the

comparison star are 10° or even 15° apart, and we desire results correct to the hundredth of a magnitude, there is absolutely no use in trying to work on any but first-class nights.

Up to the present, with our 12-inch telescope, I have not tried the selenium method on stars fainter than magnitude 3.0, and the program is therefore quite limited. Nevertheless it has seemed worth while to continue the search for variables, because of the importance of those cases where we have both the light- and velocity-curve of an eclipsing system. I have therefore made a list of spectroscopic binaries, and at the beginning of each month I have marked the nights when each star should be tested. The work of observation is slow and laborious, but there is always the hope of picking up a variable, and any such found will be both interesting and important.

The times of minima are easily computed with the aid of the "Tables for the True Anomaly in Elliptic Orbits" by Schlesinger and Miss Udick.[1] When the eccentricity is small, we may derive a convenient check from the relation that the true anomalies of the two minima differ by 180°. Let t_1 be the epoch of eclipse of the brighter, and t_2 that of the fainter spectroscopic component, v_1, v_2, and M_1, M_2 the corresponding true and mean anomalies; P the period, e the orbital eccentricity, and ω the longitude of periastron. Then[2]

$$\left.\begin{aligned} v_1-M_1=2\Big[e\sin v_1-\tfrac{1}{2}\frac{1+2\sqrt{1-e^2}}{(1+\sqrt{1-e^2})^2}e^2\sin 2v_1+\\ \tfrac{1}{3}\frac{1+3\sqrt{1-e^2}}{(1+\sqrt{1-e^2})^3}e^3\sin 3v_1-\;\ldots\;\Big]\end{aligned}\right\} \quad (1)$$

$$\left.\begin{aligned} v_2-M_2=2\Big[e\sin v_2-\tfrac{1}{2}\frac{1+2\sqrt{1-e^2}}{(1+\sqrt{1-e^2})^2}e^2\sin 2v_2+\\ \tfrac{1}{3}\frac{1+3\sqrt{1-e^2}}{(1+\sqrt{1-e^2})^3}e^3\sin 3v_2-\;\ldots\;\Big]\end{aligned}\right\} \quad (2)$$

Subtracting (2) from (1) and noting that $v_2=v_1+\pi$, also $v_1=\frac{\pi}{2}-\omega$, we have, neglecting e^5 and higher powers:

$$M_2-M_1-\pi=4e\cos\omega-\tfrac{2}{3}e^3\cos 3\omega$$

[1] *Publications of the Allegheny Observatory*, 2, 155, 1912.

[2] Tisserand, *Mécanique céleste*, 1, 224.

or

$$\frac{(M_2-M_1)P}{2\pi}=t_2-t_1=\frac{P}{2}\left(1+\frac{4e\cos\omega}{\pi}-\frac{2}{3\pi}e^3\cos 3\omega\right) \tag{3}$$

This gives a convenient check upon the computation of t_1 and t_2, but cases may arise where we may neglect e^3 and use the times t_1 and t_2 for the determination of e. From (3) there follows

$$e\cos\omega=\frac{\pi}{2}\left(\frac{t_2-t_1}{P}-\frac{1}{2}\right) \tag{4}$$

The determination of the spectroscopic elements is often troublesome when e is small, and if it happens that a star shows eclipses which can be accurately observed, then $e\cos\omega$ may be derived from (4), and only the other component, $e\sin\omega$, need be found from the velocity measnres.

Let us note that the times, t_1 and t_2, here defined are not strictly the instants of maximum eclipse when the orbital inclination differs from 90°, but the errors introduced are quite small.

It is the custom for most observers to be reasonably conservative in the announcement of new variable stars, especially those with small range. In the case of stars here considered it is quite legitimate to publish suspicions of variability, because in any case the spectroscopic period may be assumed, and other observers will know just when to confirm the light-changes. Negative evidence is also important, and when a star has been tested without showing eclipses, that fact ought to be announced with reasonable promptness so that a second observer will not waste his time on unpromising cases. In this connection, however, it is much more difficult to prove that the light of a star is constant than that it varies. When eclipses take place, and the star decreases by a conspicuous amount, the fact is settled; but to show that there are no eclipses, one must have observations extending for some time on each side of the predicted minimum, to make sure that the spectroscopic elements and period are all right. It has appeared to some astronomers that the laborious solution for the definitive elements of a spectroscopic binary are scarcely worth while, but I have found it extremely convenient to know that the spectroscopic results are reliable.

In forming the observing program, Campbell's "Second Catalogue of Spectroscopic Binary Stars"[1] has been used, though in some cases extra decimals have been taken from the original sources. The stars have been selected principally on account of brightness. The favorable cases are presumably those systems of short period, and large range in velocity; or, what amounts to the same thing, those with large values of the quantity $\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$. Some stars have been tested on many nights and throughout the whole period, but others only a few times, and there is therefore a vast difference in the relative thoroughness with which the different objects have been studied.

In what follows, under each star the spectroscopic elements are given, and with these the hypothetical times of eclipses. Greenwich Mean Time is used throughout. In the journal of observations, the phase is expressed as a fraction of the period. A "set" of measures usually consists of two exposures on the comparison star, four to six on the tested star, and then two on the comparison star. All of the measures have been corrected for atmospheric absorption. After a discussion of the evidence in each case, I have tried to give in a short summary my best judgment of the result of the tests.

21 α Andromedae

H.R. 15, Mag. 2.15, Spectrum A0

There are independent orbits for this star by Baker[2] and by Ludendorff.[3]

	Baker	Ludendorff
P	96^d67	96^d7
T	2417882.40	2416817.
ω	$76°21$	$69°4$
e	0.525	0.50
$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	0.180	0.13

The hypothetical light-elements from Baker's orbit are:

$$\text{Min. I} = \text{J.D. } 2417883.39 + 96^d67 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 57^d26 = 0^P.592$$

[1] *Lick Observatory Bulletin*, **6**, 17, 1910.

[2] *Publications of the Allegheny Observatory*, **1**, 22, 1908.

[3] *Astronomische Nachrichten*, **178**, 23, 1908.

The same from Ludendorff are:

$$\text{Min. I} = \text{J.D. } 2416818.6 + 96^{d}7 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 60^{d}5 = 0^{p}626$$

The comparison star was β *Andromedae*, with additional measures on α *Pegasi* for the absorption correction. The difference of magnitude is in the sense: β *Andromedae minus* α *Andromedae*.

TABLE I

OBSERVATIONS OF α *Andromedae*

Date	G.M.T.	Phase		Difference of Magnitude	Sets	Remarks
		Baker	Ludendorff			
1911		p	p	Mag.		
October 22	$20^{h}\ 17^{m}$	0.994	0.000	0.37	3	
23	17 15	.003	010	.37	3	
23	18 13	.003	.010	36	3	
November 15	17 48	.241	.248	.37	2	
December 19	15 04	.592	.598	.37	3	Poor sky
22	15 00	0.623	0.629	0.32	2	Smoke

On December 19 the sky was somewhat thick, but apparently uniform, and on December 22 smoke was passing near or over the stars. In determining the light-curve of a known variable, I should not think of working under such conditions; but in this case there was a chance to make a discovery, since experience shows the smoke would not change the result by more than 0.1 or 0.2 magnitude. The agreement of the other measures is in part accidental but not extraordinary.

Result for α *Andromedae.*—Observations on five nights near primary and secondary minimum, and on one night between minima, give no evidence of eclipse variation.

13 α *Aurigae*

H.R. 1708, Mag. 0.21, Spectrum G0

The orbit is by Reese,[1] and two spectra are visible.

P	$104^{d}022$	$\dfrac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	0.184
T	2414899.5		
ω	$117^{\circ}3$	$m_1 \sin^3 i$	1.19
e	0.016	$m_2 \sin^3 i$	0.94

[1] *Lick Observatory Bulletin*, **1**, 34, 1901.

The hypothetical light-elements are:

$$\text{Min. I} = \text{J.D. } 2414995.87 + 104^{d}022 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 51^{d}52 = 0^{P}495$$

A variation of *Capella* seems improbable because of the length of period and the Class G spectrum. After waiting during two years we managed to observe it near minimum on one night but through smoke. The comparison star was the variable β *Aurigae*, which, however, was not near its eclipsing phase. The difference of magnitude is in the sense: β *minus* α.

TABLE II

OBSERVATIONS OF α *Aurigae*

Date	G.M.T.	Phase	Difference of Magnitude	Sets	Remarks
1912		P	Mag		
March 6	$15^{h}\ 34^{m}$	0.998	1.70	4	Smoke
6	16 14	.999	1.70	4	Smoke
6	16 54	.999	1.84	4	Smoke
March 6	16 14	.999	1.78	12	Mean
9	16 20	0.028	1.77	5	

Result for α *Aurigae*.—Measures on one night at phase $0^{P}999$ compared with another night at phase $0^{P}028$ give no evidence of variation.

44 ι *Orionis*

H.R. 1899, Mag. 2.87, Spectrum Oe5

The orbit is by Plaskett and Harper.[1]

P	$20^{d}136$	e	0.754
T	2417587.003	$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	1.14
ω	$113^{\circ}28$		

The hypothetical light-elements are:

$$\text{Min. I} = \text{J.D. } 2417587.815 + 20^{d}136 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 7^{d}120 = 0^{P}245$$

Eclipses of this star would be especially interesting because of its spectrum, Oe5. The comparison star was 53 κ *Orionis*, and the difference of magnitude is in the sense: κ *minus* ι.

[1] *Astrophysical Journal*, **30**, 379, 1909.

TABLE III

OBSERVATIONS OF ι *Orionis*

Date	G.M.T.	Phase	Difference of Magnitude	Sets	Remarks
1911		P	Mag		
November 24	20h 14m	0.025	0.65	2	
December 11	18 28	.606	.64	2	
1912					
February 19	14 38	.003	.64	5	Smoke
19	15 28	.004	.62	5	Smoke
March 26	14 54	0.230	0.60	3	Bright moon, poor

Result for ι Orionis.—Observations at $0^P.004$ after primary and $0^P.006$ before secondary minimum show no evidence of variation.

66 α *Geminorum*

α_1 *Geminorum, H.R.* 2890, Mag. 2.85, Spectrum A0

α_2 *Geminorum, H.R.* 2891, Mag. 1.99, Spectrum A0

The orbit for each star has been derived by Curtis.[1]

	α_1	α_2
P	$2^d 928285$	$9^d 218826$
T	2416828.057	2416746.385
ω	102°.52	265°.35
e	0.01	0.503
$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	0.0097	0.0015

The hypothetical light-elements are for α_1:

$$\text{Min. I} = \text{J.D. } 2416827.957 + 2^d 928285 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 1^d 460 = 0^P.499$$

and for α_2:

$$\text{Min. I} = \text{J.D. } 2416751.305 + 9^d 218826 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 4^d 333 = 0^P.470$$

I have studied this quadruple system in some detail, principally because of the favorable comparison star, β *Geminorum*. As both the mass functions are small, there seems to be little probability of eclipses, but the high eccentricity for α_2 might lead to some light-variation. The companion in this case is three times nearer and

[1] *Lick Observatory Bulletin*, **4**, 58 and 64, 1906.

receives nine times as much radiation at periastron as at apastron. A variation of 0.02 mag. in the combined light of α_1 and α_2 would be produced by a change of 0.064 mag. in α_1, or 0.029 mag. in α_2. There is, therefore, a possibility that the most accurate observations might detect a variation in either of the stars.

As the work on *Castor* was secondary to that on other stars, the observations were prolonged over several years, and with varying conditions of the photometer. We used different mountings of the selenium cell, and different portions of its sensitive surface. For this reason, the observations are divided into five series, and small systematic corrections have been applied to the results so as to bring the mean of each series into agreement.

Series	Correction Mag.
I	−0.001
II	+.001
III	.000
IV	+.010
V	−0.003

Since the maximum difference between series was only 0.013 mag., the experiments with the instrument introduced no great disturbance.

In Table IV are given the results of the measures, after the above systematic corrections have been applied. The weights are proportional to the number of sets, except for Series II, where the general agreement is poor, and the whole series has been given half-weight. The difference of magnitude is in the sense: α *Geminorum minus* β *Geminorum*, the measure being, of course, of the combined light of α_1 and α_2.

On February 9, 1912, I was surprised to note that the measures made *Castor* abnormally faint at a phase near minimum of α_1. The observations were therefore extended as long as possible at the next minimum, three nights later, and surely enough the light decreased again about 0.05 magnitude. I then looked over the previous selenium observations, and found another discordant result at the same phase. Also a search through some visual work gave two more discordances at the eclipsing phase of α_1. I therefore had

TABLE IV

OBSERVATIONS OF *α Geminorum*

SERIES I

Date		G.M.T.	Phase		Difference of Magnitude	Weight
			a_1	a_2		
1909			P	P	Mag.	
November	25...	18h 38m	0.706	0.524	0.237	5
December	7...	17 47	792	822	217	4
	18...	18 20	556	.018	.233	5
1910						
January	9...	18 43	075	406	.231	5
	18...	19 02	153	383	217	5
	27...	18 33	219	.358	.205	5
	31...	19 18	596	.795	207	5
February	1...	18 16	923	.899	227	5
	3...	17 15	591	111	231	5
	3...	18 20	607	116	.235	3
	4...	17 48	940	222	197	5
March	12...	16 40	.218	.122	217	5
	13...	16 44	561	231	210	5
	14...	15 21	883	333	222	5
	16...	15 03	.561	.548	213	5
	17...	15 10	0.905	0.657	0.223	5
SERIES II						
April	3...	14 06	0.695	0.497	0.217	2
	6...	14 11	.720	.822	.238	2
	9...	14 16	746	.148	.217	2
	10...	13 54	082	.255	.242	2
	10...	14 49	.095	259	206	2
	13...	13 58	.108	.581	199	2
	13...	14 48	120	.585	168	2
	13...	16 08	.138	.591	.209	2
	20...	15 29	.520	.347	251	2
	20...	16 06	0.529	0.350	0.253	2
SERIES III						
November	24...	19 42	0.026	0.013	0.209	4
December	12...	20 41	187	.970	.231	5
	12...	21 38	200	.975	.221	5
1911						
January	9...	16 25	.688	.988	.218	5
	9...	17 15	.700	.992	.246	5
	9...	18 07	.712	.996	.220	5
	9...	19 01	725	000	.208	5
	9...	19 52	.737	.004	.203	5
February	24...	17 17	409	982	.197	5
	24...	18 10	422	986	.189	5
March	4...	17 40	.147	.851	.227	4
	13...	16 41	.206	823	.222	5
	20...	17 33	0.609	0.587	0.244	5

TABLE IV—*Continued*

SERIES III—*Continued*

Date		G M T		Phase α_1	Phase α_2	Difference of Magnitude	Weight
1911				P	P	Mag	
March	22	15^h	45^m	0 200	0 795	0 259	5
	24	15	20	944	010	215	5
	27	15	39	973	337	220	5
	31	16	22	349	774	191	5
April	1	15	10	673	877	212	5
	9	14	49	400	744	231	5
	15	15	16	456	396	220	5
	22	14	46	839	153	214	5
	22	15	41	852	158	207	5
	23	14	48	181	202	210	5
	23	15	43	194	206	201	5
	24	15	41	535	375	228	5
May	7	14	34	959	780	240	5
	7	15	26	0 971	0 784	0 248	5
				SERIES IV			
November	26	20	33	0 367	0 827	0 233	5
	26	21	26	380	831	220	5
December	1	21	21	086	373	237	5
	3	22	13	782	504	240	5
	3	23	22	798	509	221	5
1912							
January	8	18	48	027	483	224	3
	12	19	40	405	921	196	5
	12	20	59	424	927	199	5
	18	18	55	444	569	230	5
	18	19	49	457	573	220	5
February	9	20	12	975	961	202	5
	9	21	39	995	968	240	5
	12	18	02	968	277	199	5
	12	18	58	981	281	222	5
	12	19	53	994	285	229	5
March	5	16	40	462	657	212	5
	5	18	07	483	663	235	5
	5	19	25	501	669	214	5
	15	15	29	860	736	199	5
	15	16	24	0 873	0 740	0 225	5
				SERIES V			
March	24	15	37	0 936	0 713	0 250	5
	25	16	20	287	825	219	5
	30	15	57	989	305	218	4
	30	16	45	001	309	233	4
April	7	14	42	703	228	200	3
	8	15	16	053	330	224	5
	10	14	38	727	553	227	5
	15	14	20	430	094	189	3
	22	14	29	823	854	220	5
	26	14	38	0 191	0 288	0 211	4

five independent measures all showing that this is an eclipsing variable. On the theory of probabilities, such an agreement could scarcely be the result of chance, but nevertheless there were some suspicious circumstances. Further search revealed one or two good measures which did not confirm the variation, and there was also the possibility that a systematic error had crept in, due to increased atmospheric absorption with large zenith distances, or with a poor sky. Tests for this effect had been taken again and again on other stars, but no measurable variation with zenith distance had been found; this means, of course, any outstanding variation after the usual correction for differential absorption is applied. Here, however, we have the complication that both components of *Castor* are of spectrum Class A, while the comparison star, *Pollux*, is of Class K. It is natural to suppose that the light of the bluer star will suffer greater absorption near the horizon. Also it is possible that the color sensibility of selenium changes with varying light-intensity. Either of these effects would account for the discordances.

The amount of the absorption at any time is indicated by the size of the galvanometer deflections, and a graphical treatment showed clearly the dependence of the difference of magnitude upon the deflections. The matter was further tested by observing under extreme conditions, and in Table V are given the results of these tests. On these dates, the sky was "thick," but apparently uniform. The deflection is that of *Pollux* in terms of the mean of adjacent dates. For instance, the first two deflections of 0.6 and 0.4 were small compared with the remainder of the series, where all other deflections ranged from 0.8 to 1.2, the unit being 30 mm. The most discordant observation, on March 30, 1912, was taken at zenith distance 73°, and with the smallest deflections.

The results in Table V are not very accordant as to the amount of the absorption effect, but a good agreement could not be expected with the character of the sky. The point to emphasize is that a small deflection always corresponds to a positive residual, i.e., *Castor* faint. The measures in Table IV have already been corrected for the absorption effect, where necessary, most of the corrections being only one- or two-hundredths of a magnitude. The test observations of Table V are not used in the further discussion.

The next step is to form normal magnitudes from Table IV on the basis of the phase of α_1. In Table VI each normal is usually from three observations, or the weighted mean of 12–15 sets. With a mean difference of 0.220 mag., the residuals give the probable

TABLE V

TEST OBSERVATIONS OF α *Geminorum*

Date		G.M.T.	Phase α_1	Phase α_2	Difference of Magnitude	Residual	Deflections	Sets
			P	P	Mag.	Mag.		
1911 May	2	16^h08^m	0.273	0.244	0.290	+0.070	0.6	5
	4	15 57	.054	.461	.344	+ .124	.4	5
1912 February	12	20 34	.004	.288	.266	+ .046	.6	3
	12	21 18	.015	.291	.253	+ .033	.4	3
March	8	19 16	.524	.094	.317	+ .097	.6	5
	8	20 13	.537	.098	.314	+ .094	.4	5
	30	19 12	.036	.380	.463	+ .243	.35	3
April	5	13 54	.009	.007	.312	+ .092	.7	5
	5	15 04	.026	.012	.350	+ .130	.5	4
	5	16 26	0.045	0.018	0.421	+0.201	0.4	4

TABLE VI

NORMAL MAGNITUDES, α_1 *Geminorum*

Phase	Difference of Magnitude	Residual	Phase	Difference of Magnitude	Residual
P	Mag.	Mag.	P	Mag.	Mag.
0.038	0.210	−0.001	0.597	0.223	+0.003
.082	.231	+ .011	.657	.225	+ .005
.139	.210	− .010	.702	.232	+ .012
.186	.218	− .002	.719	.218	− .002
.200	.215	− .005	.734	.215	− .005
.234	.227	+ .007	.791	.227	+ .007
.334	.214	− .006	.838	.214	− .006
.395	.216	− .004	.872	.215	− .005
.418	.195	− .025	.921	.233	+ .013
.445	.217	− .003	.948	.217	− .003
.467	.222	+ .002	.971	.222	+ .002
.520	.230	+ .010	.981	.214	− .006
0.559	0.210	−0.001	0.996	0.236	+0.016

error of a single normal $= \pm 0.006$ mag. Any variation must therefore be quite small. Since α_1 has a short period of less than three days, it has seemed worth while to test for the possibility of ellipticity of figure, which would give a sine curve with period one-

half that of the orbital revolution. In Table VII, the combined normals are each the mean of two normals from Table VI, the phase being reduced to the first half of the period. The probable error of a combined normal is ±0.003 mag., so there is no evidence of ellipticity of figure. From the number and size of the residuals in

TABLE VII

COMBINED NORMALS, α_1 *Geminorum*

Phase	Difference of Magnitude	Residual	Phase	Difference of Magnitude	Residual
P	Mag.	Mag.	P	Mag.	Mag.
0.029	0.224	+0.004	0.336	0.214	−0.006
.070	.225	+ .005	.384	.216	− .004
.118	.216	− .004	.420	.214	− .006
.172	.222	+ .002	.446	.217	− .003
.201	.224	+ .004	.469	.222	+ .002
.226	.222	+ .002	0.488	0.225	+0.005
0.262	0.221	+0.001			

TABLE VIII

NORMAL MAGNITUDES, α_2 *Geminorum*

Phase	Difference of Magnitude	Residual	Phase	Difference of Magnitude	Residual
P	Mag.	Mag.	P	Mag.	Mag.
0.005	0.209	−0.011	0.581	0.218	−0.002
.035	.214	− .006	.596	.227	+ .007
.116	.227	+ .007	.650	.223	+ .003
.154	.212	− .008	.700	.221	+ .001
.227	.205	− .015	.753	.216	− .004
.262	.211	− .009	.786	.232	+ .012
.281	.223	+ .003	.814	.234	+ .014
.322	.218	− .002	.828	.224	+ .004
.349	.225	+ .005	.861	.219	− .001
.369	.230	+ .010	.916	.207	− .013
.385	.222	+ .002	.966	.226	+ .006
.473	.230	+ .010	.981	.202	− .018
0.557	0.223	+0.003	0.992	0.228	+0.008

Tables VI and VII, it seems fair to assume that any variability of *Castor* due to α_1 must be less than 0.02 mag., which means that α_1 itself is constant within about 0.06 mag.

The normals on the basis of phase of α_2 are given in Table VIII. Here again the probable error of a single normal comes out ±0.006 mag., and there are certainly no eclipses. There may be a faint

suspicion that the star has a continuous variation, but we may safely put this at less than 0.02 mag., which corresponds to 0.03 mag. in α_2. The normals for both α_1 and α_2 are shown in Fig. 1.

The accordance of the results for *Castor* is disappointing, for the probable error of ± 0.006 mag. is no better than in the first work on *Algol*, and not so good as that of *β Aurigae*. The brightness and proximity of *Castor* and *Pollux* made the conditions seem very

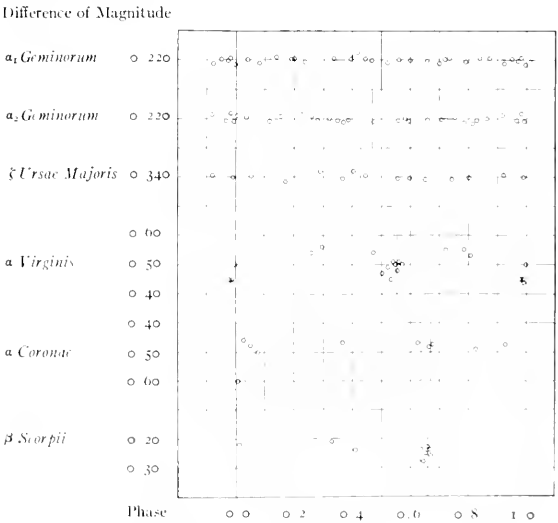

FIG. 1.—Normal magnitudes of spectroscopic binaries. (The vertical scale is uniformly 1 space = 0.10 magnitude.)

favorable, and in spite of the absorption error, I hoped that the probable error of a normal would not exceed ± 0.003 mag. Indeed, I am almost forced to the conclusion that the light of *Pollux* is not constant. If the sun, of spectrum Class G, is a variable star with range as much as 5 or 10 per cent, there is every reason to suppose that a Class K star may vary even more. As shown in Table IV, there are nights with systematic discordances of about 0.03 mag.,

and it may easily be that these are due to the variability of *Pollux*. One of the future problems of stellar photometry is a test of the various spectral types, and it will not be surprising to find an increasing tendency to irregular light-variation, as we pass along the scale from spectral classes B to M.

Result for α Geminorum.—An exhaustive test shows that the light of this star is probably constant within 0.02 mag., corresponding to a limit of 0.06 mag. for α_1 *Geminorum*, and to 0.03 mag. for α_2 *Geminorum*.

79 ζ *Ursae Majoris*

H.R. 5054, Mag. 2.40, Spectrum Ap

The orbit is by Vogel[1] and the period by Ludendorff.[2]

P	20^d536	e	0.502
T	2415472.84	$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	0.486
ω	$101^\circ.3$		
	$m_1 = m_2 =$	2.0	

The hypothetical light elements are:

$$\text{Min. I} = \text{J.D. } 2415472.66 + 20^d536 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 8^d79 = 0^P428$$

This was the first spectroscopic binary, discovered at Harvard, and eclipses would be unusually interesting as both components are bright. When the combined light is measured, the presence of the fainter visual component would reduce any variation of the binary by about one-fifth.

The photometric observations were taken in three series. In Series I two comparison stars were used, ε and η *Ursae Majoris*, while in Series II and III the measures all depend upon ε alone. The systematic corrections are:

Series	Correction Mag.
I	+0.048
II	0.000
III	+0.011

In Table IX, the difference of magnitude is in the sense: ζ *minus* ε. The weights are on the basis of the general accordance of the series.

[1] *Astrophysical Journal*, **13**, 328, 1901.

[2] *Astronomische Nachrichten*, **180**, 276, 1909.

TABLE IX

OBSERVATIONS OF ζ *Ursae Majoris*

SERIES I

Date		G. M. T.	Phase	Difference of Magnitude	Weight
	1910		P	Mag.	
July	10	15^h 14^m	0.562	0.358	3
	20	17 10	.614	.368	3
	23	15 01	.756	.323	3
	24	15 36	.806	.333	3
	26	15 15	.903	.388	3
	29	15 01	.048	.318	3
	30	15 00	.097	.338	3
August	2	15 14	.243	.303	3
	3	14 48	.291	.298	3
	4	14 59	.340	.358	3
	7	15 01	.486	.363	3
	9	15 08	.584	.303	3
	11	14 44	0.681	0.373	3

SERIES II

Date		G. M. T.	Phase	Difference of Magnitude	Weight
	1911				
March	19	15 57	0.396	0.351	4
	19	17 06	.398	.342	4
	19	19 31	.403	.320	4
	19	20 49	.406	.319	4
	19	21 59	.408	.302	4
	20	18 20	.450	.327	4
	22	16 41	.544	.344	4
	23	16 19	.592	.363	4
	24	16 14	.640	.345	4
	27	16 37	.787	.322	4
	31	16 59	.982	.362	4
	31	20 36	.990	.314	4
April	1	16 26	.030	.339	4
	9	15 49	.418	.317	4
	15	16 15	.711	.353	4
	22	16 39	.053	.348	4
	23	16 40	.102	.356	4
	24	16 36	.150	.371	4
May	2	17 05	.541	.366	4
	3	15 08	.586	.330	4
	7	16 28	.783	.346	4
	10	16 46	0.930	0.324	1

SERIES III

Date		G. M. T.	Phase	Difference of Magnitude	Weight
May	19	16 03	0.367	0.337	5
	24	16 04	.610	.339	5
	25	15 47	.658	.325	5
	27	15 50	.756	.362	5
June	1	15 16	.998	.347	5
	6	15 58	.243	.345	5
	7	16 04	.292	.330	5
	8	16 04	.341	.349	5
	10	15 42	.437	.348	3
	18	15 51	.827	.361	5
	19	15 22	.875	.325	5
	21	15 30	0.972	0.309	5

The results are combined into normals in Table X, each normal having a weight of from 12 to 16. The normal magnitudes are represented in Fig. 1. Adopting a constant difference, 0.340 mag., the 13 residuals give a probable error of a single normal = ±0.007 mag.

Result for ζ Ursae Majoris.—Observations throughout the spectroscopic period give no evidence of eclipses, nor of continuous variation. The star's light is constant within 0.02 or 0.03 mag.

TABLE X

NORMAL MAGNITUDES, ζ *Ursae Majoris*

Phase	Difference of Magnitude	Residual
P	Mag.	Mag.
0.055	0.337	−0.003
.171	.356	+.016
.292	.323	−.017
.366	.345	+.005
.404	.321	−.019
.446	.336	−.004
.556	.344	+.004
.597	.344	+.004
.649	.349	+.009
.741	.349	+.009
.802	.342	+.002
.920	.333	−.007
0.991	0.341	+0.001

67 α *Virginis*

H.R. 5056, Mag. 1.21, Spectrum B2

The orbit is by Baker,[1] and both spectra are measurable.

P	$4^{d}01416$	$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	0.823
T	2417955.846		
ω	328°	$m_1 \sin^3 i$	9.6
e	0.10	$m_2 \sin^3 i$	5.8

The hypothetical light-elements are:

$$\text{Min. I} = \text{J.D. } 2417957.004 + 4^{d}01416 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 2^{d}224 = 0^{P}554$$

With the selenium photometer, two comparison stars have been used: *4 γ Corvi*, *H.R.* 4662, mag. 2.78, spectrum B8; *27 β Librae*,

[1] *Publications of the Allegheny Observatory*, 1, 72, 1909.

TABLE XI

OBSERVATIONS OF *α Virginis*

Date		G.M.T.	Phase	Difference of Magnitude	Comparison Star
1912			P	Mag.	
March	25	17h 28m	0.309	0.54	γ
	25	17 52	.313	.50	γ
April	3	15 32	.531	.42	γ
	3	16 00	.536	.48	γ
	3	16 26	.540	.50	γ
	3	17 02	.546	.48	γ
	3	17 26	.550	.45	γ
	3	18 00	.556	.47	γ
	7	15 45	.529	.46	γ
	7	16 10	.534	.46	γ
	7	16 34	.538	.51	γ
	7	16 58	.542	.52	γ
	7	17 21	.546	.52	γ
	7	17 44	.550	.51	γ
	7	18 16	.556	.53	β
	7	18 38	.559	.53	β
	7	19 02	.564	.55	β
	7	19 36	.570	.49	β
	7	19 58	.573	.52	β
	8	16 12	.783	.53	γ
	8	16 35	.787	.57	γ
	8	16 58	.791	.52	γ
	8	18 51	.811	.54	β
	8	19 22	.816	.54	β
	15	15 25	.519	.47	γ
	15	19 13	.558	.44	β
	22	15 17	.261	.54	γ
	22	15 40	.265	.54	γ
	22	16 03	.269	.58	γ
May	7	14 20	.088	.34	γ
	7	14 43	.092	.41	γ
	7	15 06	.096	.48	γ
	9	16 39	.510	.47	β
	9	17 03	.514	.45	β
	9	17 22	.518	.55	β
	19	15 27	.989	.39	β
	19	16 24	.999	.48	β
	19	16 48	.003	.47	β
	23	14 39	.077	.46	γ
	23	15 18	.084	.50	β
	23	15 42	.088	.51	β
	23	16 18	.094	.57	β
	31	16 32	.990	.52	β
June	2	14 44	.469	.56	β
	2	15 07	.473	.48	β
	2	15 30	.477	.57	β
	2	15 54	.481	.48	β
	4	15 40	.977	.43	β
	4	16 04	.981	.46	β
	7	15 36	.724	.54	β
	7	16 00	.729	.52	β
	11	14 59	0.714	0.60	β

H.R. 5675, mag. 2.74, spectrum B8. It had been my intention to cut down the light of *Spica* by means of a screen, but a few experiments showed that it was feasible to shorten the exposures. The bright star was therefore given a 3-second exposure, as against 10 seconds for the comparison stars. This procedure is not without some risk, but the outcome seems to be satisfactory. On a good night, as the stars approached the meridian, the measures could be made with γ *Corvi* for several hours. Then when the absorption correction became too large, a pause of an hour or so was necessary until β *Librae* rose to a sufficient altitude.

In Table XI the difference of magnitude is in the sense: comparison star 10 seconds *minus Spica* 3 seconds. Each observation comprises two sets of measures. A discussion of the suitable observations gave not more than 0.01 mag. difference between the two comparison stars, and they are assumed to be equal. These measures, while not extending over the entire period, are sufficiently numerous to combine into normals, as shown in Table XII.

TABLE XII

NORMAL MAGNITUDES, α *Virginis*

Phase	Difference of Magnitude	Phase	Difference of Magnitude
P	Mag.	P	Mag.
0.263	0.54	0.560	0.51
.297	.56	.571	50
473	.54	.722	55
.502	47	.785	55
.522	49	.806	.53
.533	45	979	45
.540	51	987	45
.547	50	990	44
0.554	0.48	0.998	0.50

The normals are represented in Fig. 1, and it will be seen that either *Spica* is a variable, or some extraordinary error has crept in. The star was measured as faint near both predicted minima, and the drop was greater near primary minimum. The same variation is shown with each comparison star. A curious feature is that the eclipses are coming ahead of the times predicted from the spectroscopic elements, but a similar discrepancy in the same direction has been noted in other stars. A discussion of the probable errors

shows that the deviations from constant light are much greater than could be expected from the internal agreement of the measures. However, this is not a question to be settled by probable errors. The measures are admittedly difficult because of the low altitude of the stars, but it is my deliberate judgment that *Spica* is a variable.

Needless to say, it will be worth while to continue the observations, and fix the variation beyond a possibility of doubt. In *Spica* we have again the favorable case of an eclipsing variable with spectroscopic elements for both components, and in view of the early B2 spectrum, the computed dimensions in the system will be of special interest and importance.

Result for α Virginis.—The measures indicate that this is an eclipsing variable with two minima, the extreme range being something like 0.10 magnitude.

5 *α Coronae Borealis*

H.R. 5793, Mag. 2.31, Spectrum A0

There are two independent orbits by Cannon[1] and Jordan.[2]

	Cannon	Jordan
P	$17^{d}355$	$17^{d}36$
T	2417725.054	2417742.55
ω	$303^{\circ}68$	$312^{\circ}2$
e	0.277	0.387
$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	0.057	0.060

The hypothetical light-elements from Cannon are:

$$\text{Min. I} = \text{J.D. } 2417731.09 + 17^{d}355 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 10^{d}41 = 0^{P}600$$

and from Jordan:

$$\text{Min. I} = \text{J.D. } 2417730.04 + 17^{d}36 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 11^{d}64 = 0^{P}670$$

There is a difference of more than a whole day in the time of primary minimum predicted from the two orbits; but, with the different

[1] *Journal of the Royal Astronomical Society of Canada*, 3, 152, 1909.

[2] *Publications of the Allegheny Observatory*, 1, 89, 1909.

periods, this is reduced to about half a day at the epoch of the photometric observations.

The possibility of testing this star was overlooked for some time perhaps because the period of 17 days is not especially short. A few measures were taken in 1911, and as some were pretty close to minimum phase, it was thought that there was little need of further tests. However, on April 7, 1912, we had a fine night, and were making the most of it in observing several stars. Knowing that α *Coronae* was very near a predicted minimum, we started to measure it at about 14^h local mean time. I was at once amazed to find it faint, and the observations were therefore continued until stopped by dawn. On the next night, the star was back at normal light, and a second opportunity to test the minimum has not occurred, though measures have been made at other points along the light-curve.

The measures in Table XIII are all referred to η *Ursae Majoris*, though some of the earlier comparisons were with ϵ *Boötis*. The difference of magnitude is in the sense: α *Coronae* minus η *Ursae Majoris*. Each observation comprises three sets of measures, and there is practically no difference between the means for the two years.

The normals are shown in Fig. 1, where the dotted lines represent Cannon's minima. Excluding the first normal, we get 0.477 mag. for the mean. The residuals give a probable error of a single normal $= \pm 0.008$ mag., and the first residual, due to the eclipse, is fifteen times the probable error. If this minimum of α *Coronae* were a phenomenon observed once for all, it might be worth while to give in detail all of the measures on the one critical night. But we have the spectroscopic period, and anybody may test for the light change at his convenience.

There is no evidence of a secondary minimum, though there are three normals close to this phase. As only one spectrum is visible, the companion may not be very bright. It is not probable, therefore, that we shall be able to derive the approximate dimensions in this system, though they would be interesting since α *Coronae* is a member of the extended *Ursa Major* group.

TABLE XIII

Observations of *α Coronae Borealis*

Date	G.M.T.	Phase: Cannon	Phase: Jordan	Difference of Magnitude	Remarks
1911		P	P	Mag.	
May 24	17^h 05^m	0.585	0.622	0.48	ε Boötis
31	15 54	.986	.022	.44	ε Boötis
31	18 15	.991	.028	.48	ε Boötis
31	19 08	993	.030	46	
31	20 48	998	.034	.46	
June 1	10 06	044	.080	50	
1	16 48	.046	082	52	ε Boötis
13	15 42	.734	.770	.49	
18	16 34	.024	.060	.49	
1912					
April 7	20 42	975	006	59	Eclipse
7	21 23	.976	.007	.61	Eclipse
7	22 07	.978	.009	.60	Eclipse
8	20 08	.031	.062	.48	
8	20 50	033	.064	48	
18	20 59	.609	640	.45	
18	21 42	611	.642	.48	
22	16 43	830	.861	.51	
23	19 58	.895	926	.49	
23	20 40	.897	927	.46	
May 2	16 40	.406	.436	46	
2	17 18	407	.438	44	
5	16 21	578	608	48	
6	18 26	640	671	.48	
6	19 02	642	672	49	
6	19 42	643	674	.50	
6	20 18	.645	676	45	
9	18 11	813	843	.47	
June 2	16 42	0.192	0.222	0.50	

TABLE XIV

Normal Magnitudes, *α Coronae Borealis*

Phase: Cannon	Phase: Jordan	Difference of Magnitude	Residual
P	P	Mag.	Mag.
0.976	0.008	0.600	+0.123
990	027	460	−017
.018	052	477	000
.041	075	500	+023
.335	365	467	−.010
591	623	470	−007
631	662	483	+006
644	675	475	−.002
792	824	400	+.013
0.896	0.926	0.475	−0.002

Result for α Coronae.—Observations were made at principal minimum on one night under first-class conditions. The star was then about 0.12 mag. fainter than normal, and is therefore an eclipsing variable.

8 *β Scorpii*

H.R. 5984, Mag. 2.90, Spectrum B1

The orbit is by Daniel and Schlesinger.[1] A second orbit by Duncan[2] is practically identical, and gives the same times of eclipses. Two spectra are visible.

P	6^d8283	$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	1.26
T	2419163.923		
ω	$20°09$	$m_1 \sin^3 i$ ..	13.0
e	0.270	$m_2 \sin^3 i$	8.3

The hypothetical light-elements are:

$$\text{Min. I} = \text{J.D. } 2419164.74 + 6^d8283 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 4^d510 = 0^P660$$

Although this star is rather far south it can be compared conveniently with *δ Scorpii*. The difference of magnitude is in the sense: *β minus δ*. Each observation comprises two sets.

These normals are shown in Fig. 1. The normal at phase 0^P012 does not indicate an eclipse, but the results near secondary minimum make the star faint. Here, as in the case of *Spica*, we should have to assume that the eclipses are coming ahead of time.

Result for β Scorpii.—The results are not conclusive, but lead to a suspicion that the star is an eclipsing variable with small range.

DISCUSSION

With the stars tested with the selenium photometer, we may include *β Aurigae*[3] and *δ Orionis*. I have finished the light-curve for the latter, but some further spectroscopic data are necessary for a final discussion. Let it suffice to state that *δ Orionis* is an eclipsing variable with two minima, the total range being about 0.12

[1] *Publications of the Allegheny Observatory*, **2**, 135, 1912.

[2] *Lowell Observatory Bulletin*, **2**, 21, 1912.

[3] *Astrophysical Journal*, **34**, 112, 1911.

TABLE XV

OBSERVATIONS OF β *Scorpii*

Date	G.M.T.	Phase	Difference of Magnitude
1912		P	Mag
March 25	21ʰ 31ᵐ	0.326	0.20
25	21 48	.328	.22
25	22 05	.330	.20
30	20 45	.054	.22
30	21 04	.056	.26
April 3	20 48	.040	.19
3	21 10	.042	.26
3	21 36	.045	.28
3	21 56	.047	.30
3	22 22	.049	.30
10	18 58	.054	.25
10	19 16	.056	.21
10	19 34	.057	.23
10	19 52	.059	.24
10	20 14	.061	.23
10	20 34	.063	.30
10	20 56	.066	.22
10	21 15	.068	.19
10	21 34	.070	.28
10	21 53	.071	.23
26	21 06	.010	.21
26	21 28	.012	.20
26	21 43	.014	.24
May 14	17 40	.025	.30
14	18 01	.027	.26
14	18 22	.029	.27
14	18 42	.031	.26
14	19 04	.034	.28
25	17 41	.236	.24
25	18 02	.238	.21
25	18 21	.240	.20
June 2	17 40	.408	.21
2	18 00	0.410	0.26

TABLE XVI

NORMAL MAGNITUDES, β *Scorpii*

Phase	Difference of Magnitude	Phase	Difference of Magnitude
P	Mag	P	Mag
0.012	0.22	0.035	0.24
.055	.24	.045	.28
.238	.22	.053	.25
.328	.21	.059	.23
.409	.24	.066	.24
0.027	0.28	0.071	0.26

magnitude. We have then eleven stars which may be divided as shown in Table XVII. It seems fair to place *β Scorpii* in the first list, and *α Virginis* in the second. It appears then that 4 or 5 out of the 11 stars are eclipsing variables. Among the constant stars, the only one with a short period and large mass function is *β Scorpii*. Of the variables, all are favorable cases except perhaps *α Coronae*.

TABLE XVII

Constant Stars				Eclipsing Stars			
Star	Period	Spectrum	$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	Star	Period	Spectrum	$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$
	d				d		
α Andromedae	96.67	A0	0.18	*β Aurigae*...	3.96	Ap	0.54
α Aurigae..	104.02	G0	0.18	*δ Orionis*.....	5.73	B0	0.60
ι Orionis...	29.14	Oe5	1.14	*α Virginis*....	4.01	B2	0.82
α₁ Geminorum..	2.93	A0	0.0097	*α Coronae*...	17.36	A0	0.06
α₂ Geminorum..	9.22	A0	0.0015				
ζ Ursae Majoris	20.54	Ap	0.49				
β Scorpii....	6.83	B1	1.20				

Of course, we can draw no general conclusions from a few stars, but the desirability of testing other objects is apparent. This has already been done with success by Hertzsprung[1] and Shapley.[2] I have shown[3] that any reasonable assumption as to the densities and size of companions in short-period binaries leads to a large proportion of systems whose eclipses may be detected. In fact, one is somewhat appalled at the number of variables which are awaiting discovery. The known spectroscopic binaries are increasing by leaps and bounds; and in addition to the eclipsing variables among these, there are no doubt many of other periodic types, not to mention those with irregular light-changes similar to the sun.

The tests for small light-variations can be made only with the most accurate forms of photometer, and while the possibilities of the selenium method are still very great, it looks as if the

[1] *Astronomische Nachrichten*, **195**, 307, 1913.

[2] *Ibid.*, **196**, 383, 1913.

[3] *Astrophysical Journal*, **34**, 105, 1911.

photo-electric cell would furnish the means for the next improvements. About a year ago, Professor W. F. Schulz[1] tried a potassium cell on our 12-inch telescope with encouraging results, and the experiments are being continued by Messrs. Kunz, Schulz, and myself. As is well known, photo-electric cells like those of Elster and Geitel have been used successfully by Meyer and Rosenberg[2] at Tübingen, and by Guthnick[3] at Berlin-Babelsberg.

The observations described in the present paper were secured with the assistance of Messrs. Percy F. Whisler and H. F. Zoller. I am also indebted to Mr. J. D. Bond for checking some of the reductions. This work is a portion of that made possible by several grants from the Rumford Fund of the American Academy of Arts and Sciences.

UNIVERSITY OF ILLINOIS OBSERVATORY
URBANA, ILL.
February 6, 1914

[1] *Astrophysical Journal*, **38**, 187, 1913.

[2] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **48**, 210, 1913.

[3] *Astronomische Nachrichten*, **196**, 357, 1913.

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SUBJECTS

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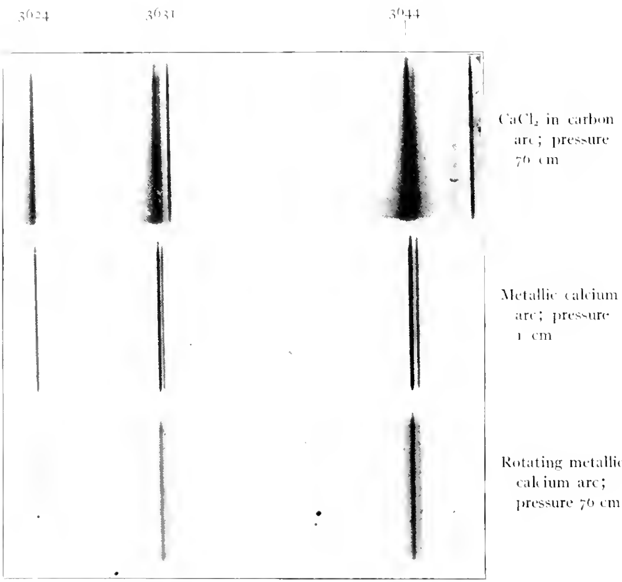
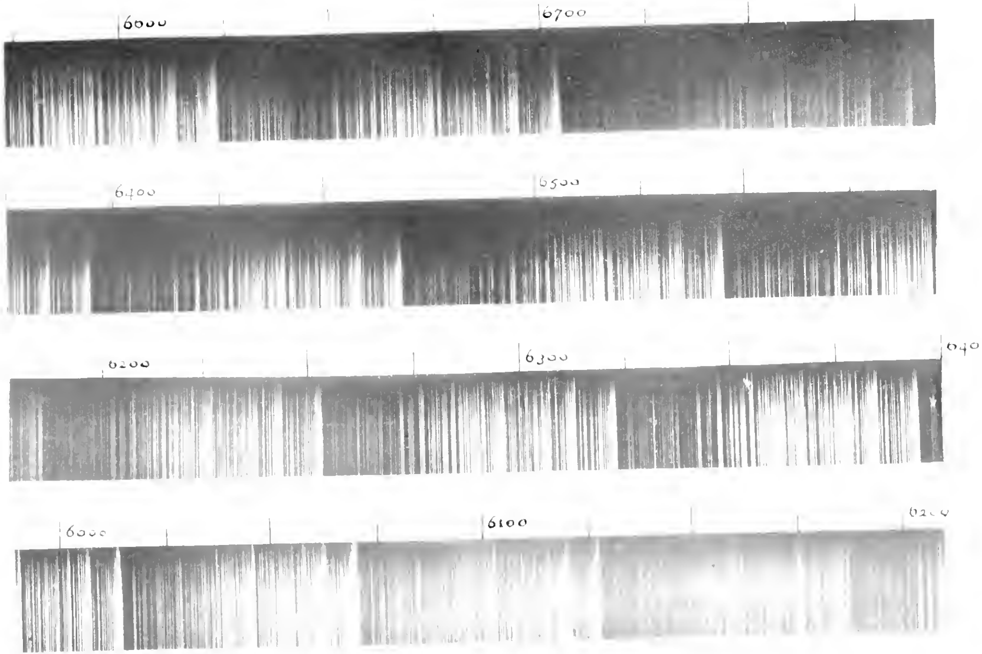
PLATE I



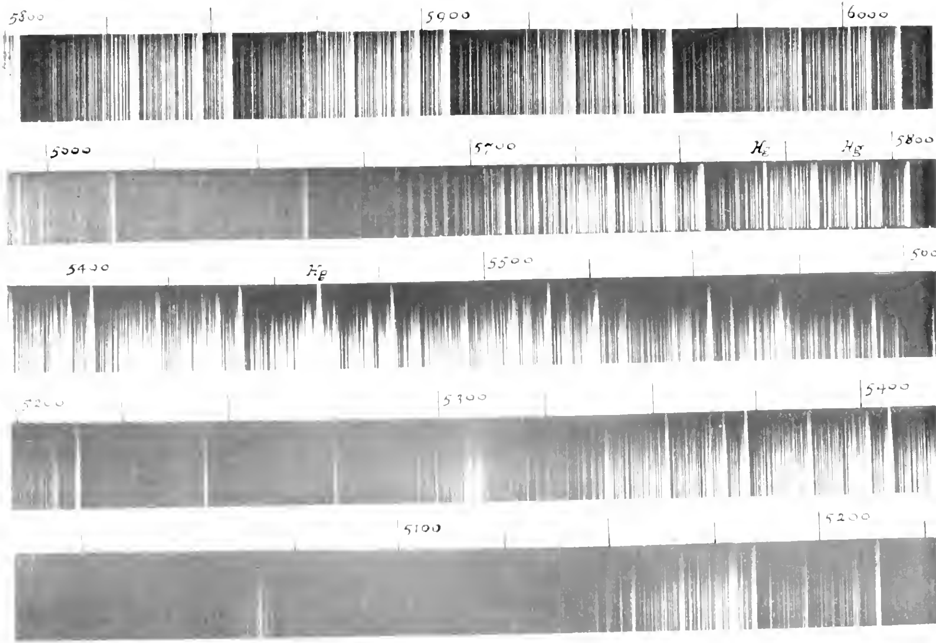
CALCIUM TRIPLET AT λ 3644 RESOLVED ONLY AT LOW PRESSURE

PLATE II



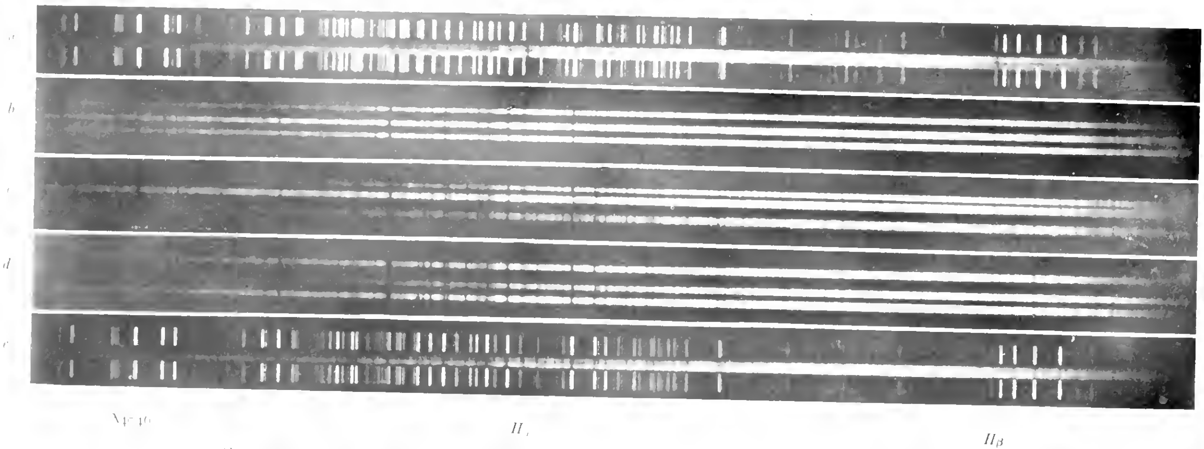
Positive Band Spectrum of Nitroge

PLATE III



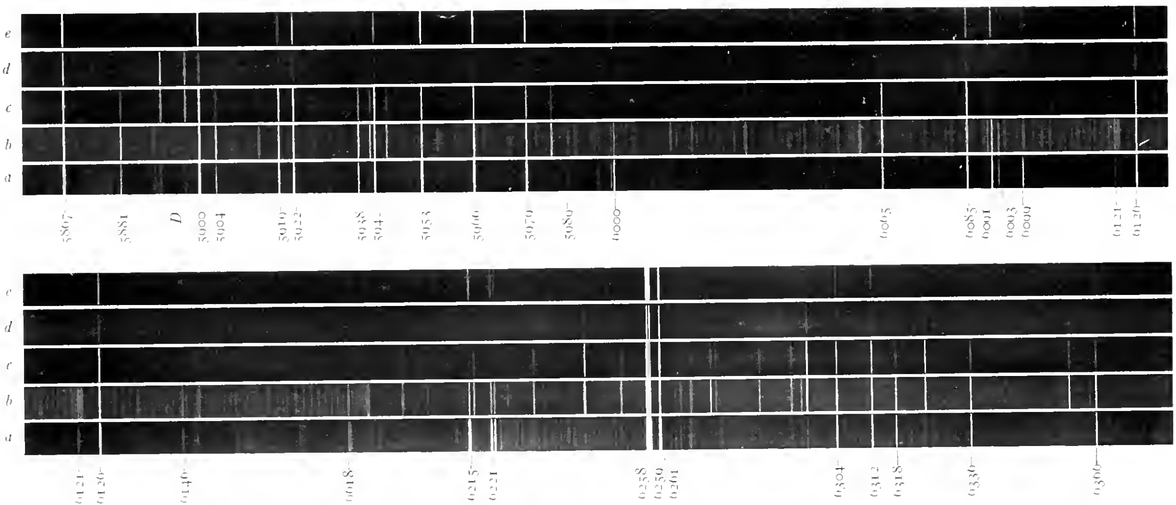
POSITIVE BAND SPECTRUM OF NITROGEN

PLATE IV



Comparison of Intensities in the Violet of Spectra of Stars of Large and Small Proper Motion

PLATE V



FURNACE AND ARC SPECTRA OF TITANIUM

a. Titanium in carbon arc, exposure 4[m].
b. Spectrum of the electric furnace at 2000° C.
c. Spectrum of the electric furnace at 2400° C.
d. Spectrum of the electric furnace at 2150° C
e. Titanium in carbon arc, exposure 1[m].

PLATE VI

Magnesium

4481 4571

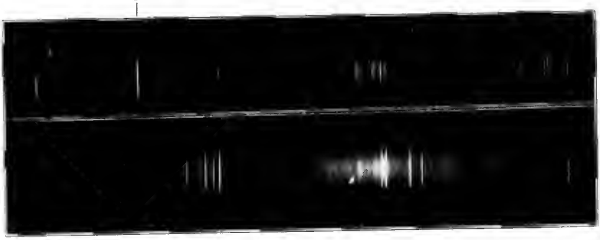
Discharge

Air *in vacu*

Arc in air

6573

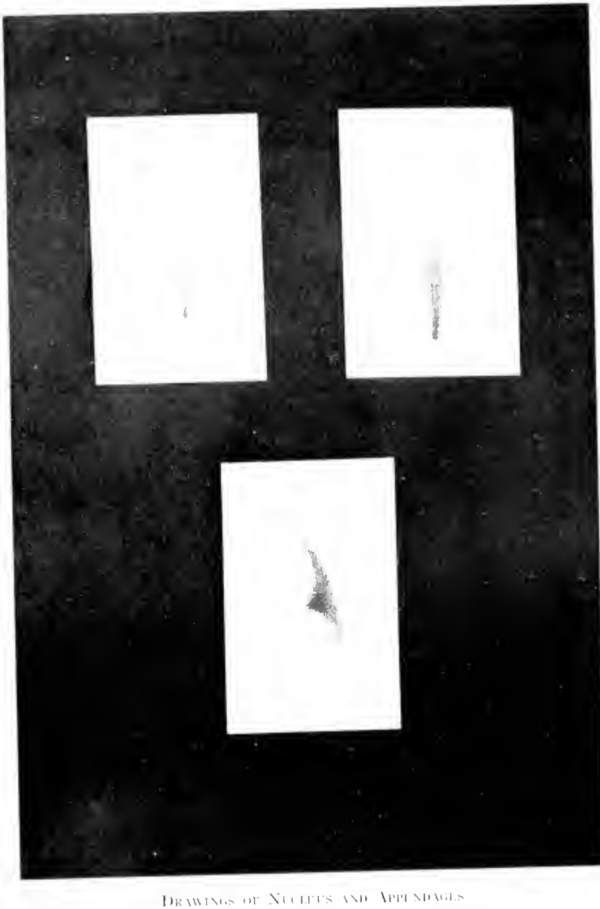
Calcium



Dischar

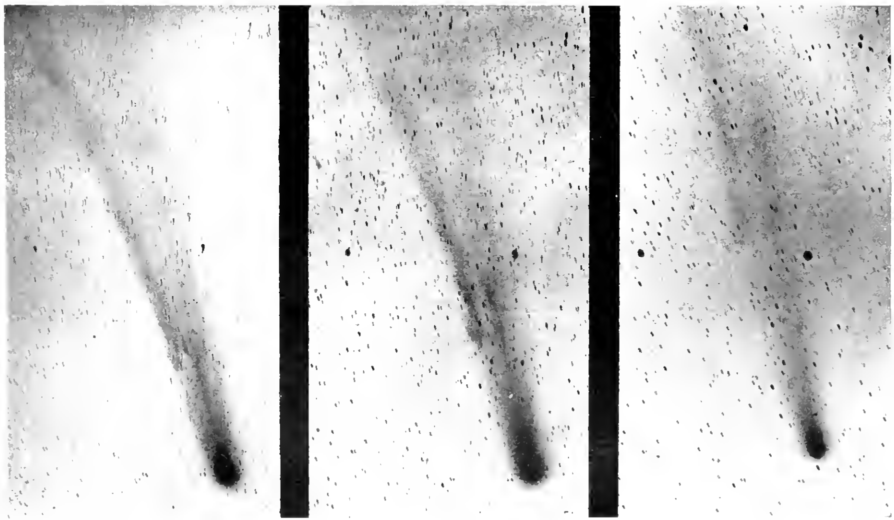
Arc in a

PLATE VII



Drawings of Nucleus and Appendages
Upper figures, May 3; lower figure, May 4

PLATE VIII

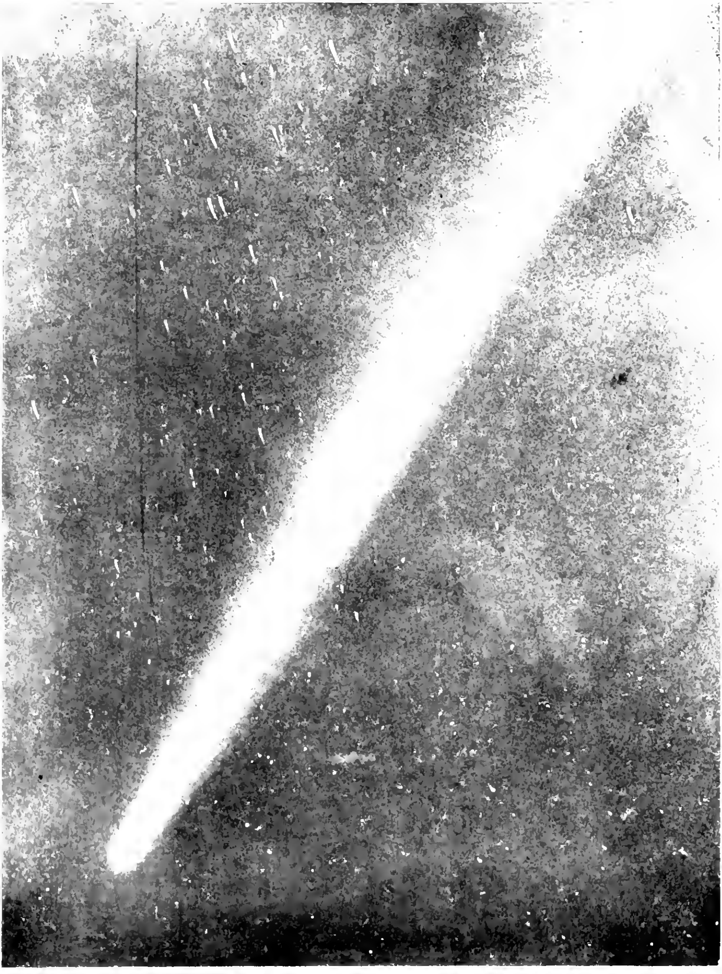


Yerkes: June 6, $15^{h}8$ G.M.T. Honolulu, June 6, $18^{h}5$ G.M.T. Beirut, June 7, $7^{h}0$ G.M.T.

DISCARDED TAIL RECEDING FROM HALLEY'S COMET

PLATE IX

North



[illegible] May 4 [illegible] G.M.T. Exposure 15 [illegible]

Scale [illegible] 1 cm [illegible]

PLATE X

May 17, G.M.T. [illegible]

May [illegible], G.M.T. [illegible]

DRAWINGS OF TAIL OF HALLEY'S COMET

PLATE XI

North

[illegible] Mar. [illegible] G. M. T. Exposure [illegible]

Scale [illegible]

PLATE XII

North

[illegible] June [illegible] G.M.T. [illegible]

Scale [illegible] cm [illegible]